P424 Assignment 3 Solutions

1) Interaction of particles in matter

a) The amount of energy a 1.5 GeV particle loses when traversing material depends on the particle type. Assume the particles *e, μ, π, K,* and *p*, each with 1.5 GeV momentum, traverse a medium of thickness 0.01 X_0. Which, on average, loses the most energy (order them from largest to smallest energy loss).

b) The particles from (a) enter a quartz bar with index of refraction n = 1.5. Which particles will emit Cherenkov radiation?

c) Consider an electromagnetic shower initiated by a 1 GeV positron in Lead. If we model the shower process as a series of bremsstrahlung events and subsequent pair production from the photons, estimate (crudely) how many positrons we expect to produce. (Hint: the critical energy in Lead is about 1 MeV.)

(a) The lower the mass, the more bremsstrahlung radiation will be created (bremsstrahlung power is proportional to m^{-6}). This dominates for particle with γ>~100, i.e. the electron, and the electron will have the greatest energy loss. The others are dominated by ionization energy loss, which has a form given by the Bethe-Bloch equation (e.g. p.2 of https://particle.phys.uvic.ca/~jalbert/424/lect6_424.pdf). The ordering of largest to smallest energy loss should start with *e*, then *p*. But the ordering of μ, π, and *K* actually does depend on the density and *Z* of the material in question, so as long as you have a properly worked-out answer in regard to those 3, I give full credit.

(b) Particles that are moving faster than the speed of light in the medium, c/1.5, will emit Cherenkov radiation, thus $\gamma > \frac{1}{\sqrt{1-1/1.5^2}} = 1.34$. $\gamma = E/m \Rightarrow \frac{\sqrt{m^2+1.5^2}}{m} > 1.34 \Rightarrow$

$\frac{m^2+2.25}{m^2} > 1.8 \Rightarrow m < 1.67$ GeV/c^2 for any particle that radiates Cherenkov radiation.
This holds for all of the above particles so they *all* will radiate Cherenkov light.

(c) As discussed in section 2.4.1 of the Perkins handout, there are $N = 2^t$ particles after t radiation lengths, with γ, e^-, and e^+ approximately equal in number. The energy per particle at depth tX_0 is $E(t)=E_0/2^t$. The process continues until $E = E_c$, thus $E_c = E_0/2^{tmax}$ => $t_{max} = \ln(E_0/E_c)/\ln 2$ => $N_{max} = 2^{tmax} = e^{tmax\ln 2} = e^{\ln(E_0/E_c)} = E_0/E_c$. Thus $N_{e+} = N_{max}/3$ = 1/3 * 1 GeV/1 MeV = ~333.

2) Particle detectors

Consider the BaBar detector (see http://www.slac.stanford.edu/BFROOT/www/doc/workbook/detector/detector.html). Describe how you would expect the decay $B \rightarrow J/\psi\, \pi^0$ to be recorded in the detector when the π^0 decays to $\gamma\gamma$ and the J/ψ decays to $\mu^+\mu^-$. Note that the decay lifetimes of both the J/ψ and the π^0 are less than 10^{-16} s. You should say where and how each particle is detected, and what properties are measured in each relevant device.

Both the J/ψ and the π^0 will decay inside the beampipe, producing the 2 muons and 2 photons. The muons and photons will move through the tracking detectors, first the silicon detector (SVT) and then the drift chamber (DCH). The muons will leave hits, and thus tracks, in both of these subdetectors, while the photons will move through the trackers without leaving any signal. The muons and photons will then pass through the DIRC particle ID device. The muons will emit some Cherenkov light which will be detected, while the photons will pass through the DIRC without typically emitting any signal. Then both the muons and the photons will enter the electromagnetic calorimeter (EMC). The muons will pass through the EMC emitting a small signal corresponding to a small fraction of their energy, while the photons will pair-produce, which will then brem, etc, creating a shower. All photon energy will be delivered to the EMC crystals and be recorded as signals. Thus only the muons will pass through the solenoid magnet and into the instrumented flux return (IFR). The muons will emit small signals at each active layer of the IFR (the active layers are essentially low-cost tracking devices) and lose small fractions of their energies in the steel and brass absorber layers but will continue to pass all the way through the IFR and exit the detector.

3) An experiment searching for proton decay in the mode $p \rightarrow e^+ + \pi^0$ is carried out using a cubical tank of water as the proton source. Possible decays are to be detected using the Cherenkov light emitted when the electromagnetic showers from the decay products traverse the water. (a) How big should the tank be in order to contain such showers if they start in the centre? (b) Estimate the total track length integral (TLI) of the showers from a decay event and hence the total number of photons emitted in the visible region (λ = 400-700 nm). (c) If the light is detected by means of an array of photomultipliers at the water surface, the effective optical transmission of the water is 50% and the photocathode efficiency is 20%, what fraction of the surface must be covered by photocathode to give an energy resolution of 5%?

(a) A proton decaying at rest will produce a positron and a π^0, each with momentum 459 MeV. The π^0 will then promptly decay into two photons, each with momentum 239 MeV. The radiation length and critical energy of water are approximately 433 mm and 70 MeV respectively, thus the tank must have a radius of at least $\frac{\ln(459/70)}{\ln 2}$*433 mm = 1175 mm = ~1.2 m.

(b) The total TLI of the positron part of the showers will be approximately (459/70)*433 mm = 2839 mm, and the two photon showers will each have TLIs of (239/70)*433 mm = 1478 mm, thus the total TLI will be approximately 5.8 m.

For the positron, the total number of particles in the range 400 – 700 nm (i.e. 1.77 – 3.1 eV) is $\frac{\frac{4.59\times10^8}{1.77}-\frac{4.59\times10^8}{3.1}}{\ln 2} \approx 1.6\times10^8$ and the photon showers will each have

$\frac{\frac{2.39\times10^8}{1.77}-\frac{2.39\times10^8}{3.1}}{\ln 2}\approx 0.8\times10^8$ particles in that range. There will be a total of approximately 3.3×10^8 particles in that energy range, of which 1.1×10^8 will be photons.

(c) Of these, 10% multiplied by the fraction of the surface covered by phototubes will be detected, thus the fraction F that must be covered is $0.05=\frac{\sqrt{F\times1.1\times10^7}}{F\times1.1\times10^7}$ $\Rightarrow$

$F=\frac{1}{0.05^2\times1.1\times10^7}=3.6\times10^{-5}$.