Quark model

Quark model of hadrons

- Developed long before QCD was recognized as the appropriate quantum field theory of the strong interactions
- Postulate that
  1. All baryons are composed of 3 quarks (anti-baryons of 3 anti-quarks)
  2. All mesons are composed of a quark and an anti-quark

- We now know that quarks really do exist and that they carry a "color" charge that can take three values (we call them red, green and blue)
- Only color-neutral combinations ($q\underline{q}$ or rgb) exist as observable bound states

Low-lying baryons

- The quark model assignments for some common baryons (all with *l*=0) are

Particle	Mass (MeV)	I, J^P	quark content
p, n	938, 939	1/2, $1/2^+$	uud, udd
Λ	1116	0, $1/2^+$	sud
Σ^+, Σ^0, Σ^-	1189, 1193, 1197	1, $1/2^+$	suu, sud, sdd
Ξ^0, Ξ^-	1315, 1321	1/2, $1/2^+$	ssu, ssd
Δ^-, Δ^0, Δ^+ Δ^{++}	1230-1234	3/2, $3/2^+$	ddd, udd, uud, uuu
Σ^{*-}, Σ^{*0}, Σ^{*+}	1379-1383	1, $3/2^+$	sdd, sud, suu
Ξ^{*-}, Ξ^{*0}	1532-1535	1/2, $3/2^+$	ssd, ssu
Ω^-	1672	0, $3/2^+$	sss

Fermi-Dirac statistics

- The overall wavefunction describing any baryon should be anti-symmetric under the interchange of identical particles
- Consider the Ω^-, consisting of three s quarks
  - Total spin 3/2 → all quark spins are aligned; spin wavefunction is symmetric under interchange
  - Flavor wavefunction is symmetric (sss)
  - Angular momentum (l=0); spatial wavefunction is symmetric
- This "problem" pointed to another hidden quantum number in which the wavefunction is anti-symmetric: color
- The Ω^- is a color *singlet*, therefore anti-symmetric under interchange: $(r_1g_2b_3+ g_1b_2r_3+ b_1r_2g_3- g_1r_2b_3- r_1b_2g_3- b_1g_2r_3)/\sqrt{6}$
- This provided the starting point for the theory of QCD

Low-lying mesons

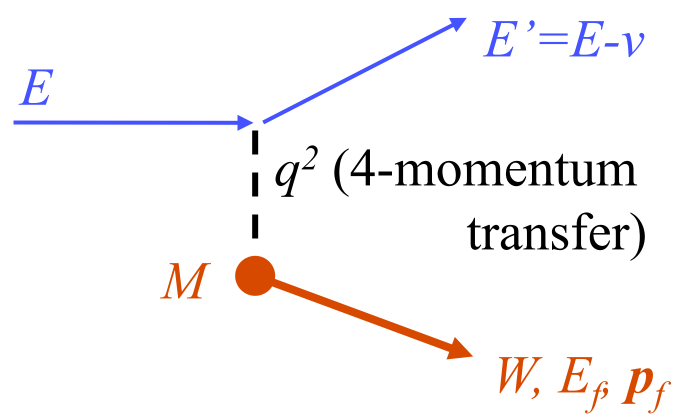
- The quark model assignments for some common mesons are given below. Note the mixing of flavorless ($q\underline{q}$) combinations; also, vectors are heavier than pseudo-scalars, and strange mesons are heavier than non-strange mesons

Particle	Mass (MeV)	I, J^P	quark content
π^+ π^- π^0	139, 135	1, 0^-	$u\underline{d}$, $d\underline{u}$, $(u\underline{u}-d\underline{d})/\sqrt{2}$
η	548	0, 0^-	$(u\underline{u}+d\underline{d}-2s\underline{s})/\sqrt{6}$
K^- K^+ $\underline{K}^0$ K^0	495 498	1/2, 0^-	$\underline{u}s$, $\underline{d}s$, $u\underline{s}$, $d\underline{s}$
η'	958	0, 0^-	$(uu+dd+s\underline{s})/\sqrt{3}$
ρ^+ ρ^- ρ^0	776	1, 1^-	$u\underline{d}$, $d\underline{u}$, $(u\underline{u}-d\underline{d})/\sqrt{2}$
Ω	783	0, 1^-	$(uu+dd)/\sqrt{2}$
K^{*-} K^{*+} $\underline{K}^{*0}$ K^{*0}	892-896	1/2, 1^-	$\underline{u}s$, $\underline{d}s$, $u\underline{s}$, $d\underline{s}$
φ	1020	0, 1^-	$s\underline{s}$

Electron-nucleon scattering

- Electron-nucleon scattering with high momentum projectiles should be sensitive to quarks
- Scattering kinematics in the rest frame of the target particle:

$$q^2 \equiv (p - p')^2 = (p_f - p_i)^2 = (E_f - M)^2 - \vec{p}_f^{\,2}$$
$$= -2M\nu + W^2 - M^2 < 0$$
$$\nu \equiv E_f - M = E - E'$$
$$W^2 \equiv p_f^2 = E_f^2 - \vec{p}_f^{\,2}$$



- Elastic scattering → $W^2 = M^2$ and thus $E' = E - \nu = E + q^2/(2M)$
- Define $X = -q^2/(2M\nu)$; $X=1$ for elastic scattering
- Inelastic scattering: $W^2 > M^2$, and $0<X<1$

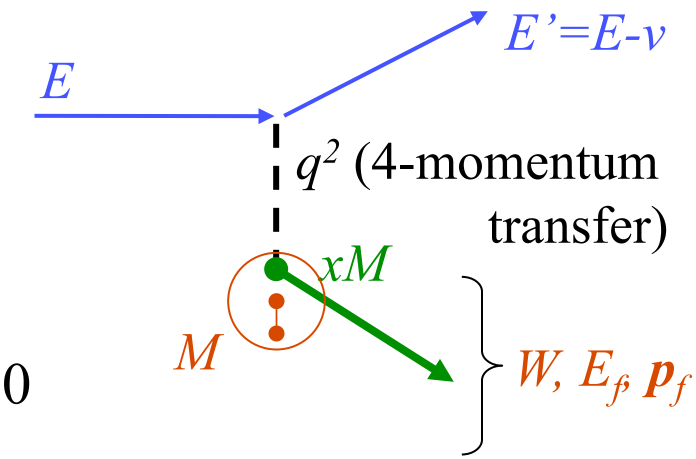
Electron-quark scattering

- Elastic scattering from a quark with momentum fraction x:

$$p_q = xp_f$$

$$q^2 = -2xM\nu + W^2 - x^2M^2$$

$$\nu = E_f - xM$$



- The 4-vector of the final quark gives

$$(xp_i + q)^2 = (xM)^2 + q^2 + 2xp_i \cdot q = m_q^2 \approx 0$$

- When $(xM)^2 << -q^2 \Rightarrow x = \dfrac{-q^2}{2M\nu}$; same as X from last slide, so picture of "quark momentum fraction" is sensible
- Nucleon is at rest but the quarks are not (Fermi motion)
- Quark momentum fractions are distributed over $0<x<1$

Related idea – electron-He scattering

- He nucleus is composed of 2p and 2n (as seen by a probe of appropriate momentum, i.e. $\hbar$ / 1 fm ~ 200 MeV
- Narrow peak is due to elastic scattering from He nucleus
- Elastic scattering from individual protons would give dashed peak, but is smeared out by Fermi momentum of the protons

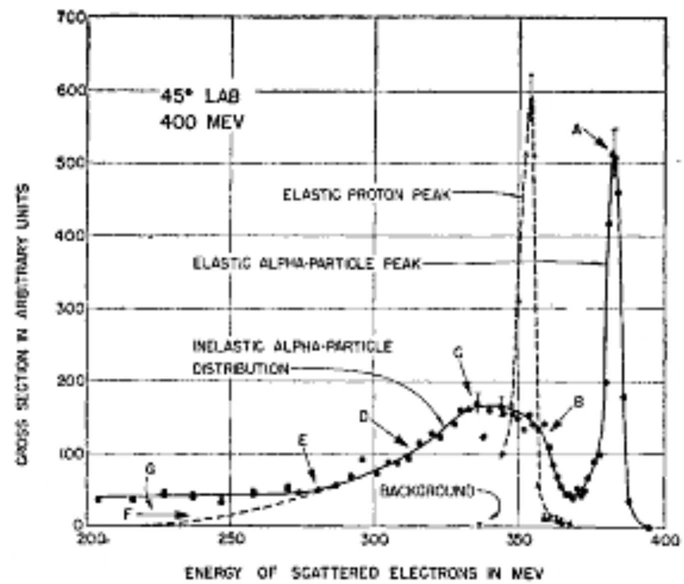
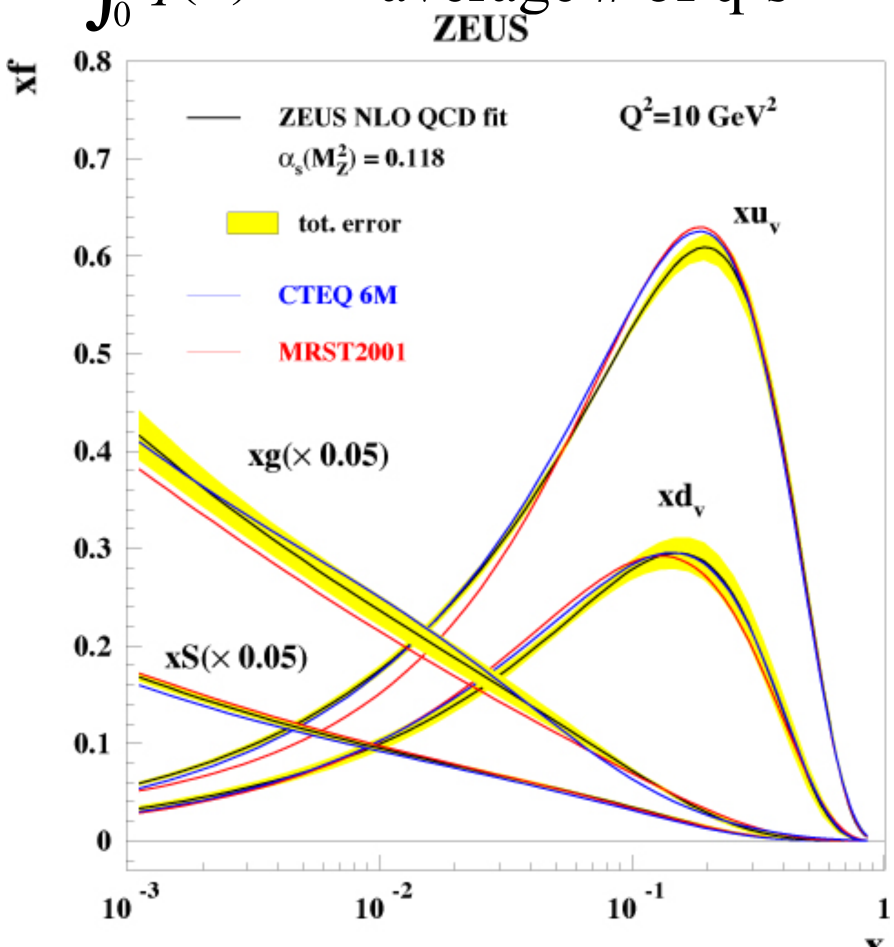
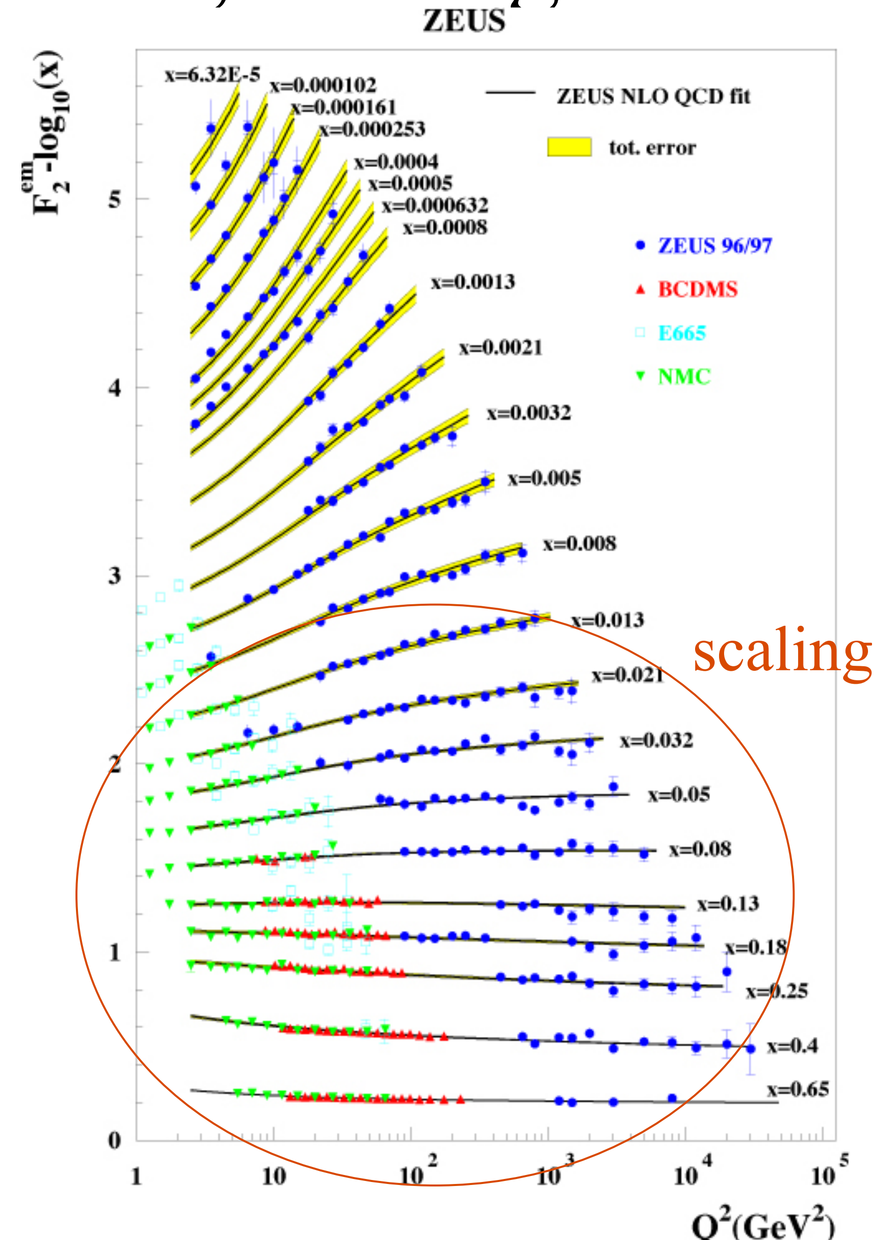


FIG. 12. Electrodisintegration of the alpha particle at 400 Mev and 45°. The elastic peak is shown at *A*. The inelastic continuum *BCDE* is related to the momentum distribution of nucleons within the alpha particle. *G* points to negative pion production. (See Sec. IVc in text.)

Parton distributions, scaling

- $\sum_i \langle x_i \rangle = 1$ where i=u, d, g…
- $\int_0^1 q(x)dx$ = average # of q's





Quark charges, spin

- Deep inelastic scattering can also teach us about the charges and spins of quarks
- Photon couples to charge; cross-section ($\sim |\psi^*\psi|$) to charge^2
- Compare scattering cross-sections from proton (uud) and isoscalar (e.g. C_{12}) targets → extract *p* and *n* separately
- Expect (and see) scattering from *u* 4 times that from *d*
- Magnetic moment proportional to spin; angular distribution allows discrimination
- We'll see details after we study QED (after reading break)

Proton-proton scattering

- Lepton-nucleon scattering has only one composite particle to consider
- At LHC, proton-proton scattering has this complexity squared
- Recall the parton distributions functions: collisions will be between quarks and quarks, or quarks and gluons, or gluons and gluons, or anti-quarks and quarks, or…
- Occasionally two high-x particles will collide, providing an enormous centre-of-mass energy → chance to produce particles never seen before
- However, the probability of "softer" collisions between low-x partons is enormously higher → lots of junk to reject

What did we learn from the quark model?

- The scores of observed hadrons can be classified based on a much smaller set of more fundamental objects
- There is a hidden quantum number based on FD statistics. It is consistent with quarks carrying an SU(3) color charge
- Certain combinations of quantum numbers are incompatible with coming from color-neutral $\underline{qq}$ or qqq states,
  e.g. $J^{PC} = 0^{+-}, 1^{-+}, \dots$
- Quarks are real (as verified by deep inelastic scattering)

What can’t we learn from the quark model?

- Low-energy QCD cannot be solved perturbatively, so we don’t know how to write the wavefunction for these bound states
- The simple atom-like picture of two (or three) constituent quarks is too simplistic; the strong force is strong, and a sea of gluons and virtual $q\underline{q}$ pairs exists inside a hadron
- The reason for which we don’t see colored combinations of quarks (called “confinement” in QCD) lies beyond the quark model itself

Hadron modeling

- Can we understand hadron masses based on quarks?
- Yes, at some level, but
  - Quark masses cannot be directly measured
  - We cannot solve low-energy QCD, so we must introduce ad-hoc potential models to describe the binding of quarks into a hadron
  - Mass predictions based on potential models have modest (few %) accuracy (not like in QED!)
- What about other properties, e.g. spin?
  - Here it's even worse; detailed experiments show that the constituent quarks in the proton account for only ~30% of the proton spin
  - The remainder must be carried by gluons and/or "sea" quarks, i.e. virtual $q\bar{q}$ pairs

Other quarks

- We only talked about u, d and s quarks
- Three heavier quarks, c, b and t exist
- We'll see those next time