P424 Assignment 8 SOLUTIONS

1 The matrix element for the decay $H \to f\bar{f}$ of the Higgs boson H to a fermion (quark or lepton) f and its antiparticle $\bar{f}$ is

$$M_{fi} = \frac{g_W m_f}{2m_W} u(p_2) v(p_3)$$

where g_W is the weak decay constant, m_W is the mass of the $W^{\pm}$ boson, m_f is the mass of the fermion f, and p_2 and p_3 are the 4-momenta of f and $\bar{f}$, respectively.

In the rest frame of the Higgs boson, with the fermion travelling in the $+z$ direction, show that $u_i(p_2)v_j(p_3)$ $(i, j = 1, 2)$ is non-zero for only two of the four possible combinations and interpret this result.

Given that the decay rate is

$$\Gamma = \frac{p^*}{8\pi m_H^2} \langle |M_{fi}|^2 \rangle$$

where p^* is the centre of mass momentum of either final state particle, show that

$$\Gamma = N_c \frac{G_F}{\sqrt{2}} \frac{m_f^2 m_H}{4\pi} \left(1 - \frac{4m_f^2}{m_H^2}\right)^{3/2}$$

where N_c is the number of colour degrees of freedom of the fermion f.

List all possible decays of the type $H \to f\bar{f}$ for a Higgs mass $m_H = 100$ GeV. Neglecting the (small) contributions from other types of decay, and taking suitable approximate values for fermion masses where needed, estimate the total width and lifetime of a Higgs boson of this mass, and the three largest branching ratios to $f\bar{f}$ final states.

Draw a leading order Feynman diagram for the Higgs boson production process $e^+e^- \to HZ^0$ and describe the various types of final state which result once the H and Z^0 bosons have decayed. Explain how events of this type can be identified and distinguished from events due to non-Higgs background processes, indicating the detection techniques which are important in this respect.

Summarise the current experimental knowledge of the Higgs boson mass, both from direct searches for Higgs boson production and from indirect approaches.

State the Higgs boson decay modes which are expected to be of importance for the experimental detection of Higgs bosons at future hadron colliders, and draw a Feynman diagram illustrating a possible Higgs boson production mechanism at such a machine.

[You may require the following information:
$G_F = 1.166 \times 10^{-5}\,\text{GeV}^{-2}$, $G_F/\sqrt{2} = g_W^2/8m_W^2$.
$\hbar = 6.582 \times 10^{-25}$ GeV.s.

The Higgs is a scalar (spin 0), so it can only decay to a spin-up + spin-down, or spin-down + spin-up, combination in order to conserve angular momentum, thus only two of the four combinations are allowed.
We have the free particle solutions to the Dirac equation:

$$u_1 = \sqrt{E+m}\begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \end{pmatrix}, \quad u_2 = \sqrt{E+m}\begin{pmatrix} 0 \\ 1 \\ \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix}$$, and the free antiparticle solutions

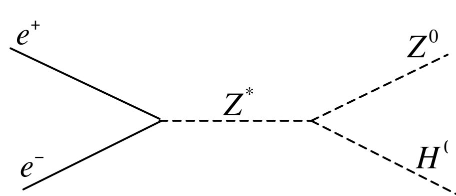
$$v_1 = \sqrt{E+m}\begin{pmatrix} \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \\ 0 \\ 1 \end{pmatrix}, \quad v_2 = \sqrt{E+m}\begin{pmatrix} \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \\ 1 \\ 0 \end{pmatrix}.$$ Thus $u(p_2)v(p_3) = (E_f + m_f)\left(\frac{2p^*}{E_f + m_f}\right) =$

$2p^* \quad \Rightarrow \quad \mathcal{M}_{fi} = \frac{g_W m_f p^*}{m_W} \quad \Rightarrow \quad \Gamma = \frac{g_W^2 m_f^2 (p^*)^3}{8\pi m_H^2 m_W^2} \times 2\,\text{spin states} \times N_C\,\text{colour states} =$

$N_C \frac{2G_F m_f^2 (p^*)^3}{\pi\sqrt{2} m_H^2}$. But $E^* = {m_H}/{2} \quad \Rightarrow \quad (E^*)^2 = (p^*)^2 + m_f^2 = {m_H^2}/{4} \quad \Rightarrow$

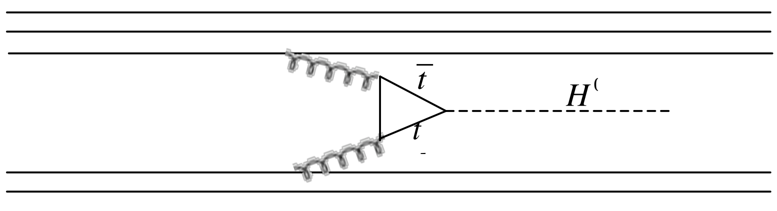
$$p^* = \left({m_H^2}/{4} - m_f^2\right)^{1/2} \quad \Rightarrow \quad \Gamma = N_c \frac{2G_F m_f^2}{\sqrt{2}\pi m_H^2}\left({m_H^2}/{4} - m_f^2\right)^{3/2} = N_c \frac{G_F m_f^2 m_H}{4\sqrt{2}\pi}\left(1 - {4m_f^2}/{m_H^2}\right)^{3/2}.$$

A 100 GeV $H \to f\bar{f}$ decay could have any of $\{e,\mu,\tau,u,d,s,c,b\}$ as the fermion. The top quark is forbidden. Neutrinos would be an almost infinitesimally small contribution. The largest of these will be *b*, then tau, then charm, as the Higgs couples to mass. From the formula above, and with $m_b \approx 5\,\text{GeV}$, $m_\tau \approx m_c \approx 1.8\,\text{GeV}$, the total width of the Higgs will be about 6.4 MeV, implying a lifetime of
$\left(6.4\times10^6 \cdot 1.6\times10^{-19}\right)^{-1} \cdot 1.055\times10^{-34} =$ 1.0 x 10^{-22} s.
A leading-order Feynman diagram for $e^+e^- \to HZ^0$ is the "Higgsstrahlung" diagram:



From this, one could end up with 2 leptons + 2 jets, 6 leptons, 4 leptons + missing E_T, 2 leptons + 2 photons, etc. Superb tracking momentum resolution (as well as very good energy resolution in the calorimeter) is the key to both separating these decays from background processes such as *Z* pairs as well as extracting the Higgs mass and other parameters from these decays.
From LEP direct searches, the standard model Higgs boson must have mass greater than 113.5 GeV at 95% CL. From indirect constraints from the top and *W* masses (measured

at the Tevatron and at LEP), a standard model Higgs should have mass below approximately 200 GeV at 95% CL.
At hadron colliders, the decay modes most relevant for standard model Higgs searches are, for a low mass, less than approximately 130 GeV Higgs: $H \to \gamma\gamma$, and for a higher mass Higgs: $H \to WW \to \ell\nu\ell\nu$ and $H \to ZZ \to \ell^+\ell^-\ell^+\ell^-$. The primary Higgs production mechanism at hadron colliders, especially proton-proton colliders, is gluon-gluon fusion:



2. In the CHOOZ experiment, a neutrino detector was positioned a distance $L \approx 1\,\text{km}$ from a nuclear reactor emitting neutrinos (actually antineutrinos) of mean energy $E \approx 3\,\text{MeV}$. The number of neutrino interactions observed was consistent with the number expected assuming no neutrino oscillations, giving the result $P(\nu_e \to \nu_e) = 1.01 \pm 0.04$.

a) Show that neutrino oscillations associated with the (solar) mass-squared difference $|\Delta m_{12}^2| \approx 7 \times 10^{-5}\,\text{eV}^2$ can be neglected for the CHOOZ experiment, and that

$$P(\nu_e \to \nu_e) \approx 1 - \sin^2 2\theta_{13} \sin^2 \Delta_{23}$$

where

$$\Delta_{23} \equiv \frac{\Delta m_{23}^2 L}{4E}\,.$$

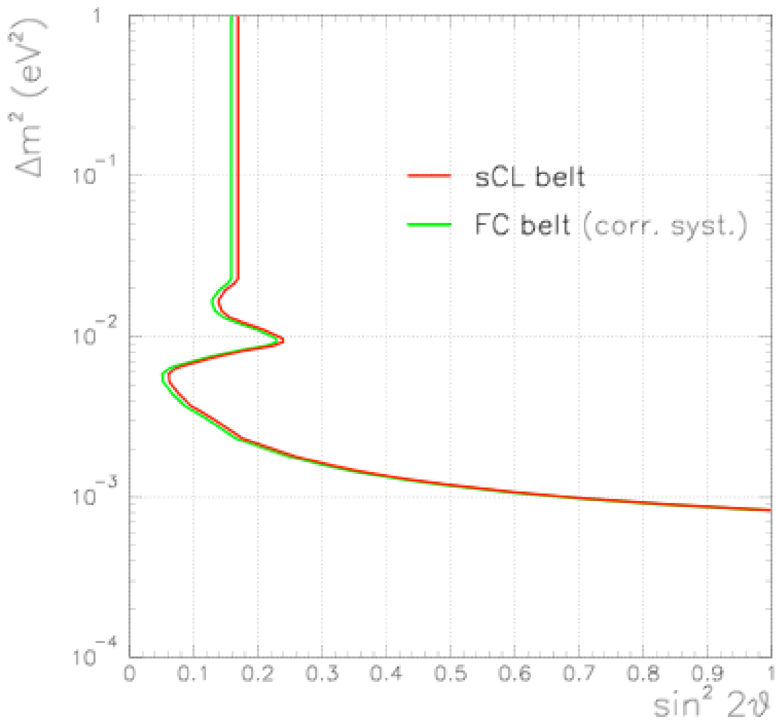
b) In the limit $|\Delta m_{23}^2| \gg (E/L)$, explain why a given measurement, P, of the survival probability $P(\nu_e \to \nu_e)$ determines the neutrino mixing to be $\sin^2 2\theta_{13} = 2(1-P)$.

c) In the limit $|\Delta m_{23}^2| \ll (E/L)$, show that a given measurement, P, of the survival probability $P(\nu_e \to \nu_e)$ determines the neutrino mixing to be $\sin^2 2\theta_{13} \propto 1/(\Delta m_{23}^2)^2$, with constant of proportionality $(1-P)(4E/L)^2$.

d) The null result from the CHOOZ experiment, $P(\overline{\nu}_e \to \overline{\nu}_e) = 1.01 \pm 0.04$, can be used to exclude a region of the $(\sin^2 2\theta_{13}, \Delta m_{23}^2)$ parameter space. This is conventionally presented as the region which can be excluded at "90% Confidence Level", which for the CHOOZ measurement encompasses all values of $(\sin^2 2\theta_{13}, \Delta m_{23}^2)$ which would give a survival probability $P(\nu_e \to \nu_e) < 0.92$ (which is about twice the rms precision of the measurement below unity: $P < 1 - 2 \times 0.04$). In the plot overleaf, published by CHOOZ, the curves correspond approximately to the contour $P = 0.92$ and the excluded region lies above and to the right of the curves. [The two similar curves correspond to slightly different statistical approaches to the analysis of the data.]

Use the results derived above to justify approximately the shape and position of the exclusion contour in the regions close to the intercepts with the upper horizontal and right-hand vertical axes. Give a qualitative explanation of the shape of the remainder of the contour.

Evaluate the survival probability $P(\nu_e \to \nu_e)$ for some representative points lying on either side of the contour, specifically for $(\sin^2 2\theta_{13}, \Delta m_{23}^2) = (0.5, 5 \times 10^{-4}\,\text{eV}^2)$ and $(0.5, 5 \times 10^{-3}\,\text{eV}^2)$.



e) Experiments studying atmospheric neutrino oscillations indicate a mass-squared splitting in the range $|\Delta m^2_{23}| \approx 2 - 3 \times 10^{-3}\,\mathrm{eV}^2$. What constraint can now be placed on the angle θ_{13} ?

a. In class (in the lecture notes) we showed that $P(\nu_e \to \nu_e) = 1 - 4|U_{e1}|^2|U_{e2}|^2 \sin^2\Delta_{12} - 4|U_{e1}|^2|U_{e3}|^2 \sin^2\Delta_{13} - 4|U_{e2}|^2|U_{e3}|^2 \sin^2\Delta_{23}$. We have $\Delta_{12} = \dfrac{\Delta m^2_{12} L}{4E} = 1.27\dfrac{7\times 10^{-5}\times 1}{0.003} = 0.03$, so the $\sin^2\Delta_{12}$ term is negligible. Also, we know that $\Delta m^2_{13} \simeq \Delta m^2_{23}$, so $\Delta_{13} \simeq \Delta_{23}$, thus $P(\nu_e \to \nu_e) \simeq 1 - 4|U_{e3}|^2\left(|U_{e1}|^2 + |U_{e2}|^2\right)\sin^2\Delta_{23}$ $= 1 - 4\cos^2\theta_{13}\sin^2\theta_{13}(\sin^2\theta_{12} + \cos^2\theta_{12})\sin^2\Delta_{23} = 1 - \sin^2 2\theta_{13}\sin^2\Delta_{23}$.

b. If $|\Delta m^2_{23}| \gg E/L$, the $\sin^2\Delta_{23}$ term will just average out to 1/2 (the detector itself will likely span multiple oscillations of the sin term), and thus we end up with $\sin^2 2\theta_{23} = 2(1-P)$.

c. If $|\Delta m^2_{23}| \ll E/L$, then $\sin\Delta_{23} \approx \Delta_{23} \quad\Rightarrow\quad \sin^2\Delta_{23} \approx \Delta^2_{23} \quad\Rightarrow$

$$P = 1 - \left(\frac{L}{4E}\right)^2 \left(\Delta m^2_{23}\right)^2 \sin^2 2\theta_{13} \quad\Rightarrow\quad \sin^2 2\theta_{13} = (1-P)\left(\frac{4E}{L}\right)^2 \left(\frac{1}{\Delta m^2_{23}}\right)^2.$$

d. Near the top of the plot, $\sin^2 2\theta_{23}$ is independent of Δm_{23}^2, which is consistent with the answer to part b. above (with the measured *P* being approximately 0.92). On the lower right side of the plot, the curve becomes nearly linear, which (with the y-axis being a log scale) is consistent with a power law relationship between $\sin^2 2\theta_{23}$ and Δm_{23}^2 (in particular $\sin^2 2\theta_{13} \propto \left(\frac{1}{\Delta m_{23}^2}\right)^2$). In between, there are oscillations, consistent with a trigonometric relationship $\sin^2 2\theta_{13} = (1-P)\sin^2\left(\frac{4E}{L\Delta m_{23}^2}\right)$.

e. From the plot above, looking only at the range $\left|\Delta m_{23}^2\right| \approx 2-3\times 10^{-3}\,\text{eV}^2$, we have that $\sin^2 2\theta_{13} < \sim 0.2$. That implies that $\theta_{13} < 0.2$ or so.