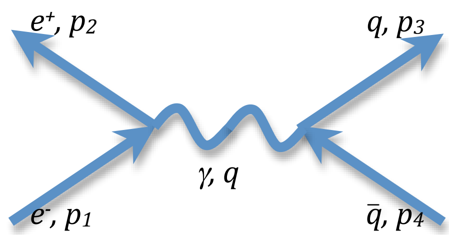
P424 Assignment 6 SOLUTIONS

1) Calculate the scattering cross-section for the reaction $e^+e^- \to q\bar{q}$, where q is an individual quark species of charge e_q. Assume that the incoming particles are not polarized and that the spin projections of the outgoing quarks are not measured. Further assume that the energy of the incoming electrons in the centre-of-mass frame is much larger than the electron or quark masses.

(a) Draw the relevant Feynman diagram(s) for the lowest order process



(b) Use the Feynman rules to determine the matrix element

Taking the product of the external lines, the vertex factors, and the photon propagator, we have $\mathcal{M} = \frac{Qg_e^2}{(p_1+p_2)^2}\left[\bar{v}(p_2)\gamma^\mu u(p_1)\right]\left[\bar{u}(p_3)\gamma_\mu v(p_4)\right]$, where Q is the quark charge, in units of e.

(c) Use Casimir's trick to calculate the spin-averaged square of the matrix element, $\langle|\mathcal{M}|^2\rangle$. You can use the approximation $E_e \gg m_q$.

Using Casimir's trick, we have that

$\langle|M|^2\rangle = \frac{1}{4}\left[\frac{Qg_e^2}{(p_1+p_2)^2}\right]^2 Tr\left[\gamma^\mu(\not{p}_1 + m_e)\gamma^\nu(\not{p}_2 - m_e)\right]Tr\left[\gamma_\mu(\not{p}_4 - m_q)\gamma_\nu(\not{p}_3 + m_q)\right]$. Using the trace formulae as per the lecture notes, or section 7.7 in the book, this equals

$\frac{4Q^2g_e^4}{(p_1-p_3)^4}\left\{p_1^\mu p_2^\nu + p_2^\mu p_1^\nu - g^{\mu\nu}(p_1\cdot p_2)\right\}\left\{p_{3\mu}p_{4\nu} + p_{4\mu}p_{3\nu} - g_{\mu\nu}(p_3\cdot p_4)\right\}$. Multiplying this all out (note that the dot products are dot products of 4-momenta, not of 3-momenta), this becomes the simpler expression

$\frac{8Q^2g_e^4}{(p_1-p_3)^4}\left\{(p_1\cdot p_3)(p_2\cdot p_4) + (p_1\cdot p_4)(p_2\cdot p_3)\right\}$. We have

$p_1\cdot p_2 = p_3\cdot p_4 \approx 2E_e^2,\ p_1\cdot p_3 = p_3\cdot p_4 \approx 2E_e^2(1-\cos\theta),\ p_2\cdot p_3 = p_1\cdot p_4 \approx 2E_e^2(1+\cos\theta)$, thus we can express the spin-averaged matrix element simply as $Q^2 g_e^4 (1+\cos^2\theta)$.

(d) Calculate the scattering cross-section under the conditions listed above. Give your result in the centre-of-mass frame. Don't forget that quarks come in three distinct colors.

The scattering cross-section $\sigma = \frac{1}{64\pi^2}\int \frac{\langle |M|^2 \rangle}{(E_1+E_2)^2}\frac{|\vec{p}_f|}{|\vec{p}_i|}d\Omega =$

$\frac{Q^2 g_e^4}{256\pi^2 E_e^2}\left\{4\pi + 2\pi\int \sin\theta\cos^2\theta d\theta\right\} = \frac{Q^2 g_e^4}{256\pi^2 E_e^2}\frac{16\pi}{3} = \frac{Q^2 g_e^4}{48\pi E_e^2} = \frac{Q^2\alpha^2\pi}{3E_e^2}$, per colour per flavour, or $\frac{Q^2\alpha^2\pi}{E_e^2}$ total per flavour.

(e) Assume the centre-of-mass energy is 30 GeV; what are the total cross-sections for scattering into each of the six quark types? How does the total cross section for scattering into quarks (all flavors) compare with the cross-section for $e^+e^- \rightarrow \mu^+\mu^-$ at this energy?

For the down-like flavours (*d*, *s*, *b*), the cross-sections will each be approximately $\frac{\alpha^2\pi}{9E_e^2}$, whereas for the up-like flavours (*u*, *c*) the cross-sections will be approximately $\frac{4\alpha^2\pi}{9E_e^2}$. The top is too heavy to be produced at this energy. Thus there is a total hadronic cross-section of approximately $\frac{11\alpha^2\pi}{9E_e^2}$. This is $\frac{11}{3} = 3.67$ times the di-muon cross-section, $\frac{\alpha^2\pi}{3E_e^2}$.