"But don't you see, Gershon - if the particle is too small and too short-lived to detect, we can't just take it on faith that you've discovered it."

Tuesday Recap

- The Lagrangian formulation of mechanics is the best way to study QFT.
- All Standard Model interactions can be obtained from the requirement of local gauge invariance under various symmetries applied to the (fermionic) matter fields.
- The resulting gauge bosons must be massless.
- The Feynman rules can be obtained directly from the Lagrangian:
  1. Free Lagrangian $\Rightarrow$ Propagators
  2. Interaction Terms $\Rightarrow$ Vertex Factors

Today: The Origin of Mass

- The Mass Term in $\mathcal{L}$
- Spontaneous Symmetry Breaking
- The Higgs Mechanism

The Mass Term

- Recall that the Lagrangian for a free spin-0 particle is

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \tfrac{1}{2}m^2\phi^2$$

- The first term, containing derivatives of the field, is the kinetic term.
- The second term, without any derivatives, is the mass term.
- Both the kinetic and the mass terms are quadratic in ϕ.
- These points are straightforward, yet important, as oftentimes the mass term is hiding.

Strange Mass Terms: Example 1

- Consider the Lagrangian

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + e^{-(\alpha\phi)^2}$$

  At first glance, it looks like there is no mass term.

- Expanding the exponential, we find that

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \left[1 - \alpha^2\phi^2 + \tfrac{1}{2}\alpha^4\phi^4 - \tfrac{1}{6}\alpha^6\phi^6 + \ldots\right]$$

- Ignoring the 1, we see that there *is* a mass term, along with some higher-order couplings. In particular the mass can be determined from

$$-\alpha^2\phi^2 = -\tfrac{1}{2}m^2\phi^2 \qquad \Rightarrow \qquad m = \sqrt{2}\alpha$$

Strange Mass Terms: Example 2

- Now consider the Lagrangian

  $$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \tfrac{1}{2}\mu^2\phi^2 - \tfrac{1}{4}\lambda^2\phi^4 \qquad \text{(for real } \mu \text{ and } \lambda)$$

  We don't have to expand anything to find the ϕ^2 term, but upon comparison with $-\frac{1}{2}m^2\phi^2$, it looks like the mass is imaginary!

- To sort this out, we need to go back to the construction of a Lagrangian from

  $$\mathcal{L} = \mathcal{T} - \mathcal{U}$$

  In this case our potential term is

  $$\mathcal{U} = -\tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{4}\lambda^2\phi^4$$

Finding the Ground State

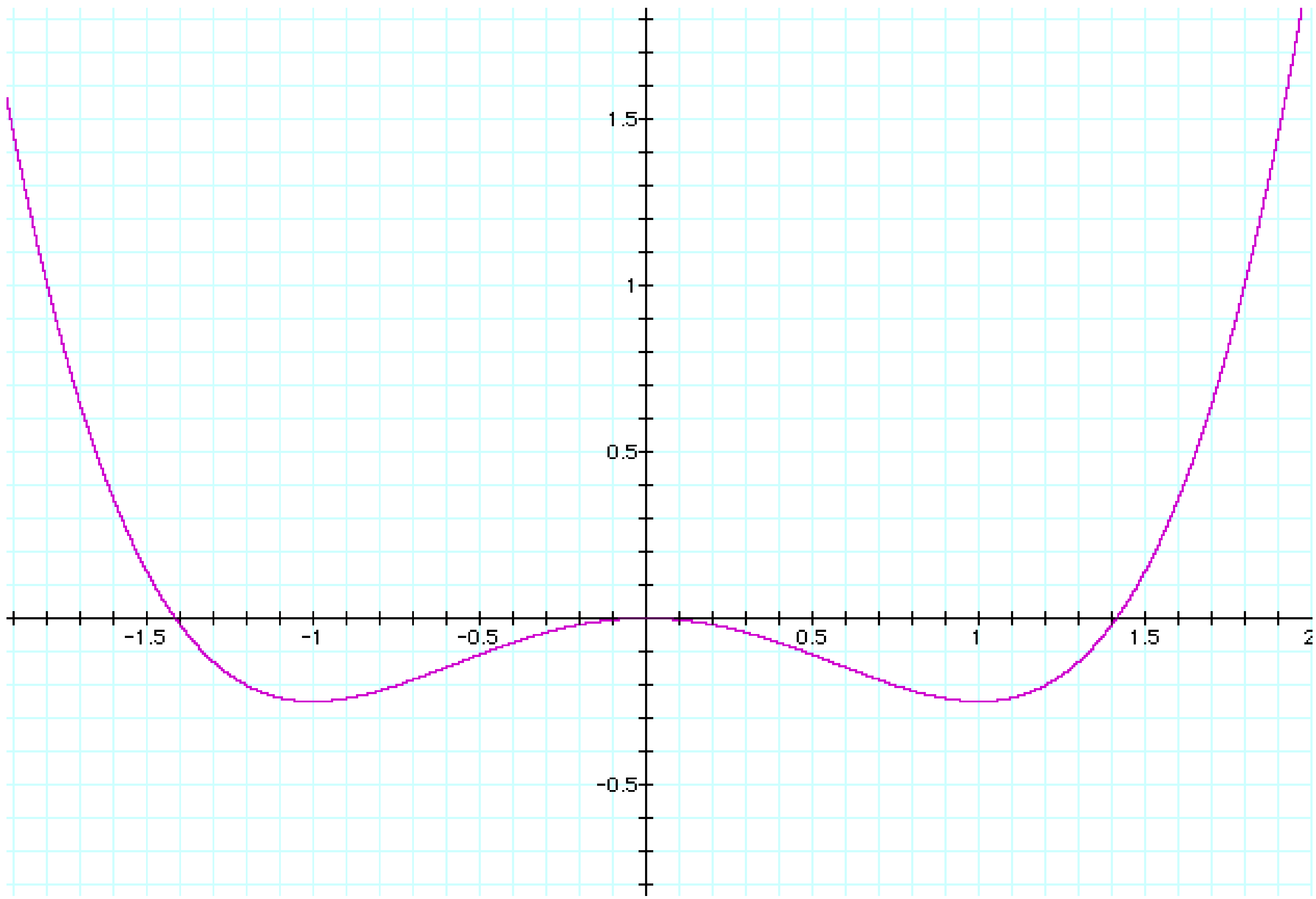
- Perturbative QFT means that we start with the vacuum and then add a couple of particles. This means that our physical fields represent displacements *away from* the vacuum. If the vacuum is at ϕ_0, the physical degree of freedom is not ϕ, but $\phi - \phi_0$.
- We determine the vacuum by minimizing $\mathcal{U}$. For

$$\mathcal{U} = -\tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{4}\lambda^2\phi^4$$

we find that the minimum occurs at

$$\phi_0 = \pm\mu/\lambda$$

$\mathcal{U} = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda^2\phi^4$ ($\mu = 1$ and $\lambda = 1$ below)



Expanding About the Minimum

- Define a new (physical) field variable

$$\eta \equiv \phi \pm \mu/\lambda$$

- In terms of η, the Lagrangian becomes

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) + \tfrac{1}{2}\mu^2\phi^2 - \tfrac{1}{4}\lambda^2\phi^4 \\ &= \tfrac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) + \tfrac{1}{2}\mu^2(\eta \mp \mu/\lambda)^2 - \tfrac{1}{4}\lambda^2(\eta \mp \mu/\lambda)^4 \\ &= \tfrac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) + \tfrac{1}{4}\mu^4/\lambda^2 - \mu^2\eta^2 \pm \mu\lambda\eta^3 - \tfrac{1}{4}\lambda^2\eta^4\end{aligned}$$

- We see that there is a sensible mass of $m = \sqrt{2}\mu$, along with 3- and 4-particle self-coupling interactions.

Spontaneous Symmetry Breaking

- In the previous example, our Lagrangian was symmetric under $\phi \to -\phi$. Once we expanded about either of the 2 vacuum states using the physical field η, this symmetry was lost.
- When the vacuum does not share the same symmetry as the Lagrangian, we call this *spontaneous symmetry breaking* (SSB).
- The true symmetry of such a system is *hidden* by the arbitrary selection of a particular asymmetrical ground state.
- Classic example of SSB: ferromagnets.

SSB of Continuous Symmetries

- In the previous example, we had one spin-0 field and 2 degenerate vacuum states. Things become more interesting when we look at continuous symmetries.

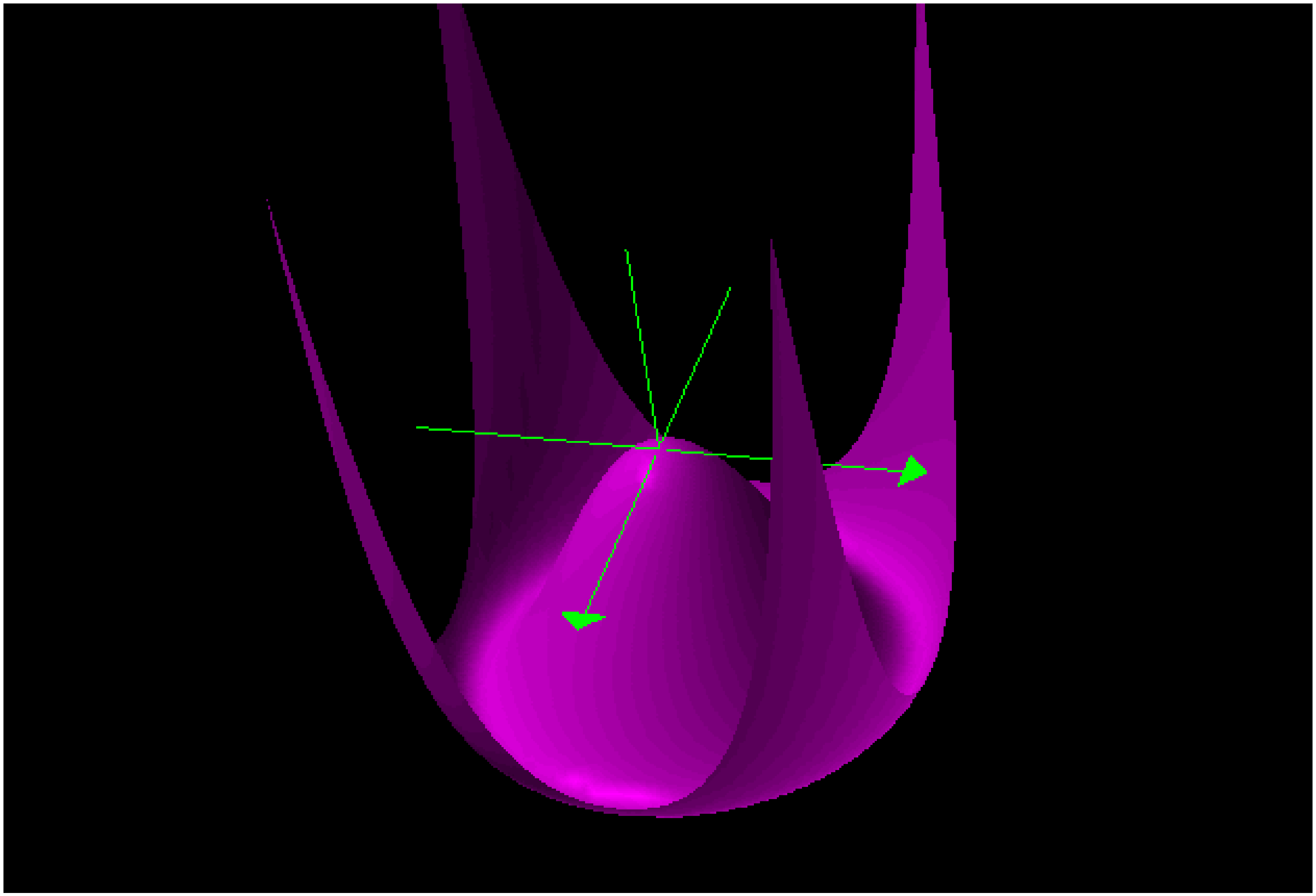
- Consider a Lagrangian with two scalar fields:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi_1)(\partial^\mu\phi_1)+\frac{1}{2}(\partial_\mu\phi_2)(\partial^\mu\phi_2)+\frac{1}{2}\mu^2(\phi_1^2+\phi_2^2)-\frac{1}{4}\lambda^2(\phi_1^2+\phi_2^2)^2$$

This Lagrangian is symmetric under $(SO(2))$ rotations in ϕ_1-ϕ_2 space, i.e.

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

$\mathcal{U} = -\frac{1}{2}\mu^2(\phi_1^2+\phi_2^2)+\frac{1}{4}\lambda^2(\phi_1^2+\phi_2^2)^2$ ($\mu = \sqrt{3}$ and $\lambda = 1$ below)



Expanding About the Minimum

- Now there is an entire circle of minima:

$$\phi_1^2 + \phi_2^2 = \mu^2/\lambda^2$$

- To proceed further, we have to choose a particular ground state and then define our physical fields as expansions away from this ground state. If we choose

$$(\phi_1)_{min} = \mu/\lambda \qquad (\phi_2)_{min} = 0$$

then our physical fields will be

$$\eta \equiv \phi_1 - \mu/\lambda \qquad \xi \equiv \phi_2$$

The New Lagrangian

- Substituting for ϕ_1 and ϕ_2 in terms of η and ξ, it is straightforward (but tedious) to show that the Lagrangian becomes

$$\begin{aligned}\mathcal{L} &= \left[\tfrac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \mu^2\eta^2\right] + \left[\tfrac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi)\right] \\ &\quad + \mu^4/(4\lambda^2) + \mu\lambda(\eta^3 + \eta\xi^2) - \tfrac{1}{4}\lambda^2(\eta^4 + \xi^4 + 2\eta^2\xi^2)\end{aligned}$$

- η is evidently a scalar field with mass $m_\eta = \sqrt{2}\mu$, corresponding to radial oscillations up and down the ridges near the ground state.
- ξ is a massless scalar field, corresponding to motion along the ring of minima.
- The Lagrangian also contains various 3- and 4-particle interactions involving η and ξ.

Goldstone's Theorem

- It is not an accident that ξ turned out to be massless.
- *Goldstone's theorem* asserts that we will *always* get one or more massless scalars when we spontaneously break a continuous symmetry.
- We call these massless scalars *Goldstone bosons*.
- The closest thing we have to a massless scalar particle in nature is the π. How is it, then, that spontaneous breaking of the electroweak symmetry is going to provide masses to the $W^{\pm}$ and Z^0 without leaving behind a Goldstone boson for us to detect?

Changing the Notation

- In our previous example, rather than starting with 2 separate *real* scalar fields ϕ_1 and ϕ_2, we can create 1 *complex* scalar field:

$$\phi \equiv \phi_1 + i\phi_2$$

so that the initial Lagrangian can be written as

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^*(\partial^\mu \phi) + \frac{1}{2}\mu^2(\phi^*\phi) - \frac{1}{4}\lambda^2(\phi^*\phi)^2$$

- This substitution means that the rotational $SO(2)$ symmetry of ϕ_1 and ϕ_2 becomes a $U(1)$ symmetry of ϕ:

$$\phi \longrightarrow e^{i\theta}\phi$$

This is precisely the same symmetry which we *gauged* (i.e., promoted the symmetry from global to local) yesterday to obtain QED.

Gauging the Symmetry

- We promote the global $U(1)$ invariance of the Lagrangian to a local invariance by introducing a massless gauge field A_μ via the covariant derivative:

$$\mathcal{D}_\mu = \partial_\mu + iqA_\mu$$

- The Lagrangian is then

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{2}\left[(\partial_\mu - iqA_\mu)\phi^*\right]\left[(\partial^\mu + iqA^\mu)\phi\right] \\ &\quad +\tfrac{1}{2}\mu^2(\phi^*\phi) - \tfrac{1}{4}\lambda^2(\phi^*\phi)^2 - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}\end{aligned}$$

- Next, we spontaneously break the symmetry by picking out a vacuum state. This gives rise, as before, to the fields

$$\eta \equiv \phi_1 - \mu/\lambda \qquad \xi \equiv \phi_2$$

After a LOT of Algebra...

$$\begin{aligned}\mathcal{L} \;=\; & \left[\tfrac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \mu^2\eta^2\right] + \left[\tfrac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi)\right] \\ & + \left[-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}(q\mu/\lambda)^2 A_\mu A^\mu\right] - 2i(q\mu/\lambda)(\partial_\mu\xi)A^\mu \\ & +(\mu^2/2\lambda)^2 + q[\eta(\partial_\mu\xi) - \xi(\partial_\mu\eta)]A^\mu + (\mu q^2/\lambda)\eta A_\mu A^\mu \\ & +\tfrac{1}{2}q^2(\xi^2+\eta^2)A_\mu A^\mu - \lambda\mu(\eta^3+\eta\xi^2) - \tfrac{1}{4}\lambda^2(\eta^4+2\eta^2\xi^2+\xi^4)\end{aligned}$$

- As before the SSB leads to a massive scalar (η) and a massless Goldstone boson (ξ), along with a pile of 3- and 4-particle interactions.
- Wait a minute... the massless gauge boson has picked up a mass! And what's that $(\partial_\mu\xi)A^\mu$ term supposed to mean?

Getting Rid of the $(\partial_\mu \xi) A^\mu$ Term

- Whenever we see a bilinear term like $(\partial_\mu \xi) A^\mu$ involving 2 different fields, it means that we have incorrectly identified the physical particles in the theory.
- How do we fix this? Since $\mathcal{L}$ is gauge invariant, let's perform a gauge transformation to make $\xi = 0$ (i.e., ϕ real). Then

$$\begin{aligned}\mathcal{L} &= \left[\tfrac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \mu^2 \eta^2\right] \\ &\quad + \left[-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \tfrac{1}{2}(q\mu/\lambda)^2 A_\mu A^\mu\right] \\ &\quad + (\mu^2/2\lambda)^2 + (\mu q^2/\lambda)\eta A_\mu A^\mu \\ &\quad + \tfrac{1}{2} q^2 \eta^2 A_\mu A^\mu - \lambda\mu\eta^3 - \tfrac{1}{4}\lambda^2\eta^4\end{aligned}$$

The gauge boson (A) is still massive and the Goldstone boson (ξ) is gone!

The Higgs Mechanism

1. Start with at least a couple of scalar fields related by a continuous symmetry.
2. Promote the symmetry from global to local (i.e., "gauge it") by introducing one or more massless gauge bosons.
3. Spontaneously break the symmetry by choosing a particular ground state, about which the symmetry is not manifest.
4. Rewrite the Lagrangian in terms of fields centered on the ground state.
5. Use the symmetry to eliminate (i.e. "gauge away") the Goldstone bosons.
6. Voila! The gauge bosons have a mass. The surviving scalar fields are massive *Higgs bosons*.

Counting the Degrees of Freedom

- Before we gauge away the ξ, we have

Particle	Spin	Mass?	# of D.O.F.
η	0	Yes	1
ξ	0	No	1
A^μ	1	No	2

- After we gauge away the ξ, we have

Particle	Spin	Mass?	# of D.O.F.
η	0	Yes	1
A^μ	1	Yes	3

- Either way, there are 4 degrees of freedom.

The Higgs Mechanism in the Standard Model

- We have 3 gauge bosons (the W^+, W^-, and Z^0) to provide masses to, therefore we will need to create 3 Goldstone bosons.
- The simplest way to do this is to start with a *complex scalar Higgs doublet*:

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$$

  These are not physical particles at this stage; they correspond to the ϕ fields.
- After SSB, only the real part of

$$h^0 = H^0 - v/\sqrt{2}$$

  survives. This is the Higgs boson.

Counting the Degrees of Freedom

- Before we gauge away the Goldstone bosons, we have

Particle	Spin	Mass?	# of D.O.F.
H doublet	0	Yes/No	$4\times1=4$
W_μ^i	1	No	$3\times2=6$

- After we gauge away the Goldstone bosons, we have

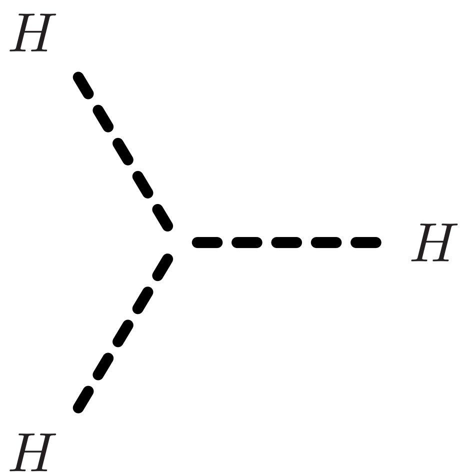
Particle	Spin	Mass?	# of D.O.F.
$\mathrm{Re}\, h^0$	0	Yes	1
W^+, W^-, Z^0	1	Yes	$3\times3=9$

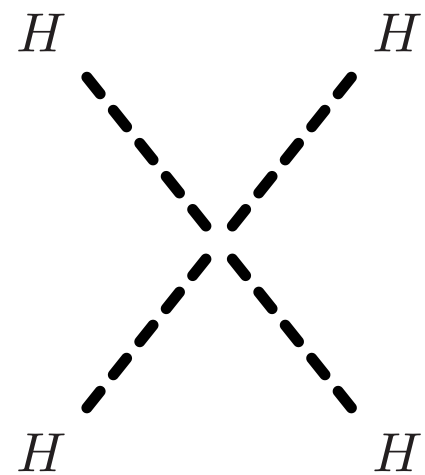
- Either way, there are 10 degrees of freedom.

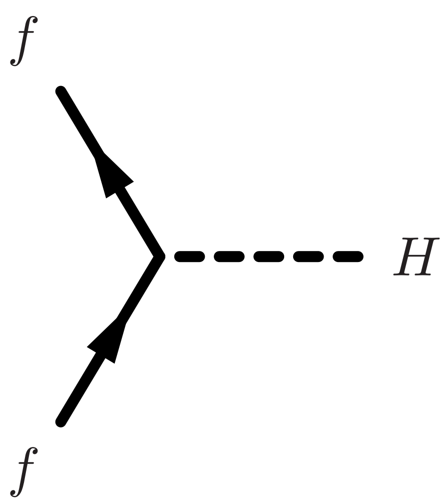
The Standard Model Higgs

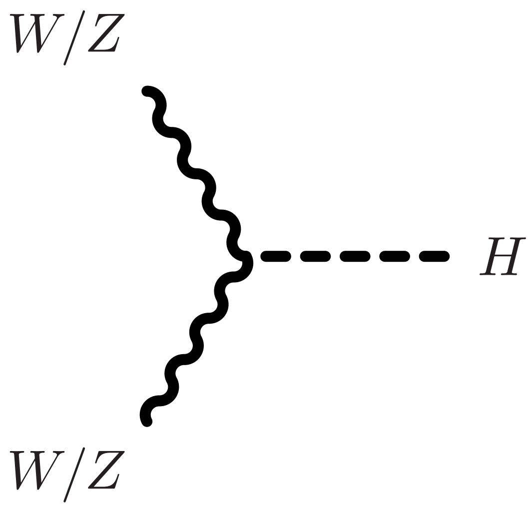
- We usually assume that the Higgs mechanism arises from a complex scalar doublet (more complicated arrangements are also possible).
- This leads to 1 massive Higgs boson.
- The Higgs couples to *every* massive particle in the Standard Model. In fact, in more sophisticated formulations of the Standard Model, all mass terms are generated by *Yukawa couplings* to the Higgs.

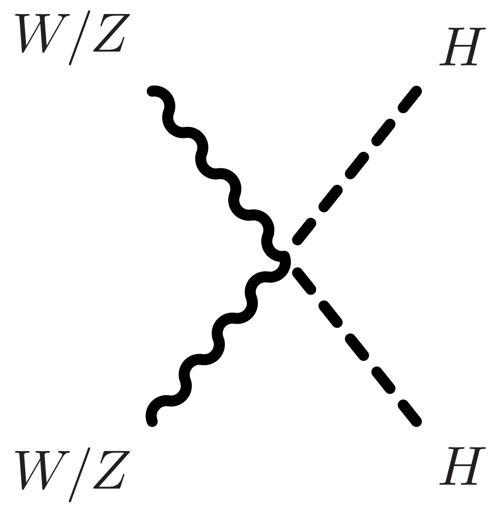
Higgs Boson Vertices









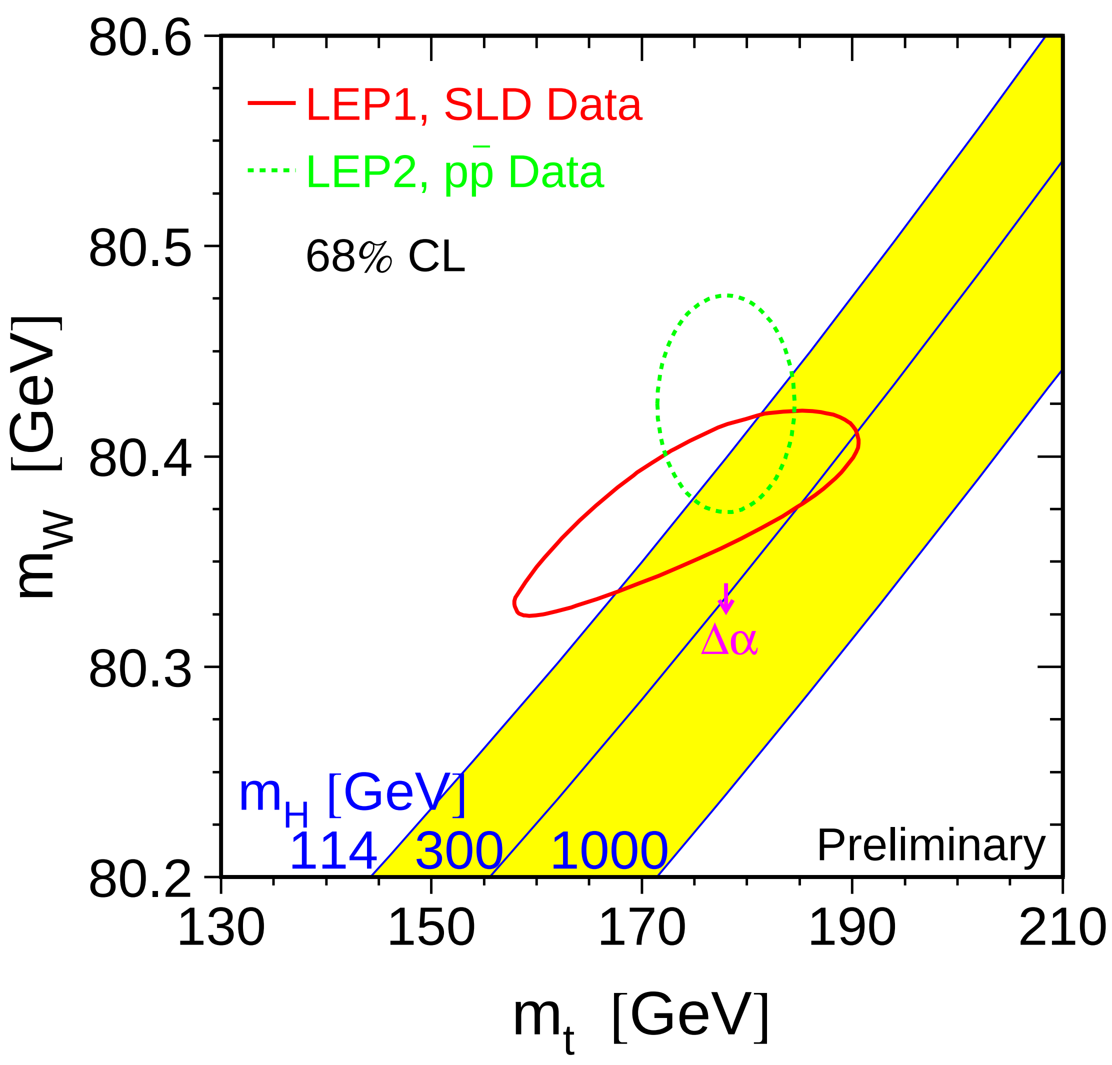


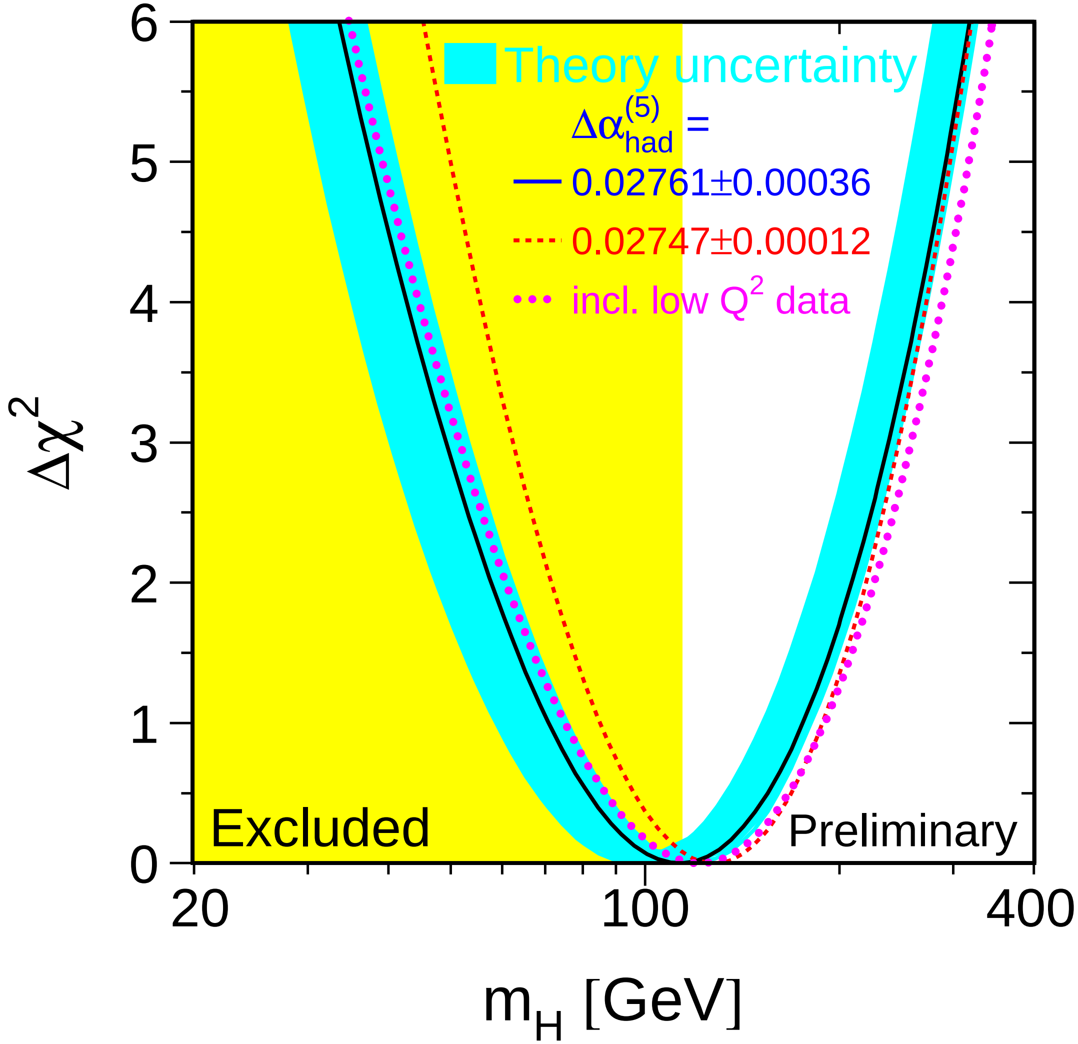
What Makes Us So Sure the Higgs Exists?

- One word: *unitarity*.
- Just as we inferred the existence of the $W^{\pm}$ and the Z^0 based on the pathological high-energy behavior of certain scattering cross sections, we find that high-energy divergences in $W^+ W^- \to W^+ W^-$ scattering are cured by the Higgs boson.
- Technically, this doesn't mean that the disease *has to* be cured by the Higgs boson, but there had better be *something* new before 1 TeV. The Higgs just happens to be the "simplest something new".

Constraints on m_H

- The mass of the Higgs is $m_H = v\sqrt{\lambda/2}$ where $v = 247$ GeV is fixed by G_F and λ is an unknown dimensionless coupling constant.
- m_H can't be too large, lest we violate *unitarity*.
- m_H can't be too small, lest the weak vacuum become unstable (i.e., an even lower-energy state exists elsewhere).
- Even at energies below m_H, the Higgs appears in Standard Model loop diagrams. This allows us to infer the most likely mass of the Higgs (as we'll soon see).
- In a nutshell, the Standard Model Higgs has a mass somewhere between 115 GeV (where LEP stopped looking) and about 200 GeV. LHC will soon sort this out.


80.6
80.5
80.4
80.3
80.2
130
150
170
190
210
m_W [GeV]
m_t [GeV]
LEP1, SLD Data
LEP2, $p\bar{p}$ Data
68% CL
$\Delta\alpha$
m_H [GeV]
114
300
1000
Preliminary


6
5
4
3
2
1
0
Theory uncertainty
$\Delta\alpha^{(5)}_{had}$ =
0.02761±0.00036
0.02747±0.00012
incl. low Q^2 data
$\Delta\chi^2$
Excluded
Preliminary
20
100
400
m_H [GeV]

Summary

- Perturbative QFT requires that we expand away from the ground state.
- Spontaneous Symmetry Breaking implies that the ground state does not share the same symmetries as the Lagrangian.
- When we spontaneously break a continuous symmetry, we obtain one or more massless Goldstone bosons.
- When we combine local gauge invariance and SSB, we get the Higgs mechanism, whereby the gauge bosons acquire mass by "eating" the Goldstone bosons.
- The Standard Model contains 1 Higgs boson and (knock on wood) it will be found soon at the LHC.

Tuesday: Beyond the Standard Model

- Neutrino Oscillations
- Grand Unification