P424 Midterm SOLUTIONS

75 minutes, 100 points total

1) Short answer (a few lines each): 8 points each

a) What is a typical value for the critical energy of a medium? What processes dominate the energy loss above/below the critical energy?

The critical energy is the energy at which the ionization loss and the bremsstrahlung radiation loss for electrons are equal. The critical energy is approximately equal to $600/Z$ MeV, where Z is the atomic number of the material, thus a typical value (for, say, carbon) is of the order of 100 MeV. Above the critical energy, bremsstrahlung is the dominant source of energy loss for electrons, and below the critical energy ionization is the dominant source of energy loss for electrons.

b) True or false (plus give a brief explanation): a calorimeter can determine the energy of an electromagnetic shower by measuring either the number of particles produced or by summing the total energy deposited.

True. Clearly one can measure the energy by summing the energy deposited. Calorimeters can also measure the amount of energy by measuring the number of particles produced, as the number of electrons, positrons, and photons present after t radiation lengths are each approximately equal to $2^t/3$.

c) Give examples of both fundamental and composite particles that are eigenstates of charge conjugation.

Any particle that is its own antiparticle is an eigenstate of charge conjugation. For fundamental particles, this includes the photon, Z^0, and (presumably) the Higgs and the graviton. For composite particles, there are several, including the π^0, ρ^0, η, and others.

d) What symmetry leads to the conservation of (i) angular momentum? (ii) energy? (iii) baryon number?

(i) Rotational symmetry, (ii) time translation symmetry, (iii) none that we know of.

e) What final-state particle is an unambiguous signal of the weak interaction?

An (anti-)neutrino is an unambiguous signal of the weak interaction; there's no way to produce them in any other interaction.

f) Nuclear physicists typically work with half-life ($t_{1/2}$) instead of mean lifetime (τ); $t_{1/2}$ is the time it takes for half the members of a large sample to decay. For exponential decay, what is $t_{1/2}$ (as a multiple of τ)?

We have $N(t) = N_0 e^{-\left(t_{1/2}/\tau\right)}$, thus $t_{1/2} = \tau \ln 2$.

A bit longer (but not much) answer problems:

2) An electron scatters elastically from a proton in the proton rest frame. The initial and final electron energies are E_e and E_e', and the scattering angle is θ. You can assume the electron energies are large compared with m_e. Calculate:

a) The invariant mass of the final state system 7 points

We have $\begin{pmatrix} E_e \\ E_e \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} m_p \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} E_e + m_p \\ E_e \\ 0 \\ 0 \end{pmatrix}$, so the invariant mass of the final state is $\sqrt{E_e^2 + 2m_p E_e + m_p^2 - E_e^2} = \sqrt{2m_p E_e + m_p^2}$.

b) The electron scattering angle in the centre-of-mass frame 7 points

In the c.m. frame the 4-vectors of the electron and proton after interaction are $\begin{pmatrix} p' \\ p'\cos\phi \\ p'\sin\phi \\ 0 \end{pmatrix}$ and $\begin{pmatrix} \sqrt{p'^2 + m_p^2} \\ -p'\cos\phi \\ -p'\sin\phi \\ 0 \end{pmatrix}$ respectively, where ϕ is the scattering angle in the c.m. frame. Since the invariant mass of the final state system is invariant (hence its name), we have that $p' + \sqrt{p'^2 + m_p^2} = \sqrt{2m_p E_e + m_p^2}$ which implies, after a few lines of algebra, that $p' = \dfrac{m_p E_e}{\sqrt{m_p^2 + 2m_p E_e}}$. In the lab frame the 4-vector of the electron after interaction is $\begin{pmatrix} E_e' \\ E_e'\cos\theta \\ E_e'\sin\theta \\ 0 \end{pmatrix}$. The boost between the two frames is in the x-direction, so there is no change to the y-coordinate of the 4-vector, hence we have that $p'\sin\phi = E_e'\sin\theta$, thus $\phi = \sin^{-1}\left(\frac{E'_e \sin\theta\sqrt{m_p^2 + 2m_p E_e}}{m_p E_e}\right)$. (Note that one could further eliminate θ from this formula by noting that in the lab frame we have

$$\begin{pmatrix} E_e + m_p \\ E_e \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} E'_e \\ E'_e \cos\theta \\ E'_e \sin\theta \\ 0 \end{pmatrix} + \begin{pmatrix} \sqrt{E'^2_e + E_e^2 - 2E_e E'_e \cos\theta + m_p^2} \\ E_e - E'_e \cos\theta \\ -E'_e \sin\theta \\ 0 \end{pmatrix}$$

, so equating the energies we get that $E_e E'_e \cos\theta = E_e E'_e + m_p E'_e - m_p E_e$, thus

$$\theta = \cos^{-1}\left(\frac{E_e E'_e + m_p E'_e - m_p E_e}{E_e E'_e}\right)$$

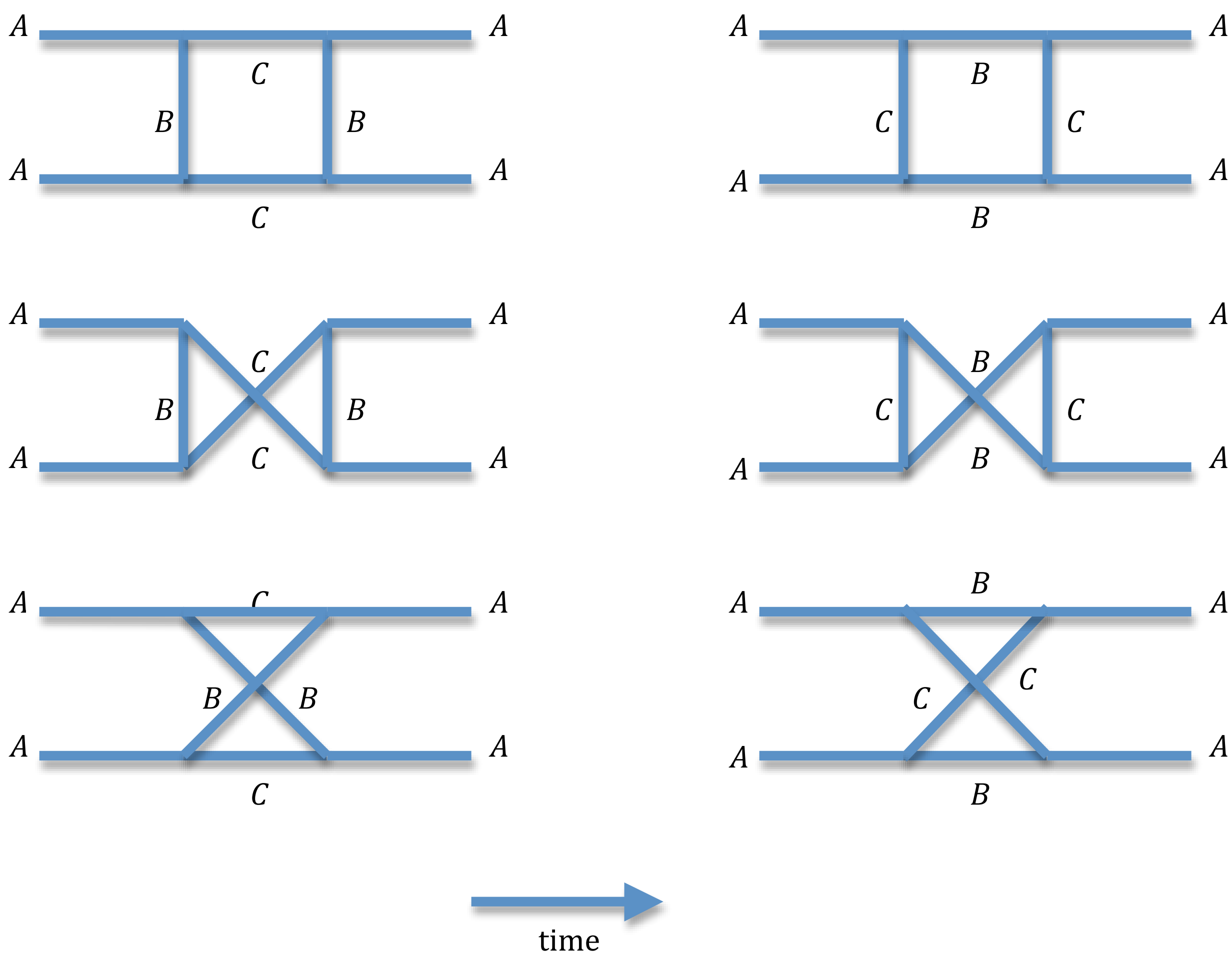
.)

3) Can the decay $\eta \to \pi^+\pi^-$ proceed via any of the known interactions? (The η has $I^G(J^{PC}) = 0^+(0^{+-})$ and the π^+ has $I^G(J^P) = 1^-(0^-)$.) 10 points

Big error on my part – I wrote that the η has $I^G(J^{PC}) = 0^+(0^{+-})$, which really should have been $I^G(J^{PC}) = 0^+(0^{-+})$. In reality, the decay is forbidden in the strong and electromagnetic interactions due to parity violation, which can be seen from the fact that the η parity is -1 (not +1), whereas two pions have a parity -1 x -1 x $(-1)^l$, with l necessarily being 0 as the J of the η is zero. But if the η actually had $I^G(J^{PC}) = 0^+(0^{+-})$, then the decay would be forbidden due to charge conjugation, since a system consisting of a spinless particle and its antiparticle will constitute an eigenstate of C with eigenvalue $(-1)^l$, and l would have to be 0 since the η is spinless. Of course, I accept either answer.

4) In the Feynman toy theory:

a) Draw all the lowest-order diagrams for $A + A \to A + A$. (There are 6 of them! Do the twist ♪ ♪ ♬ ...) 6 points



b) Set up the formula (but don't calculate/integrate/etc.!) for the amplitude for this process. 9 points

From the Feynman rules for the toy theory, the amplitude for, for example, the first of the above diagrams, would be obtained by integrating

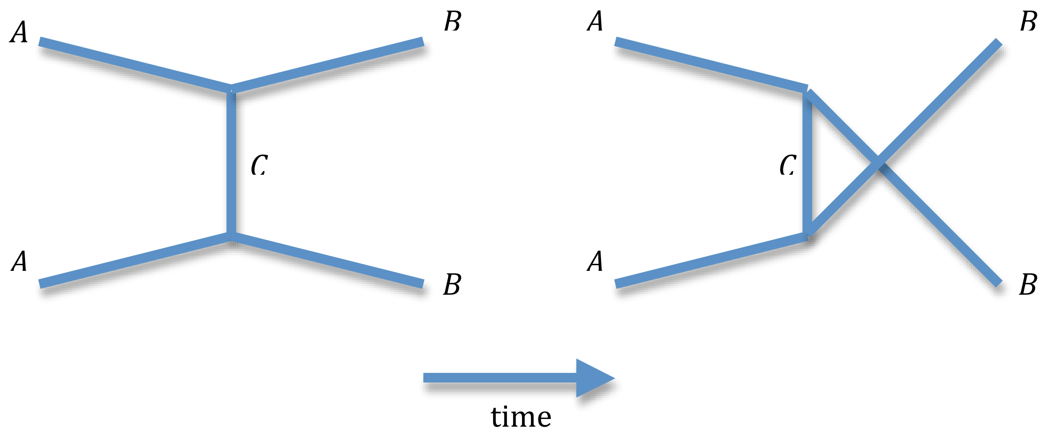
$$i(-ig)^4\left(\frac{i}{q_{C1}^2 - m_C^2}\right)\left(\frac{i}{q_{C2}^2 - m_C^2}\right)\left(\frac{i}{q_{B1}^2 - m_B^2}\right)\left(\frac{i}{q_{B2}^2 - m_B^2}\right) \times$$

$$(2\pi)^{16}\delta^{(4)}(p_{A1} - q_{B1} - q_{C1})\delta^{(4)}(p_{A2} + q_{B1} - q_{C2})\delta^{(4)}(-p_{A3} - q_{B2} + q_{C1})\delta^{(4)}(-p_{A4} + q_{B2} + q_{C2})$$

$$\times \frac{1}{(2\pi)^{16}} d^4q_{B1} d^4q_{B2} d^4q_{C1} d^4q_{C2}$$ and dropping the extra $(2\pi)^4\delta^{(4)}$ function.

To obtain the total amplitude for the process, one would sum all 6 of the amplitude contributions.

c) Draw the two lowest-order diagrams for the process $A + A \to B + B$. 4 points



d) Set up the formulas (but don't calculate/integrate/etc.!) for the differential cross-section for this process in the CM frame. 9 points

The amplitude for the first diagram is given by $i(-ig)^2 \frac{i}{(p_{A1} - p_{B1})^2 - m_C^2}$ $= \frac{g^2}{t - m_C^2}$ and the amplitude for the second diagram is given by $i(-ig)^2 \frac{i}{(p_{A1} - p_{B2})^2 - m_C^2} = \frac{g^2}{u - m_C^2}$, so the total amplitude is the sum of these contributions. To convert the amplitude to a differential cross section, we use Fermi's Golden Rule. In the c.m. frame, we have

$$\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi)^2} \frac{S|M|^2}{(E_1 + E_2)^2} \frac{|\vec{p}_f|}{|\vec{p}_i|}.$$

$S = ½$ because we have two identical particles in the final state, and $E_1 = E_2 = E$, so we end up with

$$\frac{d\sigma}{d\Omega} = \frac{1}{2(16\pi E)^2} \frac{|\vec{p}_f|}{|\vec{p}_i|} \left| \frac{g^2}{t - m_C^2} + \frac{g^2}{u - m_C^2} \right|^2.$$