# Heavy quarks

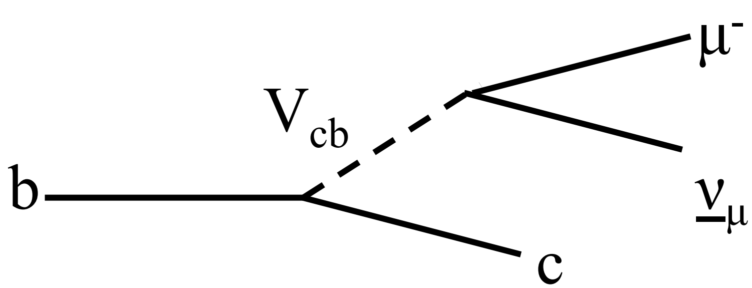
- CKM matrix
- CP violation
- Heavy quarkonium – a positronium analogue
- Heavy quark symmetry

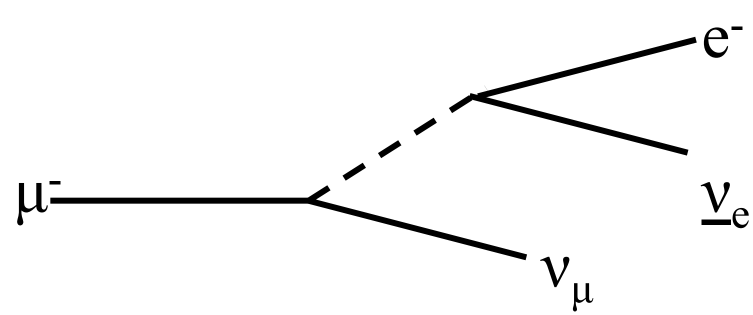
# Weak interaction doublets

- The charged weak interaction ($W^{\pm}$) makes transitions between the two states in each *weak isospin doublet*:

$$W \updownarrow \quad \begin{pmatrix} e \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix} \begin{pmatrix} d' \\ u \end{pmatrix} \begin{pmatrix} s' \\ c \end{pmatrix} \begin{pmatrix} b' \\ t \end{pmatrix} \quad \text{(left-handed only)}$$

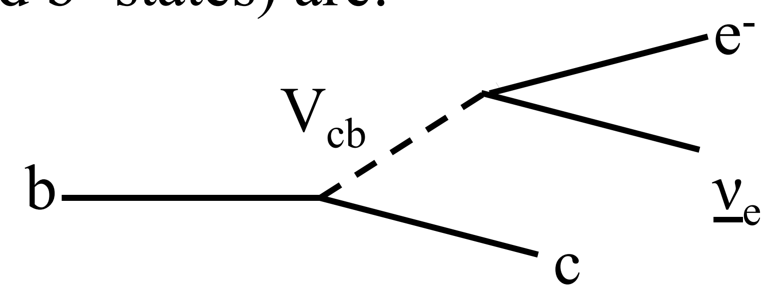
- The coupling strength in each case is the same: *universality*
- Transitions between doublets are not observed
- Weak decays look like



# Universality

- The possible weak decays of the b quark (which is a linear combination of the d', s' and b' states) are:
  1. b→c $e^-$ $\nu_e$
  2. b→c $\mu^-$ $\nu_\mu$
  3. b→c $\tau^-$ $\nu_\tau$
  4. b→c d' u (where the d' u can be any of red, green or blue)
  5. b→c s' c (where the s' c can be any of red, green or blue)
  6. b→c b' t (forbidden by energy conservation)




- Naïve expectation for the branching fraction into mode (1) is

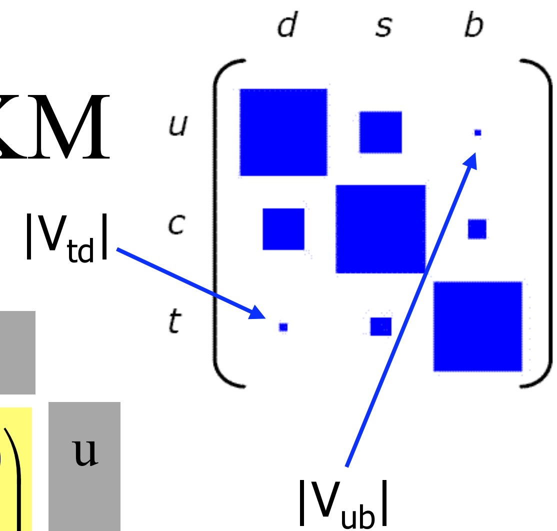
$$BF\left(b \to ce\overline{\nu}_e\right) \approx \frac{1}{1+1+1+3+3+0} = \frac{1}{9} = 0.11$$

- We observe 10.7% (although this level of agreement is somewhat fortuitous)

# Cabibbo-Kobayashi-Maskawa matrix

- The quark eigenstates that participate in the weak interaction are $\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$
- This matrix must be unitary, i.e. $\left(\mathbf{M}^*\right)^t \mathbf{M} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
- This provides 6 independent complex equations that must be satisfied. Appropriate choices of quark phases can eliminate all but one imaginary component ➔ 3 real angles, 1 phase
- This irreducible phase prompted KM to propose a 3rd generation before even the charm quark was discovered!

# Parameterization of CKM




$$
\begin{array}{cccc}
 & d & s & b \\
V_{CKM} = & \left( \begin{array}{ccc} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3\left(\rho - i\eta\right) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3\left[1-\left(\rho+i\eta\right)\right] & -A\lambda^2 & 1 \end{array} \right) & & \begin{array}{c} u \\ c \\ t \end{array}
\end{array}
$$

Buras, Lautenbacher, Ostermaier, PRD 50 (1994) 3433.

- 4 parameters; shown here to $O(\lambda^3)$ where $\lambda = |V_{us}| = 0.22$
- Parameters $A$, $\rho$ and $\eta$ are order unity ($\eta$ describes CP)
- Unitarity $\mathbf{VV}^\dagger = \mathbf{1}$ implies 6 independent complex equations
- Unitarity *triangle* of interest is

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0$$

all terms $O(\lambda^3)$

# "The" Unitarity Triangle

**$\lambda$, A, $\bar{\rho}$ and $\bar{\eta}$**

At the 1% level: $\lambda$

$$\lambda = |V_{us}| = \sin\theta_c$$

$$\lambda = 0.2265 \pm 0.0020$$

At the 3% level: A

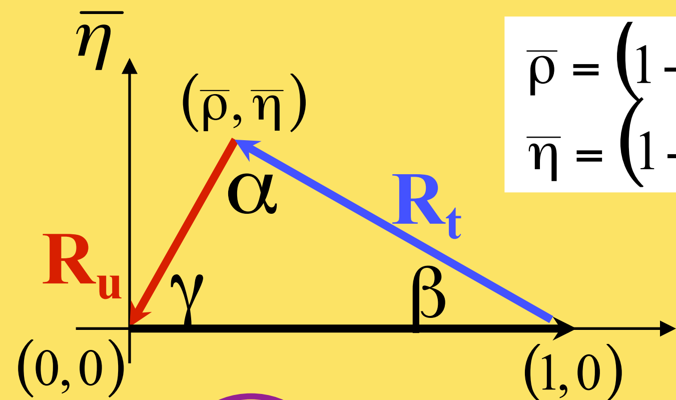
$$A = |V_{cb}|/\lambda^2$$

$$A = 0.801 \pm 0.025$$

$|V_{ub}|$ and $|V_{td}|$

$\rightarrow \bar{\rho}\text{-}\bar{\eta}$ plane

Unitarity: $1 + R_t + R_u = 0$




$$\bar{\rho} = \left(1 - \lambda^2/2\right)\rho$$
$$\bar{\eta} = \left(1 - \lambda^2/2\right)\eta$$

$$R_u = \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \approx -\sqrt{\bar{\rho}^2 + \bar{\eta}^2}\; e^{i\gamma}$$

$$R_t = \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*} \approx -\sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2}\; e^{-i\beta}$$

$$\gamma = \arg V_{ub}^*, \quad \alpha = \pi - \gamma - \beta$$

# Measuring the CKM

nuclear β decay,
K decays:

Cabibbo angle

CKM matrix

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

B lifetime,
semileptonic
decays:

$|V_{qb}|$

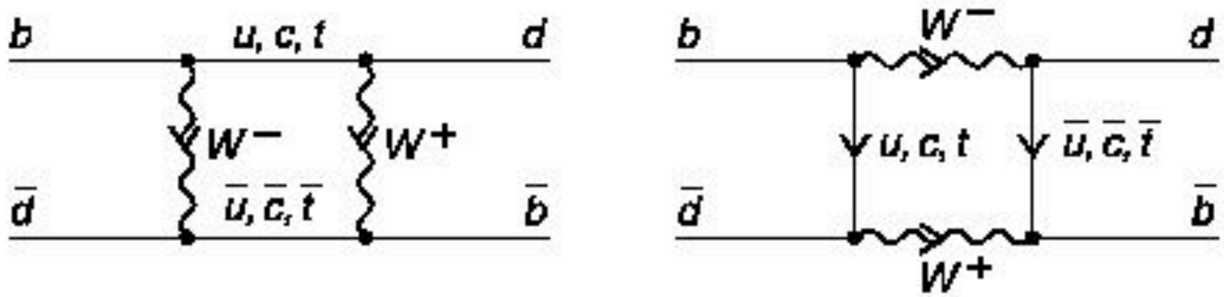
$B_d\underline{B}_d$ and $B_s\underline{B}_s$
oscillations,
radiative decays:

$|V_{tq}|$

$\cong 1$

**CP asymmetries:**
**imaginary phase**

# $B\underline{B}$ oscillations

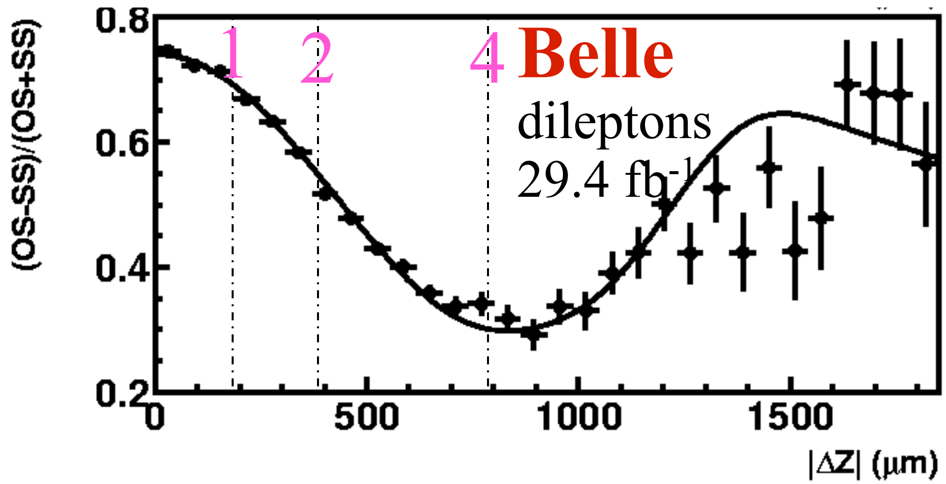


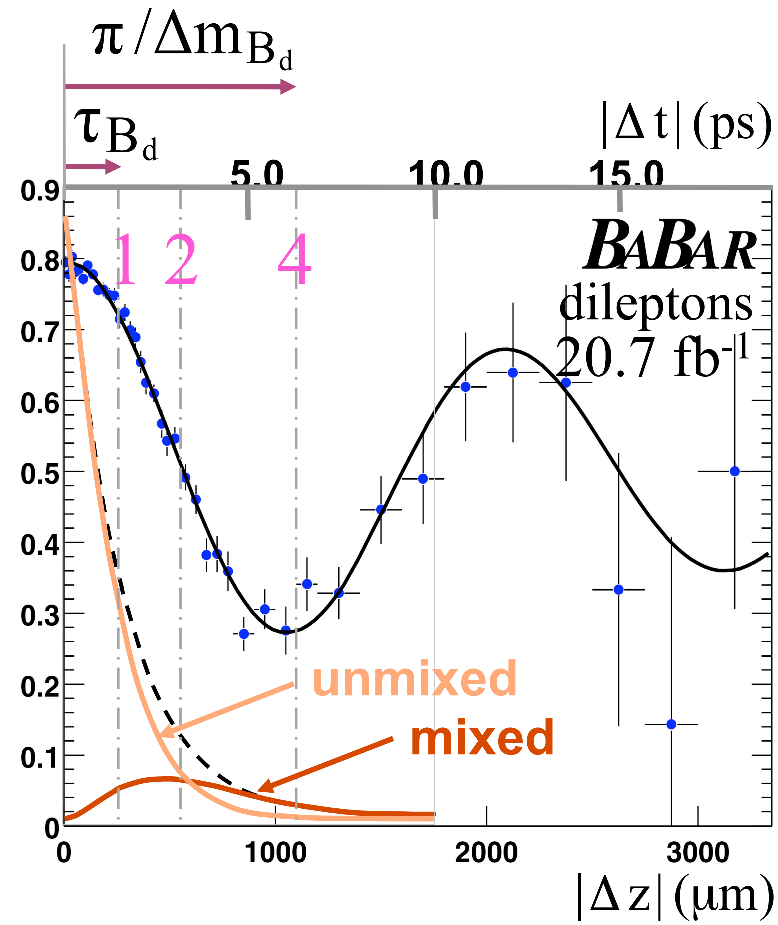

$\underline{B}^0 \rightarrow B^0$: $(\underline{B}^0 \rightarrow B^0) / \underline{B}^0 = 0.18$

- $2^{nd}$ order $\Delta b=2$ transition takes $B^0 \rightarrow \underline{B}^0$ making decay eigenstates distinct from flavour eigenstates
- In contrast to $K^0$ system, $\Lambda\Gamma \sim 0$ ($\Delta m$ leads to oscillations)
- Large $m_t$ makes up for GIM suppression

# Observations of B$\underline{\text{B}}$ oscillations

- The flavor oscillation is now mapped out over ~1.5 full periods
- $\Delta m = (0.502 \pm 0.006)\ \text{ps}^{-1}$



# CP violation

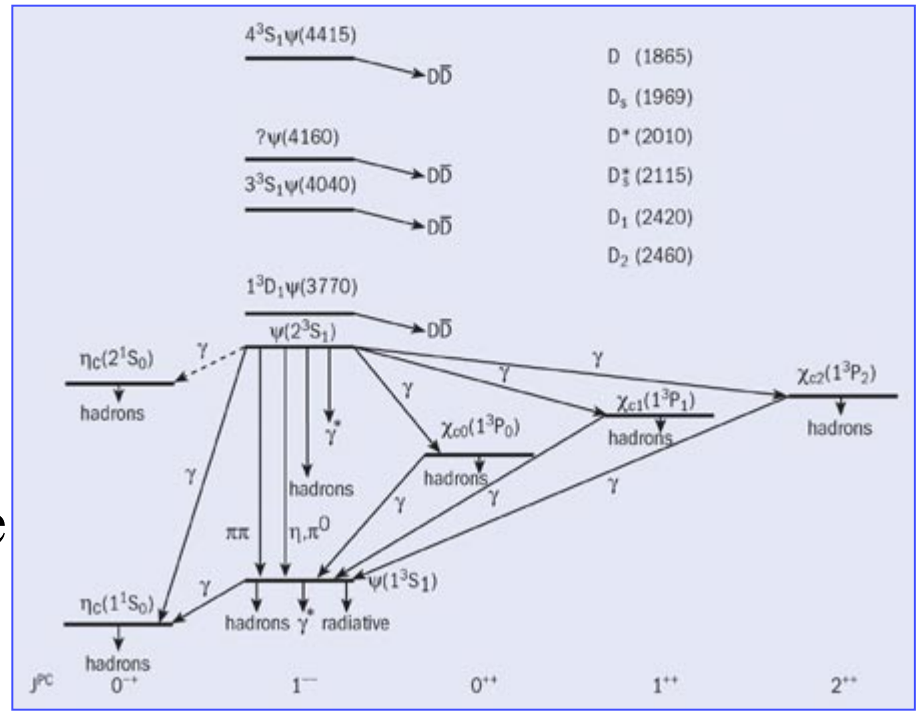
- The CKM matrix has 1 irreducible phase
- A process involving 2 or more amplitudes can see the effects of this phase
- Decay rates are $\propto |A|^2$, so $|A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + A_1^* A_2 + A_1 A_2^*$
- The operation of CP changes the sign of the weak phases (from CKM elements) but does not affect strong phases
- A phase different from $e^{in\pi}$ in a decay involving more than one amplitude can give CP violation, namely a difference between the rates of a reaction $B \rightarrow f$ and the CP conjugate $\underline{B} \rightarrow \underline{f}$
- CP violation has been observed only in $K$ and $B$ mesons; compatible with CKM mechanism

# Meson with two heavy quarks

- Mesons composed of light quarks require a full relativistic QM treatment to understand their binding, excitation spectra, ...
- Quarkonium mesons ($c\underline{c}$, $b\underline{b}$) are non-relativistic, i.e. the binding energies are small compared to the quark masses
- They are modeled by a similar system that we know in wonderful detail: positronium, a bound state of $e^+$ and $e^-$
- Let’s see how it works (refer to Blockland/Sobie notes)

# Quarkonia spectra

- Charmonium ($c\underline{c}$ bound states)
- Many of the predicted excitations (states) have been observed
- Heavier states decay strongly to open charm ($D\underline{D}$, where D is a $c\underline{u}$ or $c\underline{d}$ meson)
- Transitions can be via γ or one or more π
- Lowest states (J/ψ(3100) and $\eta_C$(2900)) decay via annihilation of the $c\underline{c}$ pair

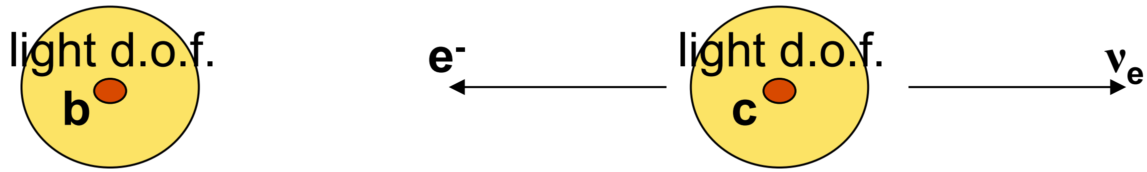


# Heavy-light systems

- Hadrons involving one heavy and one light quark cannot be treated as quarkonia
- There is a symmetry one can exploit for the case where the initial and final state quarks from a weak decay are heavy, e.g. $\underline{B} \rightarrow D e^- \underline{\nu}$ where $\underline{B} = (b, \underline{u})$ and $D = (c, \underline{u})$
- To understand it, recall the hydrogen atom
  - the proton is a ~static source of the electric field. The properties of the hydrogen atom are largely independent of the proton mass and spin
  - Replacing the hydrogen nucleus with deuterium (pn) leads to only small changes in the energy levels

# Heavy quark symmetry

- The analogue to the decoupling of the nucleon mass in atoms is that the "light quark degrees of freedom", i.e. the light valence quark and the sea of virtual quarks and gluons, can't tell the difference between one heavy quark (e.g. b) and another (e.g. c)
- This symmetry should be realized in decays like




- In practice, construct an effective theory around this symmetry and treat deviations as small perturbations
- Allows relations between mass splittings in c and b systems and calculations of some decay rates

# Standard Model (now part of the high school science curriculum in New York State)

~25 'fundamental' parameters
(masses, couplings, mixings)