P424 Assignment 2 Solutions

1) Fun with isospin

(a) For which of the following is strong decay forbidden by isospin conservation? To determine the quark content and properties of the particles listed below, refer to the Review of Particle Properties summaries of mesons and baryons:
http://pdg.lbl.gov/2005/tables/contents_tables.html

1. $\omega(783) \to \rho^+(770)\pi^-$
2. $\phi(1680) \to \phi(1020)\pi^0$
3. $K^*(892) \to K\pi$
4. $\rho^0(770) \to \pi^0\pi^0$

Note that some of these may be forbidden by other things (e.g. kinematics), and that decays forbidden by isospin can occur via electromagnetic or weak decay, so don't just look for the presence or absence of a decay mode in the PDG tables when answering this question. Look at the isospin quantum number for each particle and use angular momentum addition (or Clebsch-Gordon tables).

1. has isospin content $|0\ 0\rangle \to |1\ 1\rangle + |1\ {-1}\rangle$, which is allowed under isospin conservation, since it has Clebsch-Gordon coefficient $\sqrt{1/3}$ (but note that the decay is kinematically forbidden, as the omega is not heavy enough to decay into a rho plus a pion, at least for most of the natural width of the rho [and omega]).
2. has isospin content $|0\ 0\rangle \to |1\ 0\rangle + |1\ 0\rangle$, which is forbidden.
3. has four distinct possibilities (up to charge conjugation): $K^{*+}(892) \to K^+\pi^0$, $K^{*+}(892) \to K^0\pi^+$, $K^{*0}(892) \to K^0\pi^0$, and $K^{*0}(892) \to K^+\pi^-$. The first and third of these have isospin content $|\tfrac{1}{2}\ \tfrac{1}{2}\rangle \to |\tfrac{1}{2}\ \tfrac{1}{2}\rangle + |1\ 0\rangle$, which is allowed (with Clebsch-Gordon coefficient $\sqrt{1/3}$), whereas the second and fourth of those have isospin content $|\tfrac{1}{2}\ \tfrac{1}{2}\rangle \to |\tfrac{1}{2}\ {-\tfrac{1}{2}}\rangle + |1\ 1\rangle$, which is also allowed (with Clebsch-Gordon coefficient $\sqrt{2/3}$), so these decays are allowed.
4. has isospin content $|0\ 0\rangle \to |1\ 1\rangle + |1\ {-1}\rangle$, which has a zero Clebsch-Gordon coefficient and thus is forbidden.

2) Selection rules

Parity and Charge Conjugation are good symmetries of both the strong and the electromagnetic interactions, as is angular momentum. Use these to answer the following questions.

(a) The η decays to three pions but not to two; what prevents it?

(b) The decay $B^+ \to K^+\gamma$ is forbidden - by what?

(c) What forbids the decay $\pi(1300) \to 3\gamma$?

(d) The decay $K^+ \to \pi^+\pi^0$ violates isospin and parity (verify this for yourself). How does it occur?

(a) Parity conservation. The η, being a bound state of two fermions with spin and orbital angular momentum 0, has a parity $(-1)^{\ell+s+1} = -1$, whereas two pions have a parity $-1 \times -1 \times (-1)^{\ell}$, with ℓ necessarily being zero as J of the η is zero, enforcing positive parity. Thus the η cannot decay to two pions. A three pion decay can have either parity depending on the relative angular momenta of the pion pairs, thus the 3π decay is allowed.

(b) Angular momentum conservation. A scalar (spin 0) cannot decay to a scalar plus a photon, since the photon is polarized. There is no way (such as would occur with vector + photon decay products) to balance out the photon's angular momentum such that a scalar can produce it.

(c) Conservation of C, the charge conjugation quantum number. The $\pi(1300)$ is, like the ground state pion, +1 under charge conjugation, as it is a spin-0 state. But a system of three photons has C quantum number $(-1)^3 = -1$. Thus this decay is forbidden.

(d) This decay violates parity conservation because the kaon is a bound state of two fermions with spin 0 and angular momentum 0, therefore it has parity $(-1)^{\ell+s+1} = -1$, and it violates isospin conservation because the K^+ is a $|\tfrac{1}{2}\ \tfrac{1}{2}\rangle$ isospin state decaying to $|1\ 1\rangle + |1\ 0\rangle$. Weak interactions and decays do not in general conserve either isospin or parity, so this decay is not unexpected.