P424 Assignment 4 Solutions

1) Neutral kaon beams

Suppose you have a monochromatic beam of K^0 mesons produced at position $z = 0$. They have a momentum of 40 GeV parallel to $\hat{z}$. (Never mind that real kaon beams aren't quite so ideal.)

At what distance do you expect to see the K^0 content of the beam go through its first minimum? At this point what fraction of the kaons are in the $\overline{K}^0$ state?

The minimum occurs when the intensity $I(t)$ divided by $I(t) + \overline{I}(t)$ is a minimum, where

$$I(t) = \frac{1}{4}\left(e^{-\Gamma_L t} + e^{-\Gamma_S t} + 2\,e^{-(\Gamma_L+\Gamma_S)t/2}\,cos(\Delta m\,t)\right)$$

$$\overline{I}(t) = \frac{1}{4}\left(e^{-\Gamma_L t} + e^{-\Gamma_S t} - 2\,e^{-(\Gamma_L+\Gamma_S)t/2}\,cos(\Delta m\,t)\right)$$

This occurs at $t_{\rm min} = 4.42\times 10^{-10}\,s$. At this point the flight distance is $L = \beta\gamma c t_{\rm min} = 10.64\,\rm m$ and the fraction of the beam in the $\overline{K}^0$ state is 0.56.

2) Quark model

Explain why the J^{PC} combinations 0^{+-} and 1^{-+} are incompatible with being $q\bar{q}$ states.

The parity eigenvalue of a $q\bar{q}$ state is -1^{L+1} and the charge conjugation eigenvalue is -1^{L+S}, where L and S are the orbital and spin angular momenta, respectively. To get $J = 0$ we must have $L = S = 0$ or $L = S = 1$, giving $J^{PC} = 0^{++}$ or 0^{-+}. The combinations 0^{--} and 0^{+-} are not allowed. For the case $J = 1$ we can have $L = 1$, $S = 0$ or $S = 1$ with $L = 0$, 1 or 2. This gives $J^{PC} = 1^{+-}$, 1^{--}, 1^{++} and 1^{--}; the 1^{-+} combination is not allowed.

3) Strange physics

(a) Could the phenomenon of strangeness oscillations exist without CP violation?

(b) Why are strangeness oscillations not seen in the excited neutral strange mesons $K^{*0}(892)$ and $\overline{K}^{*0}(892)$?

(a) Yes. Even without CP violation, the two neutral kaon eigenstates can oscillate into one another. In fact, strangeness oscillations were discovered about 5 years before CP violation was discovered. The only thing required for strangeness oscillations is that the mass eigenstates differ from the weak eigenstates. Since the two neutral kaon eigenstates share quantum numbers conserved by the weak interactions such as charge and spin, and are close in mass, oscillations can occur regardless of whether there is CP violation.

(b) The excited neutral kaons decay strongly, typically into a kaon and a pion. Since the timescale of the strong interaction is typically around 10^{12} times shorter than that of the weak interaction, the excited kaons simply have no time to oscillate into one another before they decay.