

**University Of Victoria**

**HILLAIRET ANTHONY**

**MEASUREMENT OF THE  $\delta$  MUON DECAY PARAMETER  
AND OF POSSIBLE TENSOR INTERACTIONS.**

**Monday December 4th, 2006  
14:00 Elliot Building, room 160  
Candidacy Exam**

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**Abstract**

TWIST has a very high statistics spectrum of positrons from muon decay from which Michel parameters may be extracted. It is then possible to probe for deviations from the Standard Model and also test models beyond. The TWIST group has already published results with precisions improved by a factor of 3 compared to the previous experiments.

My work will be to achieve the final goal of our group on the muon decay parameter  $\delta$  characterizing the momentum dependence of the muon decay spectrum. This final goal is a precision an order of magnitude better than the results prior to TWIST.

So far only the local effective Fermi interaction has been considered, but if one abandons the locality of the interaction, then the tensor coupling constants must be taken into account. A new description of the Michel spectrum is needed and the parameter  $\kappa$  must be introduced to characterize the tensor interaction. Experiments on the pion decay have seen effects that could be explained by this kind of new interaction.

The TWIST experiment is sensitive to a tensor interaction through modification of the positron spectrum. A dedicated analysis of the spectrum is needed to optimize our sensitivity to  $\kappa$ . With an expanded analysis it will be possible to explore other sources of distortion from physics beyond the SM.



# 1 Introduction

For the full range of known particles, the Standard Model is the theory used in particle physics. This model describing 3 of the 4 fundamental interactions has proven its validity in almost all experiments. Despite its success, the Standard Model cannot answer fundamental questions such as the energy hierarchy of the particles, or the mass of the neutrino. At the same time because it is successful in explaining and predicting many phenomena, the next main theory of particle physics seems to be more likely an extension to the Standard Model, rather than a replacement. Many possible theories have been proposed to extend the Standard Model, and only results from only new experiments will give us new answers. One way is to reach higher energies to study new reactions. The other way is to have a better precision on known reactions to see any deviation from the Standard Model predictions.

The muon decay is a very interesting reaction to probe the Standard Model and test other models at low energy because it involves only the weak interaction. The Michel parametrization gives a description of the muon decay spectrum in angle and energy through 4 parameters,  $\eta$ ,  $\delta$ ,  $P_\mu\xi$  and  $\rho$ . This formalism requires only very general assumptions and is model independent, therefore the TWIST measurement is model independent.

The TWIST experiment is designed to measure this positron decay spectrum with a very high precision. Results for  $\delta$ ,  $P_\mu\xi$  and  $\rho$  have already been published [1][2][3] and further improvements of the precision up to an order of magnitude better than the experiments prior to TWIST is the goal for the final analysis. My work will be to study the systematic uncertainties of  $\delta$  for this analysis.

Recent results of two experiments deviate from the Standard Model predictions. An explanation introducing non-local tensor interaction has been developed. A new parameter called  $\kappa$  is added to the general Michel parametrization to describe this new interaction. TWIST is able to measure  $\kappa$ . I will make and carry out the analysis procedure required to measure this parameter.

Through this report, I will explain the formalism of the standard Michel parametrization and the extension. After an overview of the TWIST experiment, I will present the results I have already produced in the TWIST collaboration. I will finally explain the determination of the systematic errors for  $\delta$  and how we plan to achieve the measurement of  $\kappa$ .

## 2 Formalism of the Michel Parametrisation

### 2.1 Standard formalism

An interesting description of the muon decay is to use the general 4-fermion point interaction. We use at this point very general assumptions and the interaction is described as a free-derivative, Lorentz-invariant and lepton-number conserving. The matrix element is therefore:

$$M = 4 \frac{G_F}{\sqrt{2}} \sum_{\substack{\gamma = S, V, T \\ \epsilon, \mu = R, L}} g_{\epsilon\mu}^\gamma \langle \bar{e}_\epsilon | \Gamma^\gamma | \nu_e \rangle \langle \bar{\nu}_\mu | \Gamma_\gamma | \mu_\mu \rangle \quad (1)$$

The factor  $G_F$  is the Fermi constant. The subscript  $\gamma$  labels the type of the interaction; S for scalar, V for vector and T for tensor and  $\epsilon$  and  $\mu$  describe respectively the chirality of the electron and the muon. For each interaction there is the complex coupling constant  $g_{\epsilon\mu}^\gamma$ .

This description was introduced by L. Michel [4] and the notation used are from [5]. Only 19 real independent coupling constants are needed to completely describe the interaction because we have  $g_{RR}^T \equiv 0$  and  $g_{LL}^T \equiv 0$ , and a common phase doesn't matter.

From this matrix element we can calculate the differential decay rate:

$$\frac{d^2\Gamma}{dx d\cos\theta} = \frac{m_\mu}{4\pi^3} W_{e\mu}^4 G_F^2 \sqrt{x^2 - x_0^2} (F_{IS}(x) + P_\mu \cos\theta F_{AS}(x)) + \text{RC}. \quad (2)$$

using the reduced positron energy  $x = E_e/W_{e\mu}$  with the maximum energy for the positron  $W_{e\mu} \equiv (m_\mu^2 + m_e^2)/2m_\mu$ . The minimum positron energy is  $x_0 \equiv m_e/W_{e\mu}$ . The angle  $\theta$  is the angle between the positron momentum and the muon polarization ( $P_\mu$ ).

The isotropic and anisotropic parts of the spectrum are:

$$F_{IS}(x) = x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x) \quad (3)$$

$$F_{AS}(x) = \frac{1}{3}\xi\sqrt{x^2 - x_0^2} \left[ 1 - x + \frac{2}{3}\delta(4x - 3 + (\sqrt{1 - x_0^2} - 1)) \right]$$

The Standard Model, for which the interaction is pure V-A, predicts the following value for the Michel Parameters:

$$\rho = \frac{3}{4}, \quad \eta = 0, \quad \xi = 1, \quad \delta = \frac{3}{4}$$

### 2.2 Extended formalism for the tensor interaction

The two experiments ISTRA [6] and PIBETA [7] studying the pion decay have observed a deviation from the Standard Model predictions. One explanation for this

deviation is the contribution of a non V-A interaction. A new non-local tensor interaction has been suggested in [8] to explain these deviations. This new interaction contributes to muon decay. Besides the local 4-fermion interaction of equation (1), a new matrix element [9] must be added :

$$M_T = -\sqrt{2}G_F g_{RR}^T \langle \bar{e}_R | \sigma_{\alpha\lambda} | \nu_e \rangle \frac{4q_\alpha q_\beta}{q^2} \langle \bar{\nu}_\mu | \sigma_{\beta\lambda} | \mu_R \rangle \quad (4)$$

This matrix element, unlike the general form in the equation 1, is transfert-momentum-dependent.

This new matrix element modifies the differential decay rate

$$\frac{d^2\Gamma}{dx d\cos\theta} = \frac{m_\mu}{4\pi^3} W_{e\mu}^4 G_F^2 \sqrt{x^2 - x_0^2} (F'_{IS}(x) + P_\mu \cos\theta F'_{AS}(x)) + \text{RC.} \quad (5)$$

The isotropic and anisotropic parts of the equation 3 are modified by the new parameter  $\kappa$ :

$$\begin{aligned} F'_{IS}(x) &= x(1-x) + \frac{2}{9}\rho_\kappa(4x^2 - 3x - x_0^2) + \eta x_0(1-x) + \kappa x_0 \\ F'_{AS}(x) &= \frac{1}{3}\xi_\kappa \sqrt{x^2 - x_0^2} \left[ 1 - x + \frac{2}{3}\delta_\kappa \left( 4x - 3 + \left( \sqrt{1 - x_0^2} - 1 \right) \right) \right] \\ &\quad + \kappa x_0(2-x) \end{aligned} \quad (6)$$

The Michel parameters themselves are affected by  $\kappa$ :

$$\begin{aligned} \rho_\kappa &= \frac{3}{4}(1 - 2\kappa^2) & \xi_\kappa &= 1 + 2\kappa^2 \\ \xi_\kappa \delta_\kappa &= \frac{3}{4}(1 - 4\kappa^2) & \delta_\kappa &= \frac{3}{4}(1 - 6\kappa^2) \end{aligned} \quad (7)$$

assuming the SM model value for  $\rho$ ,  $\delta$  and  $\xi$ .

M. Chizhov has calculated from the results of ISTRA and PIBETA an expected value of  $\kappa \approx 0.013$ . The precision aimed for the last analysis of TWIST is the order of magnitude  $10^{-4}$  therefore we will be able to give a definite answer whether or not the deviation in the pion decays is related to this non-local tensor interaction.

## 3 The TWIST experiment

### 3.1 Hardware

The TWIST spectrometer is installed at TRIUMF on the M13 beamline. This beamline can provide muons as well as pions at various momenta. We have control of the M13 beamline components from the production target to the spectrometer. The components are 7 quadrupoles, 2 dipoles and 5 adjustable collimators. This allows us to select surface muons, produced by pions at rest at the surface of the production target. These muons have a polarization very close to -1.

The detector consists of 56 wire chambers, installed perpendicular to a 2Tesla magnetic field in the cradle (figure 1). The field is produced by a superconducting solenoid inside a steel yoke to insure the uniformity of the field. The tracks of the positron decays are helices contained in the space of the spectrometer.

Out of the 56 planes, 44 are drift chambers (DCs) used to reconstruct the helices. From this reconstruction we extract the momentum of each positron and the  $\cos\theta$  between the positron track and the muon polarization.

The other 22 planes are proportional chambers (PCs) used for particule identification. The event classification in our analysis software uses time windows from the PCs to identify the particles, which may be a beam positron, a pion or a muon.

At the center of the detector is a target foil. The muon momentum is adjusted using a gas degrader upstream of the tracking region so that almost all of the muons stop in the target foil. They decay at rest and the positron produced will travel in the upstream or downstream half of the detector. The energy and angle reconstruction covers a wide range and to avoid any bias, the upstream and downstream part of the detector must be identical.

The purpose of the TWIST spectrometer is to perform a high precision measurement. For this reason a great deal of care went into the detector's construction [10]. The spacing of the chambers, for example, is precisely defined by Sital ceramic spacers with a negligible coefficient of thermal expansion. This makes the z position of the chambers precise to a few microns.

In addition to the spectrometer, a Time Expansion Chamber (TEC) has been designed and installed upstream of the detector to measure the muon beam position and angle. Like the rest of the spectrometer, the TEC design was required to be low mass. The TEC box is installed directly in the vacuum of the beamline and works with DME gas at low pressure. The main sources of energy loss and multiple scattering from the TEC are the aluminum windows separating the TEC gas from the vacuum of the beamline.

### 3.2 Software

The raw data produced by the acquisition system are basically composed of the time of the signals on each wire, for each event. Our reconstruction software called MOFIA uses these times to reconstruct the event.

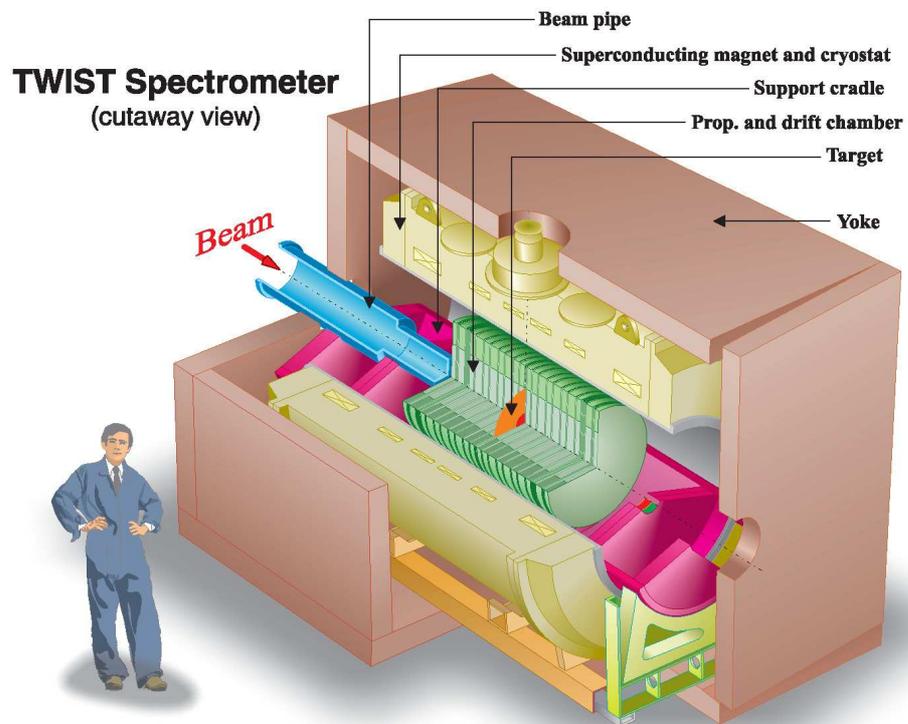


Figure 1: Schematic view of the TWIST spectrometer.

It first identifies the particles using the scintillator and PCs informations by grouping the hits in time. Each time window corresponds to different particles. Then the hits are used to reconstruct the helices for the decay positrons. First a wire center fit is performed and then the drift times from the DCs give much more precise hit position and therefore much better tracks. The fitter, based on a  $\chi^2$  minimization, takes into account the energy loss and also the multiple scattering affecting the tracks. The first cuts are applied at this stage such as a time of flight cut to select the surface muon or the selection of events containing only one muon.

MOFIA stores each event in ROOT trees [11]. These trees are summed and for each data set, an energy calibration is performed using the positron kinematic endpoint. This calibration corrects effects such as the target energy loss.

Once the energy calibration is done, the experimental Michel spectrum is ready.

TWIST determines the Michel parameters by evaluating the difference between the two-dimensional histograms of reconstructed experimental decay positron momenta and angles, with histograms of reconstructed Monte Carlo (MC) data. This way the response function of the detector is taken into account by the simulation. The difference between the two spectra is determined by fitting the MC spectrum plus the variable contribution of the derivatives for each Michel parameter to the experimental spectrum.

The decay rate for the MC spectrum plus derivatives is written as:

$$\left. \frac{d^2\Gamma}{dx d(\cos\theta)} \right|_{\rho_{MC}, \delta_{MC}, \xi_{MC}} + \sum_{\alpha=\rho, P_\mu\xi, P_\mu\xi\delta} \frac{\partial}{\partial\alpha} \left[ \frac{d^2\Gamma}{dx d(\cos\theta)} \right] \Delta\alpha \quad (8)$$

with  $\Delta\alpha$  the contribution of the parameter to the difference between the experimental and MC spectra. We actually use the derivative of  $P_\mu\xi\delta$  instead of  $\delta$  alone to keep the derivatives linear, then the derivatives are independent of the parameters value.

The constants  $\rho_{MC}$ ,  $\delta_{MC}$  and  $\xi_{MC}$  are the values used to generate the MC spectrum.

Our simulation of the spectrometer is based on GEANT3 [12]. The MC data undergo like the real data, the analysis chain. Therefore the possible biases introduced during the analysis will be largely reduced because they will have an effect only on the difference between the two spectra. Then the systematic uncertainties come from the imperfections of the MC simulation to reproduce the reality. This is why a great deal of care goes into the MC simulation.

## 4 Work of the past year

My initial work for the TWIST group was to determine the alignment of various parts of the spectrometer. To do this I gained expertise in the straight line fitter part of our analysis MOFIA.

### 4.1 DC/TEC alignment

The TEC produces a characterization of the muon beam that is used in our MC simulation which is an essential part of our final analysis. In the MC, the TEC is perfectly aligned on the "world" axis of the MC simulation which is defined as being the axis of the yoke.

The physical alignment of the TEC is performed at the beginning of the run period when the cradle containing the DCs is out of the yoke (figure 2). The axis of the detector is at that time the yoke. Crosshairs are installed at the yoke entrances and the TEC entrances. A theodolite optical axis is aligned on the crosshairs of the yoke and then the TEC is moved to align its crosshairs on this axis. This measurement has a precision of  $50\mu\text{m}$  for the position and 2 mrad for the angle.

During the final installation of the TEC, the vacuum is made in the beamline in which the TEC is installed. The forces from the differential pressure are sufficient to modify the beamline geometry and the TEC could then be misaligned. That is why we need to check the alignment of the TEC once it is fully installed and operational. Of course at that stage, the theodolite cannot be used.

One possibility is a relative alignment of the TEC with respect to the DCs. The principle is to reconstruct straight tracks in the TEC and in the DCs, and then compare their angles and positions. The difference between the two tracks is put in a histogram which is used to extract a mean value.

To have straight tracks, this measurement is performed without any magnetic field. The particles used are pions at a momentum of  $55\text{MeV}/c$ . This choice is a compromise between multiple scattering and the TEC efficiency.

This alignment technique has been tested on the 2005 data. Figure 3 shows the data measurement for each run, and the final mean value extracted.

The misalignment measured is within the precision of the alignment technique with the theodolite. This result gives us confidence in the TEC alignment and in the TEC position stability because this data have been taken at the end of a run period.

### 4.2 Beam positron characterization

For the measurement of the parameters  $\rho$  and  $\delta$  in the ongoing 2004 data analysis, Rob MacDonald needed a better simulation of the beam positrons which contaminate our muon beam. The TEC is by design incapable of measuring the beam positrons. We had to find another way to characterize the positron contribution.

We decided to characterize the beam positron using field off data, for which the drift chambers can be used to reconstruct the positron straight tracks. The beam characterization with the field off is used later in the MC simulation. We start the

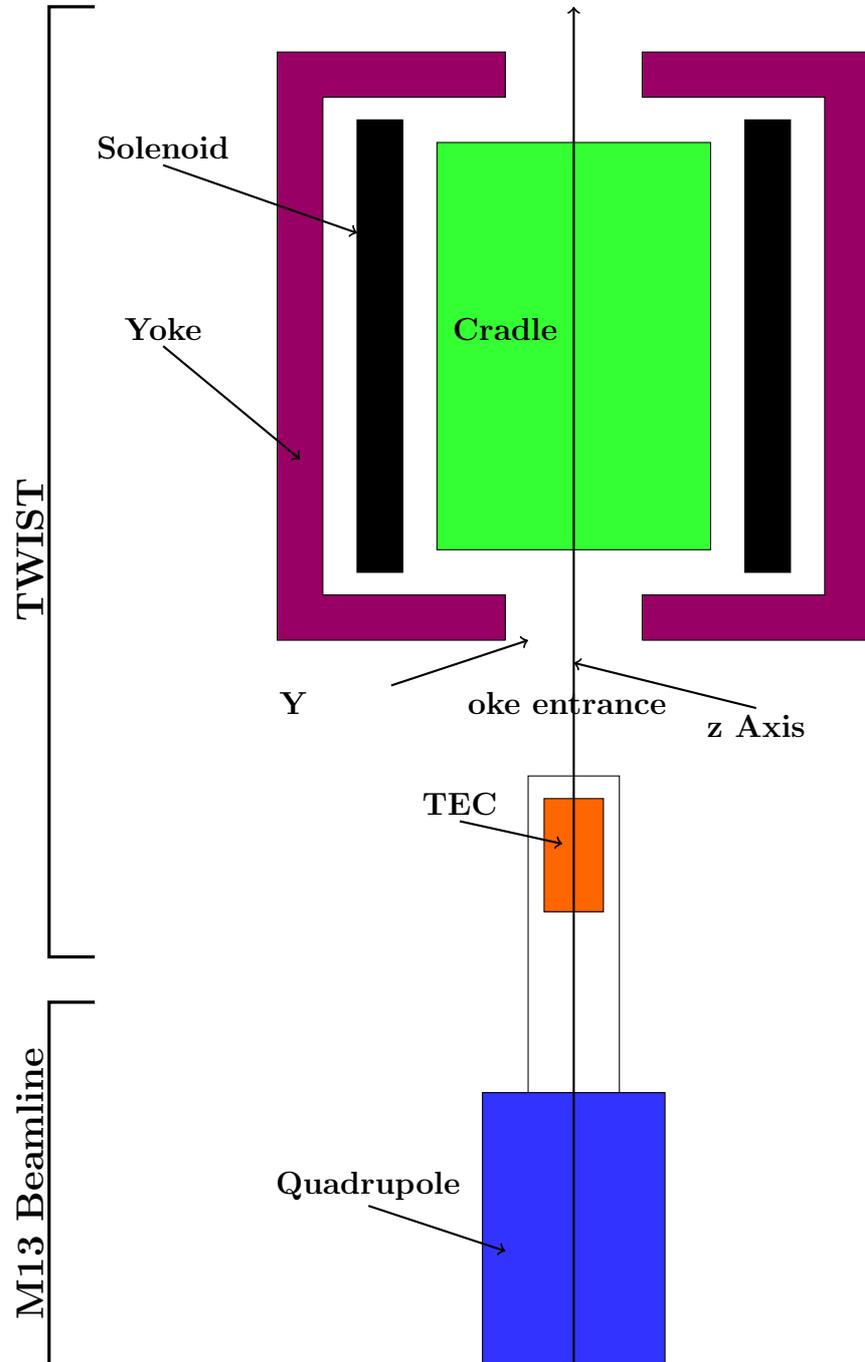


Figure 2: This diagram shows the position of various components of the experiment. Only the end of the M13 beamline is represented with the last quadrupole. The TEC is inside the beam pipe.

simulated beam positrons far away from the magnetic field map, and the simulation deduces the effect of the magnetic field on the particles.

To validate the results, the latter is simply used in a simulation reproducing the beam positron characterization and the same analysis is performed. Therefore the output characterisation should be the same as the input characterization. This validation proved the consistency of the procedure.

We can see on the figure 4 that the two characterizations are similar. They are not identical but we decided that it was good enough for the 2004 analysis. The previous Michel parameters analysis used a simple beam positron with gaussian distributions for the start position and angle without any correlation between the two. This new characterization technique extracts this correlation and pass it to the MC which is a great improvement.

### 4.3 DC/Yoke alignment

One more time to improve the MC simulation, we developped a procedure to measure the DC/Yoke relative alignment. The principle is to install at each end of the yoke a collimator (figure 2). Those collimators are two thick aluminum disks with fourteen holes forming a cross (Figure 5). We send 120MeV/c pions in the detector. These pions go through the holes and therefore their position with respect to the yoke is known. Then we use the reconstruction of the straight tracks to know where are the tracks with respect to the DCs. Therefore we can measure the relative angle and positions alignment of the yoke and the DCs.

The measurement is a simple gaussian fit of the distribution of the center hole. The other holes are not reliable because of the divergence of the incoming pion beam. The results for the misalignment of the DCs with respect to the yoke are:

- Position at the center of the detector
  - In X :  $(-0.02 \pm 0.04)$  cm
  - In Y :  $(0.04 \pm 0.04)$  cm
- Angle
  - In X :  $(1.1 \pm 0.1)$  mrad
  - In Y :  $(0.5 \pm 0.1)$  mrad

We can use the results from this alignment to make a consistency check of the alignment of the DCs, the yoke and the B field. A technique using the helices of the decay positron track in the magnetic field gives us a correction of the field misalignment with respect to the DCs. The angle between the DC and the magnetic field is zero once this correction is applied within the 0.02 mrad precision of this alignment procedure.

A previous study of the magnetic field map gives a precision of the alignment of the yoke and the B field of 2 mm and a precision of 2 mrad for the angle alignment between the yoke and the field. We see that there is no inconsistency between the three measurements, since the 1.1 mrad DC/Yoke misalignment is within the 2 mrad precision of the alignment Yoke/Bfield.

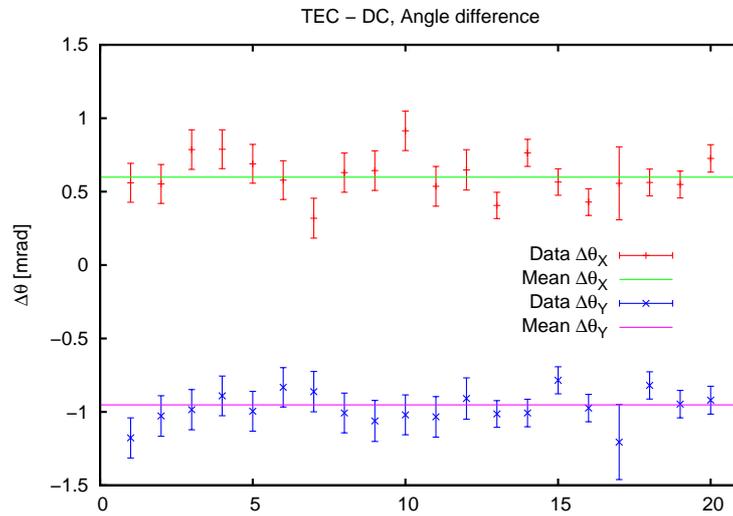


Figure 3: The DC/TEC alignment measurement shows an angle misalignment of 1 mrad in X and -1 mrad in Y which is within the precision of the alignment at the installation of the TEC.

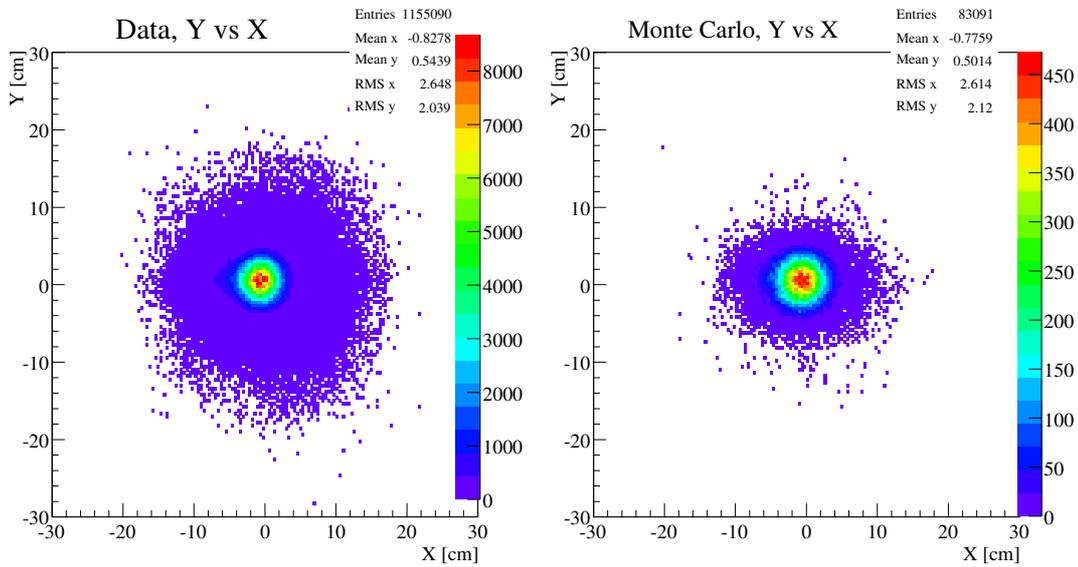


Figure 4: A characterization is extracted from the data (on the left) and used as an input to a MC simulation. The track are reconstructed exactly the same way in the MC (on the right) as they are in the data. The two characterizations are similar.

## 4.4 DC planes alignment

The knowledge of the plane position is very important as it has a direct impact on the reconstruction of the decay positron track and then on the Michel spectrum. The basic principle of the alignment is to produce straight tracks in the detector using 120MeV/c pions without any magnetic field. The tracks are fitted and the residual distance between the fitted track and the hit position is computed. The goal is to reduce the residuals for each plane by adjusting the plane position and angle in an iterative process.

The previous alignment procedures used the Kalman Filter to fit the straight tracks. This procedure was efficient but only aligned the upstream and downstream half of the detector separately. The procedure was not converging when applied to the whole detector because most of the track, even at 120MeV/c, undergo multiple scattering in the detector, especially at the target. The solution would have been to add the possibility of kinks in the straight line fit; however this never worked with the Kalman Filter.

That is why we decided to use the straight line fitter based on a  $\chi^2$  minimization of MOFIA, which has the kinks working, for the alignment procedure.

So far this improved procedure is working for the translational alignment of the plane in the directions perpendicular to the axis of the detector (figure 6). An MC study shows that the procedure has converged to an alignment close to the input offsets by  $15\mu m$  which is the defined as the minimum precision the alignment must have. The procedure will be improved for the final analysis because the convergence has not yet stopped.

This new procedure has also to be tested for the rotation of the planes around z axis too.

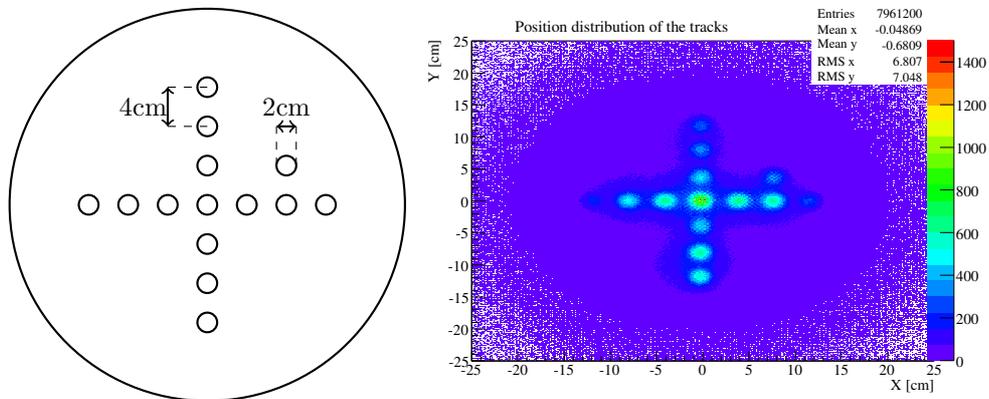


Figure 5: The DC/Yoke alignment is performed using collimators (shown schematically on the left) at each end of the yoke. The reconstructed tracks are used to measure the shifts upstream and downstream and also the angle with respect to the yoke (Track position distribution on the right).

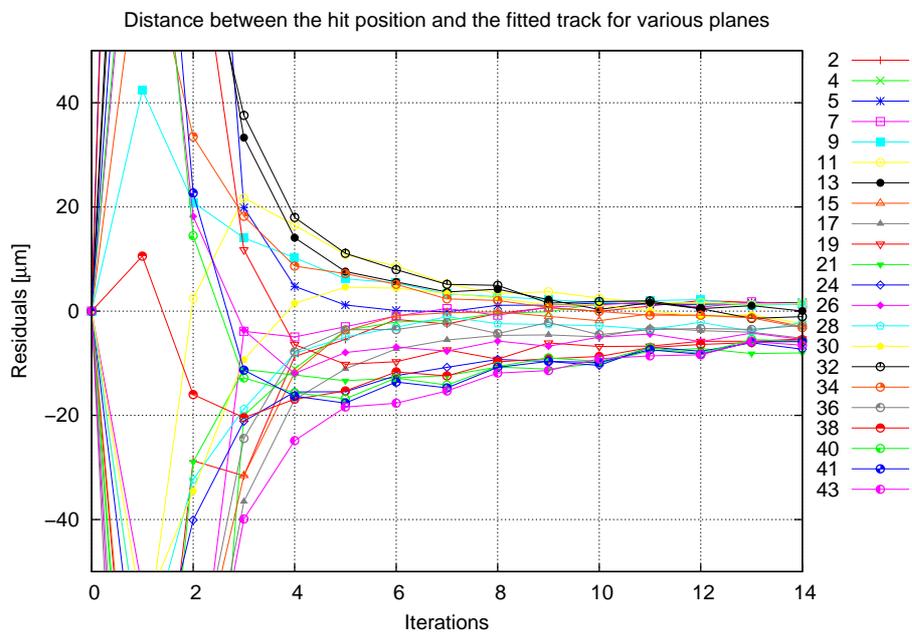


Figure 6: The residual distance is decreasing as the iterations are carried out therefore the DC alignment procedure is converging.

## 5 Analysis plan

### 5.1 Systematics study for $\delta$

Most of the work required for the next measurement of  $\delta$  will be on the determination of the systematic errors. As we have already seen in the section 3.2, the fitting procedure to determine the Michel parameters makes them virtually independent of the track reconstruction software because both experimental and MC data are analyzed with the same code. We must then determine how well the MC simulation reproduces the reality and then how the differences affect the Michel parameters precision.

Most uncertainties are determined by evaluating the sensitivity  $R = d\delta/da$  of  $\delta$  to a given systematic parameter  $a$ . The systematic uncertainty of  $\delta$  on this parameter  $a$  known to a precision of  $\pm\sigma_a$ , is then defined as being  $R\sigma_a$ .

The sensitivity is determined by fitting the difference between two MC spectra. One is generated with the the best estimation of the parameter  $a$  that we have, and the other spectrum is generated with a value  $a + S\sigma_a$ . The difference measured between the two spectra is divided by the scaling factor  $S$  to give  $R\sigma_a$ .

The systematic uncertainties for  $\delta$  are categorized as the positron interactions, the spectrometer alignment, the chamber response, the momentum calibration and the muon beam stability. More work is expected for the chamber response uncertainties.

### 5.2 Extended analysis for $\kappa$

A major difference in the  $\kappa$  analysis is the contribution of  $\kappa^2$  in the differential decay rate including the new non-local tensor interaction (equation 5). Indeed we have  $x_0 \approx 0.0097$  and an expected value of  $\kappa \approx 0.013$  so the contributions from  $\kappa x_0$  and  $\kappa^2$  are the same order of magnitude, and then  $\kappa^2$  cannot be neglected.

Therefore the derivatives for  $\kappa$  depend on  $\kappa$  and the fit is no longer linear. At the same time we notice that the effect of  $\kappa^2$  is to alter the values of the Michel parameters.

We plan to abandon the model independent analysis for  $\kappa$ . The idea is to assume that the muon decay is composed of the interactions from the Standard Model plus the non-local interaction and nothing else. So we will set  $\eta$ ,  $\rho$ ,  $\delta$  and  $P_\mu\xi$  to their values predicted by the Standard Model and fit  $\kappa$ .

The fit for  $\kappa$  cannot be done directly because of the non-linearity of the differential decay rate. First we must fit without the contribution of  $\kappa^2$  in the fit function or in the derivatives. We obtain a first value  $\kappa_1$  that we use to prepare the derivatives for  $\kappa^2$  and perform a new fit but this time with the complete parametrization. Then we start an iterative process where the  $n$ th fit of  $\kappa$  is performed using the derivatives calculated with the result  $\kappa_{n-1}$  of the previous fit. The convergence will be defined by the statistic uncertainty.

The determination of the systematic uncertainties for  $\kappa$  will depend on the iterative process. To get the sensitivity, we need to perform the fitting procedure and if it is taking too much computation resources or time, we might have to find another way to determine the systematic errors for  $\kappa$ .

## 6 Conclusion

The run period for 2006 will end at the end of December. We might run for a few weeks in 2007 if some data are still needed. We will start the first full analysis of datasets as soon as possible next year. The alignment of the DCs is required before starting this analysis.

I have now enough experience with the detector and the acquisition system and also with the tool used in the TWIST group to be able to perform a full analysis of a Michel spectrum.

New challenges will be faced when we will improve the precision on the parameter  $\delta$ . At the same time we will make a new analysis dedicated to the particularities of  $\kappa$ . A lot of work will be required to setup and also validate this new analysis.

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