Measuring a Second Class Current In τ Decays

by

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<u>Abstract</u>

This paper presents the prospect of measuring a second class current in τ decays. A study of the $\tau^- \to \eta'(958)\pi^-\nu_{\tau}$ decay, which proceeds through a second class current, will be proposed. Such decays have never been observed due to isospin symmetry. Isospin symmetry is not exact and it is expected that decays such as $\tau^- \to \eta'(958)\pi^-\nu_{\tau}$ should be observed at a small rate. The paper will begin with a review of hadronic τ decays and lead into a discussion of isospin symmetry and the mechanism for the production of a second class process. The measurement will be made using the full BaBar dataset which now has enough statistics to test current theory.

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1 Introduction

The Standard Model (SM) is a description of the fundamental constituents of matter and the forces that govern them. The particles that populate the SM are shown in Tables 1, 2 and 3. Ongoing testing of the model is done through (predominantly) high energy particle physics experiments. Subatomic particles are accelerated to nearly the speed of light, and are collided inside a large detector. A detector in high energy particle physics is a multi system apparatus designed to measure the identity, momentum and energy of charged and neutral particles.

With this information we can measure, among other things, decay probabilities of a particle into a particular final state. A final state can be composed of two or more particles which have a combined energy equal to the mass of the parent particle. Rare or unobserved decays are sometimes suppressed due to conservation laws based on imperfect symmetries such as isospin, which is exact for the strong interaction but not for the electromagnetic or weak interactions. This report will describe the plan for a search for the decay of a τ^- lepton to a $\eta'(958)\pi^-\nu_{\tau}$ final state which is suppressed according to isospin and is called a decay that proceeds through a second class current.

2 Project Overview

The rate of first and second class decays are proportional to the mass sum and difference of constituent particles (quarks) that combine to form hadrons (i.e. pions, protons, neutrons, etc.), respectively. Figure 1 shows diagrams



Figure 1: The top figure shows a Feynman diagram for the $\tau^- \to \pi^- \pi^0 \nu_\tau$ decay and the lower Feynman diagram shows the second class $\tau^- \to \eta \pi^- \nu_\tau$ decay

for two decays. The top diagram illustrates a $\tau^- \to \pi^- \pi^0 \nu_{\tau}$ decay and the lower diagram shows a $\tau^- \to \pi^- \eta \nu_{\tau}$ decay. The former is a first class current decay and is the dominant decay mode of the τ with a measured branching fraction of $(25.50 \pm .10) \times 10^{-2}$ [1]. The latter is a second class current decay and has never been observed. The decay has a 95% confidence level upper limit on the branching fraction of (1.4×10^{-4}) [1]. Although some extra suppression is expected due to the fact that the η is heavier than the π^0 , the large difference in the branching fractions is primarily due to the fact that the $\tau^- \to \pi^- \eta \nu_{\tau}$ decay proceeds through a second class current.

By using the high statistics data sample of the BaBar detector at the Stanford Linear Accelerator Center (SLAC) it is possible to probe the current theoretical predictions of the second class channels. The study of second class currents will provide insight on the fundamental understanding of weak decays in particle physics. It will also improve our understanding of hadronic τ decays, as well as testing the conserved vector current hypothesis (CVC) which is a topical issue in the g-2 anomaly. A brief discussion of CVC can be found in the Appendix.

Lepton	Charge	Mass	Principal decays
е	-1	$(0.51099892 \pm 0.00000024) \times 10^{-6} \text{ MeV}$	-
$ u_e $	0	< 3 eV	-
μ	-1	$(105.658369 \pm 0.000008)$ MeV	$e u_{\mu}ar{ u_e}$
$ u_{\mu}$	0	$< 0.19 { m ~MeV}$	-
au	-1	$1776.99^{+0.29}_{-0.26} \text{ MeV}$	$\mu \nu_{\tau} \bar{\nu_{\mu}}, e \nu_{\tau} \bar{\nu_{e}}, \rho \nu_{\tau}$
$\nu_{ au}$	0	$< 18.2 { m MeV}$	-

Table 1: Leptons (spin 1/2).

Flavor	Charge	Mass
d	-1/3	(4-8) MeV
u	+2/3	$(1.5-4) { m MeV}$
s	-1/3	(80-130) MeV
с	+2/3	$(1.15-1.35) \mathrm{GeV}$
b	-1/3	(4.1-4.4) GeV
t	+2/3	$(174.3\pm 5.1) \mathrm{GeV}$

Table 2: Quarks (spin 1/2).

Mediator	Charge	Mass(GeV)	Lifetime	Force
gluon	0	0	∞	strong
$photon(\gamma)$	0	0	∞	electromagnetic
W^{\pm}	± 1	80.425 ± 0.038	$(3.099 \pm 0.060) \times 10^{-25} \text{ s}$	(charged) weak
\mathbf{Z}^{0}	0	91.1876 ± 0.0021	$(2.6349 \pm 0.0024) \times 10^{-25} \text{ s}$	(neutral) weak

Table 3: Bosons (spin 1).

2.1 Decays of Interest

A τ can decay to a number of final states with η and $\eta'(958)$ mesons (see Table 4). The η and $\eta'(958)$ can then decay to the modes shown in Table 5. The two simplest Second Class Current τ decays involve the $\eta(549)$ and $\eta'(958)$ mesons

$$\tau^- \to a_0^- (980) \nu_\tau \to \eta \pi^- \nu_\tau$$
$$\tau^- \to a_0^- (1450) \nu_\tau \to \eta' (958) \pi^- \nu_\tau$$

where each decay proceeds through an a_0 resonance. This work focuses on the $\tau^- \to \eta'(958)\pi^-\nu_{\tau}$ decay while the $\tau^- \to \pi^-\eta\nu_{\tau}$ mode is being worked on by other BaBar members. The decay rate is proportional to the amount of available phase space. For the listed decays the phase space factor is

$$\left(1 - \frac{Q^2}{m_\tau^2}\right)^2$$

where Q is the mass of the intermediate resonance and m_{τ} is the τ mass. The rate is four times less for the $\tau^- \to \eta'(958)\pi^-\nu_{\tau}$ mode relative to the $\tau^- \to \pi^-\eta\nu_{\tau}$ mode; however, the latter decay will suffer higher backgrounds from decays such as $\tau^- \to \pi^-\pi^0\nu_{\tau}$ (see the top Feynman diagram in Figure 1). As shown in Table 4 the current limit on $\tau^- \to \eta'(958)\pi^-\nu_{\tau}$ is significantly better than $\tau^- \to \pi^-\eta\nu_{\tau}$.

Mode	Branching Fraction
$\tau^- \to \eta \pi^- \pi^0 \nu_{\tau}$	$1.77 \pm 0.24 \times 10^{-3}$
$\tau^- \to \eta \pi^- \pi^0 \pi^0 \nu_\tau$	$1.5\pm0.5\times10^{-4}$
$\tau^- \to \eta K^- \nu_{\tau}$	$2.7\pm0.6\times10^{-4}$
$\tau^- \to \eta K^* (892)^- \nu_\tau$	$2.9\pm0.9\times10^{-4}$
$\tau^- \to \eta K^- \pi^0 \nu_\tau$	$1.8\pm0.9\times10^{-4}$
$\tau^- \to \eta \bar{K}^0 \pi^- \nu_{\tau}$	$2.2\pm0.7\times10^{-4}$
$\tau^- \to \eta \pi^- \pi^+ \pi^- \nu_\tau$	$2.3\pm0.5\times10^{-4}$
$\tau^- \to \eta'(958)\pi^-\pi^0\nu_\tau$	$< 8.0 \times 10^{-5} \ 90\% \ {\rm CL}$
$\tau^- \to \eta \pi^- \nu_{\tau}$	$< 1.4 \times 10^{-4} 95\%$ CL
$\tau^- \to \eta'(958)\pi^-\nu_\tau$	$< 7.4 \times 10^{-5} \ 90\% \ {\rm CL}$

Table 4: Measured τ lepton decays involving an η or $\eta'(958)$ meson. The first set of decays proceed through a first class current, and the second set by second class current. [1]

2.2 τ Physics at BaBar

The proposed work will use data collected by the BaBar detector at the SLAC, which is an e^-e^+ (electron/positron) collider. The e^-e^+ collisions produce $\tau^+\tau^-$ pairs with a cross section of $\sigma_{\tau\tau} = (0.919 \pm 0.005)$ nb. The

Mode	Branching Fraction $(\%)$
$\eta \to \gamma \gamma$	39.39 ± 0.24
$\eta ightarrow \pi^0 \pi^0 \pi^0$	32.52 ± 0.26
$\eta \to \pi^+ \pi^- \pi^0$	22.68 ± 0.35
$\eta \to \pi^+ \pi^- \gamma$	4.69 ± 0.10
$\eta'(958) \to \pi^+\pi^-\eta$	44.5 ± 1.4
$\eta'(958) \to \rho^0 \gamma$	29.4 ± 0.9
$\eta'(958) \to \pi^0 \pi^0 \gamma$	20.8 ± 1.2
$\eta'(958) \to \omega \gamma$	3.03 ± 0.31
$\eta'(958) \to \gamma\gamma$	2.12 ± 0.14

Table 5: Shown are the main decay modes of the η and $\eta'(958)$ mesons.



Figure 2: Pictured is a general Feynman diagram for a hadronic τ decay. The τ lepton decays producing a ν_{τ} and some mesonic state X.

 τ leptons decay to other particles, which are recorded using the BaBar detector. The final sample has over 450 million τ pairs for study. This large sample of τ leptons will facilitate the study of rare decay processes, such as the second class currents. Section 6 outlines the expected sensitivity for the measurement.

3 Hadronic τ Decays

The τ lepton is a robust particle to study, as its mass and charge allow it to decay into low lying hadrons (mesons). A hadronic τ decay occurs via the weak process by coupling to the W[±] boson. The W[±] subsequently decays by coupling to a mesonic state "X[±]" which is composed of a $d'\bar{u}$ pair, where d' can be either a d or s quark. The amount of each combination produced is proportional to the cosine and sine of the Cabbibo angle, respectively. The pair of quarks hadronize into mesons, where the types of mesons produced are limited to the τ mass. A generic τ decay is shown in Figure 2. The allowed τ decays can be classified by their symmetry properties. Of particular interest in this study are decays which are suppressed by G-parity conservation, known as second class currents. G-parity is an operation which involves isospin rotation and charge conjugation.

3.1 Isospin, Charge and G-Parity

Isospin is a property of hadrons that contain u and d quarks. The u and d quarks can be symbolized with vectors in "isospin space"

$$u = \begin{pmatrix} 1 \\ 0 \end{pmatrix} d = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

or using the more formal Dirac bra-ket notation

$$u = |1/2, +1/2\rangle$$

$$d = |1/2, -1/2\rangle$$

where the first number in the ket is the isospin value of the doublet and the second is the third component of the isospin value for the u and d quarks respectively. With the above notation the 3^{rd} component of isospin for the u quark is +1/2 and -1/2 for the d quark. Applying an operation which rotates the vectors by 180° in isospin space, transforms the up quarks into down quarks and vice versa.

Charge conjugation is an operation that takes a particle and transforms it into its anti-particle.

$$C|\pi^+\rangle = |\pi^-\rangle$$

The above example shows the C operator acting on a positive pion state and transforming it into its negatively charged counterpart. All of the particles quantum numbers (charge, lepton number, strangeness, etc.) are reversed, while leaving the physical observable quantities (energy, mass, momentum, spin) unaltered.

G-parity is a combination of charge conjugation and isospin rotation about the number 2-axis 1 . The G-Parity operator is written as

$$G = CR_2$$
$$R_2 = e^{i\pi I_2}$$

This means that applying the G-Parity operator to a wave function rotates the wave function by 180° in isospin space and then applies a charge conju-

¹Where the axis is defined in Cartesian coordinates.

gation. Using the G-Parity operator is convenient because both charged and neutral states are applicable. The G-Parity of a particle can be determined with the following formula

$$G(X) = (-1)^I \mathcal{C}$$

where X is the particle in question, I is the isospin value for its multiplet, and C is the charge conjugation value of the neutral member of the multiplet. Referring back to Figure 1 we can now assign the G-parity for the two decays. For the $\tau^- \to \pi^- \pi^0 \nu_{\tau}$ decay

$$G = G(\pi^0)G(\pi^{\pm}) = (-1)(-1) = 1$$

and for the $\tau^- \to \pi^- \eta \nu_\tau$ decay

$$G = G(\eta)G(\pi^{\pm}) = (1)(-1) = -1,$$

where $C(\pi^0) = C(\pi^{\pm}) = C(\eta) = 1$. The difference in G-Parity values arises because the neutral pion belongs to a triplet of particles with isospin value I = 1 and the η is a singlet with isospin I = 0. The different G-Parity for the decays separate the decays into first class and second class.

3.2 Second Class Currents

The final states in τ decays can be ordered by the G-Parity of the "X" state in the $\tau^- \to X^- \nu_{\tau}$ (see Figure 2) decay, where X⁻ is an allowed mesonic state. It is experimentally observed that only one value of G-Parity is allowed for decays with a particular J^P value, where P is the parity and J is the spin. Final states can be classified according to J^{PG} . In τ decays, the accessible first class currents are $(J^{PG} = 0^{--}, 1^{+-}, 1^{-+})$ and the suppressed second class currents are $(J^{PG} = 0^{+-}, 1^{++})$.

The spin zero state can be divided into pseudoscalar and scalar final states according to

$$\frac{m_d + m_u}{\sqrt{2}m_{had}} |0^-\rangle + \frac{m_d - m_u}{\sqrt{2}m_{had}} |0^+\rangle.$$

The amount of the $|0^+\rangle$ final state produced (where $|0^+\rangle$ is a more formal way of writing $J^P = 0^+$), which is the state relevant to the $\tau^- \rightarrow \eta'(958)\pi^-\nu_{\tau}$ decay, is proportional to the difference in the masses of the u and d quarks. This representation assumes free quarks, which is an oversimplification at energies where quarks are strongly bound into mesons. For a more complete description one must consider a hadronic coupling to the W[±] boson.

The hadronic current couples to the W^{\pm} boson and the decay products through either a vector current or an axial-vector current.

$$V_{\mu} = \langle \Psi_{vac} | F_1(Q^2) \gamma_{\mu} + \frac{iF_2(Q^2)}{2|Q|} \sigma_{\mu\nu} q^{\nu} | \Psi_{had} \rangle$$
$$A_{\mu} = \langle \Psi_{vac} | G_1(Q^2) \gamma_{\mu} \gamma_5 + \frac{iG_2(Q^2)}{2|Q|} \sigma_{\mu\nu} \gamma_5 q^{\nu} | \Psi_{had} \rangle$$

In the above equations V_{μ} is the vector current, A_{μ} is the axial-vector current, Ψ represent the wave functions, F and G are the form factors, and $Q^2 \equiv m_{had}^2 \equiv s$. By inserting the CVC hypothesis, which states that $\partial^{\mu}V_{\mu} = 0$, into the vector current the equation collapses to

$$m_{had}F_1(Q^2)\langle\Psi_{vac}|\Psi_{had}\rangle = 0.$$

Since the vacuum state is defined as spin zero and scalar, the CVC hypothesis will only hold if there are no scalar currents, or equivalently the $|0^+\rangle$ state is not observed.

4 Existing Experimental Measurements

There have been experimental searches for second class currents in β decay studies, and τ decays. Although there have been claims of observation of these modes, none have been substantiated.

4.1 β Decays

A study measuring the β -ray emissions from polarized ¹²B and ¹²N nuclei has yielded support for the existence of second class currents [3]. In the study samples of ¹²B and ¹²N nuclei were generated and polarized. The decays studied were

$${}^{12}\mathrm{B} \rightarrow {}^{12}\mathrm{C} + e^- + \nu$$
$${}^{12}\mathrm{N} \rightarrow {}^{12}\mathrm{C} + e^+ + \nu.$$

The abundance of these decays proceed through a first class current, but it is proposed that the decays may have a second class current component. Measurements were made of the e^+ and e^- (β -rays) energies and the decay angles relative to the polarization direction of the nuclei. The experiment measured a decay asymmetry defined by

$$A = [W^+ - W^-]/[W^+ + W^-]$$

where W^+ is the number of β -rays aligned along the polarization vector of the nuclei and W^- is the number aligned against the polarization vector. The asymmetry is approximately given by

$$A \cong \mp P(p/E)(1 + \alpha_{\mp}E)$$

where P is the nuclear polarization, p is the momentum of the β -ray and E is the energy of the β -ray. In the study the coefficients " α_{\mp} " are tested against theoretical predictions given by CVC. CVC theory predicts a dependence of the asymmetry on the β -ray energy as a higher order effect, however, a second class component of the decay can also affect the asymmetry energy dependence. The predicted CVC component of the coefficient was calculated to be

$$(\alpha_{-} - \alpha_{+})_{CVC} = 0.27\%/\text{MeV}$$

whereas the results of the experiment found the asymmetry to be

$$\alpha_{-} - \alpha_{+} = 0.52 \pm 0.09\%/\text{MeV}.$$

It was suggested that the excess asymmetry could be accounted for by a second class current.

Similar tests have been conducted in β -ray emissions from polarized ¹⁹Ne

nuclei [4]

$$^{19}\mathrm{Ne} \rightarrow {}^{19}\mathrm{F} + e^+ + \nu$$

The results of this study also cannot be accounted for by CVC theory. It is proposed that a second class current contribution could make up the difference.

4.2 τ Decays

There have been experimental attempts at measuring second class currents in the $\tau^- \to \pi^- \eta \nu_{\tau}$ and $\tau^- \to \eta'(958)\pi^-\nu_{\tau}$ decays. In 1987 the HRS Collaboration reported a (0.051 ± 0.015) branching fraction of the $\tau^- \to \pi^- \eta \nu_{\tau}$ decay [5]. These claims were soon after refuted by other experiments [6] and were in contradiction to theoretical predictions [7]. To date there has been no experimental observation of the $\tau^- \to \pi^- \eta \nu_{\tau}$ decay. The CLEO Collaboration used 3.5 fb⁻¹ of data to set a 95% confidence level upper limit at 1.4×10^{-4} [2].

The $\tau^- \to \eta'(958)\pi^-\nu_{\tau}$ decay was studied by CLEO [8] which analyzed 4.68 fb⁻¹ of data, and set a 95% confidence level upper limit at 7.4×10^{-5} .

5 Theoretical Support

Calculations of second class decays have been carried out by extending chiral perturbation theory to the lowest-lying meson resonances [9]. The calculations show that the decay is dominated by the $a_0(980)$ resonance, and predict a 1.2×10^{-5} branching fraction for the $\tau^- \to \eta \pi^- \nu_{\tau}$ mode.

Previous theoretical work suggests that the expected rate of a second

class decay is proportional to the square of the up-down quark mass difference, $(m_u - m_d)^2$ [10]. The predictions are limited by the experimental uncertainty in knowing the individual quark masses, and incomplete resonance modeling.

6 Proposed Work

The focus of the proposed work will be to make the first observation of the $\tau^- \rightarrow \eta'(958)\pi^-\nu_{\tau}$ decay.

With the amount of data available at BaBar (approximately 100 times the CLEO sample) it is now possible to probe the second class current at the predicted theoretical levels. The full decay chain considered would be $\tau^- \rightarrow \eta'(958)\pi^-\nu_{\tau} \rightarrow \eta\pi^-\pi^+\pi^-\nu_{\tau}$ where the η would then be allowed to decay to either $\gamma\gamma$, $\pi^+\pi^-\pi^0$, or $\pi^0\pi^0\pi^0$. Since three different η decays are allowed, three different selection criteria will be optimized. One will be optimized for a 5 charged track signal with photons originating from a π^0 another will be optimized for a 3 charged track signal with photons originating from an η , and the third selection criteris will be optimized for a 3 charged track system with $3\pi^0$ candidates in the signal. In both cases the $\tau^- \rightarrow \eta'(958)\pi^-\nu_{\tau}$ measurement will be made by studying the $\eta\pi^+\pi^$ invariant mass distribution.

A recent BaBar study, completed as part of my M.Sc. and Ph.D. work, has placed the most precise limit on the $\tau^- \rightarrow \eta'(958)\pi^-\nu_{\tau}$ decay [11]. Figure 3 shows the fit to the data invariant $\pi\pi\eta$ mass in the region of the $\eta'(958)$ meson for data. The fit contains $19 \pm 13 \eta'(958)$ candidates. The



Figure 3: Shown is a fit to the data invariant $\pi\pi\eta$ mass distribution in the region of the $\eta'(958)$ used in setting the $\tau^- \to \eta'(958)\pi^-\nu_{\tau}$ confidence limit. The fitting function is a Gaussian summed with a polynomial.

central value branching fraction is $B(\tau^- \to \eta'(958)\pi^-\nu_\tau \to \pi^-\pi^+\pi^-\eta\nu_\tau) =$ (4.1±2.5)×10⁻⁶, where the error is the combined statistical and systematic error. The 90% CL upper limit for the $\tau^- \to \eta'(958)\pi^-\nu_\tau \to \pi^-\pi^+\pi^-\eta\nu_\tau$ decay mode was set at 7.2×10⁻⁶.

The limit was set using 350 million τ pairs, where the η was reconstructed from the $\eta \to \gamma \gamma$ mode. It is expected that by the end of running BaBar will have approximately 500 million τ pairs available. By using the full data set and reconstructing the η from $\gamma \gamma$, $\pi^+\pi^-\pi^0$, and $\pi^0\pi^0\pi^0$ it is possible to probe the $\tau^- \to \eta'(958)\pi^-\nu_{\tau}$ decay at the level of $3 - 4 \times 10^{-6}$. The theoretical prediction for $\tau^- \to \eta \pi^- \nu_{\tau}$ can be used to estimate the $\tau^- \to$ $\eta'(958)\pi^-\nu_{\tau}$ branching fraction, but the latter branching fraction is expected to be smaller due to phase space suppression.

7 Summary

Latest experimental results for the $B(\tau^- \rightarrow \eta'(958)\pi^-\nu_{\tau} \rightarrow \pi^-\pi^+\pi^-\eta\nu_{\tau})$ have set limits close to theoretical predictions. By using the full BaBar dataset it is possible to either measure a second class current or set a more stringent limit at the level of current theoretical predictions. The study of a second class process has extensions to work in Conserved Vector Current theory and the μ g-2 debate.

Appendix: The Conserved Vector Current

The conserved vector current hypothesis (CVC) is an experimentally observed symmetry. CVC is denoted by

$$\partial^{\mu}V_{\mu} = 0$$

where the equation denotes the divergence (∂^{μ}) of a vector current V_{μ} to be 0, or simply that the vector current is fully conserved [12]. This symmetry facilitates a relation of weak and electromagnetic interactions in a manner that assumes nothing about hadronic final state interactions. The removal of the hadronic interactions allows for simple, computation of complex processes. The main value of CVC lies in its ability to predict the lifetime of the $\pi^+ \to \pi^0 + e^+ + \nu_e$ decay.

The μ g-2 predictions are limited by the uncertainty of the hadronic corrections which are determined from low energy e^+e^- data. If one assumes CVC, then one can use more precise τ data to get a better estimate of the hadronic corrections. The use of τ -data instead of e^+e^- data is a topical issue that has not been completely resolved (for a detailed discussion, see [13] [14])

References

- [1] Particle Data Group, Y.-M. Yao et al., J. Phys. G 33, 1 (2006)
- [2] J. E. Bartelt *et al.* [CLEO Collaboration], Phys. Rev. Lett. **76**, 4119 (1996).
- [3] K. Sugimoto, I. Tanihata and J. Goring Phys. Rev. Lett. 34 1533 (1975).
- [4] F.P. Calaprice, S.J. Freedman, W.C. Mead and H.C. Vantine Phys. Rev. Lett. 35 1566 (1975).
- [5] M. Derrick et al., Phys. Lett. B **189** (1987) 260.
- [6] P. Baringer et al., Phys. Rev. Lett. **59** (1987) 1993.
- [7] A. Bramon, S. Narison and A. Pich, Phys. Lett. B 196, 543 (1987).
- [8] T. Bergfeld *et al.* [CLEO Collaboration], Phys. Rev. Lett. **79**, 2406 (1997) [arXiv:hep-ex/9706020].
- [9] H. Neufeld and H. Rupertsberger, Z. Phys. C 68, 91 (1995).
- [10] A. Stahl, *Physics with Tau Leptons*, ed. J. Kühn *et al.*, Springer Tracts in Modern Physics, (2000)
- [11] M. Lewczuk and R. Sobie, BaBar Analysis Note BAD1685, (2008)
- [12] R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).
- [13] J. F. de Troconiz and F. J. Yndurain, Phys. Rev. D 71, 073008 (2005)
 [arXiv:hep-ph/0402285].

[14] M. Davier, Nucl. Phys. Proc. Suppl. 169, 288 (2007) [arXiv:hepph/0701163].