Abelian Higgs Model

- Why are the W and Z boson masses non-zero?
- U(1) gauge theory with single spin-1gauge field, A_μ

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$$

- U(1) local gauge invariance:

$$A_\mu(x) \to A_\mu(x) - \partial_\mu \eta(x)$$

- Mass term for A:

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu$$

- Mass term violates local gauge invariance
- We understand why $M_A = 0$

Gauge invariance is guiding principle

Abelian Higgs Model, 2

- Add complex scalar field, φ, with charge –e:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 - V(\phi)$$

- Where $D_\mu = \partial_\mu - ieA_\mu$ $\quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$$V(\phi) = \mu^2|\phi|^2 + \lambda(|\phi|^2)^2$$

Most general potential invariant under φ→-ϕ

- L is invariant under transformations: $A_\mu(x) \to A_\mu(x) - \partial_\mu\eta(x)$

$$\phi(x) \to e^{-ie\eta(x)}\phi(x)$$

- Case 1: $\mu^2 > 0$
  - QED with $M_A=0$ and $m_\varphi=\mu$
  - Unique minimum at $\varphi=0$

Abelian Higgs Model, 3

- Case 2: $\mu^2 < 0$ $V(\phi) = -|\mu^2||\phi|^2 + \lambda(|\phi|^2)^2$
- Minimum energy state at: $<\phi> = \sqrt{-\frac{\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$
- Rewrite $\phi \equiv \frac{1}{\sqrt{2}} e^{i\frac{\chi}{v}}(v+h)$
- L becomes: $L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - evA_\mu\partial^\mu\chi + \frac{e^2v^2}{2}A^\mu A_\mu + \frac{1}{2}(\partial_\mu h\partial^\mu h + 2\mu^2 h^2) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + (h, \chi \text{ interactions})$
- Theory now has:
  - Photon of mass M_A=ev
  - Scalar field h with mass-squared $-2\mu^2 > 0$
  - Massless scalar field χ

Choosing a vacuum breaks U(1) symmetry

- What about mixed χ-A propagator?
  - Remove by gauge transformation

$$A'_\mu \equiv A_\mu - \frac{1}{ev}\partial_\mu\chi$$

$$\phi' \equiv e^{-i\frac{\chi}{v}}\phi = \frac{v+h}{\sqrt{2}}$$

- χ field disappears
  - We say that it has been eaten to give the photon mass
  - χ field called Goldstone boson

Higgs Mechanism summarized

Spontaneous breaking of a gauge theory by a non-zero VEV results in the disappearance of a Goldstone boson and its transformation into the longitudinal component of a massive gauge boson

What about gauge invariance? Choice above called unitary gauge

- No χ field
- Bad high energy behavior of A propagator: $\Delta_{\mu\nu}(k) = -\frac{i}{k^2 - M_A^2}\left(g_{\mu\nu} - \frac{k^\mu k^\nu}{M_A^2}\right)$
- R_ξ gauges more convenient: $L_{GF} = -(1/2\xi)(\partial_\mu A^\mu + \xi e v \chi)^2$
- χ field is part of spectrum with $M_\chi^2 = M_A^2$
- $\xi=1$: Feynman gauge with massive χ
- $\xi=0$: Landau gauge
  - χ massless with no coupling to physical Higgs boson

Aside on gauge boson counting

- Massless photon has 2 transverse degrees of freedom
  - $p_\gamma=(k,0,0,\vec{k})$
  - $\varepsilon_\pm=(0,1,\pm i,0)/\sqrt{2}$
- Massive gauge boson has 3 degrees of freedom (2 transverse, 1 longitudinal)
  - $\varepsilon_L=(|\vec{k}|,0,0,k_0)/M_V \rightarrow (k/M_V)+O(M_V/|\vec{k}|)$
- Count: Abelian Higgs Model
  - We start with: Massless gauge boson (2 dof), complex scalar (2 dof)
  - We end with: Massive gauge boson (3 dof), physical scalar (1 dof)

Non-Abelian Higgs Mechanism

- Vector fields $A^a_\mu(x)$ and scalar fields $\varphi_i(x)$

$$L = L_A + L_\varphi, \qquad L_\varphi = \frac{1}{2}\sum_i (D^\mu \phi_i)^2 - V(\phi), \qquad V(\phi) = \mu^2 \sum_i \phi^2 + \frac{\lambda}{2}\left(\sum_i \phi_i^2\right)^2$$

- L is invariant under the non-Abelian symmetry:

$$\phi_i \rightarrow (1 - \eta^a T^a)_{ij}\phi_j$$

- In exact analogy to the Abelian case

$$\frac{1}{2}(D^\mu \phi)^2 \rightarrow \ldots + \frac{1}{2} g^2 (T^a \phi)_i (T^b \phi)_i A^a_\mu A^{b\mu} + \ldots$$

$$\xrightarrow{\phi \rightarrow \phi_0} \ldots + \frac{1}{2} g^2 (T^a \phi_0)_i (T^b \phi_0)_i A^a_\mu A^{b\mu} + \ldots$$

- $T^a \varphi \neq 0$ $\Rightarrow$ Massive vector boson + Goldstone boson
- $T^a \varphi = 0$ $\Rightarrow$ Massless vector boson + massive scalar field

Goldstone theorem in general

- ***When a continuous symmetry is spontaneously broken (ie, it is not a symmetry of the physical vacuum), the theory has one massless scalar particle for each broken generator***
- Consider L with N_G real scalar fields ϕ

$$L = L_A + L_\phi, \qquad L_\phi = \frac{1}{2}\sum_i (D^\mu \phi_i)^2 - V(\phi), \qquad V(\phi) = \mu^2 \sum_i \phi^2 + \frac{\lambda}{2}\left(\sum_i \phi_i^2\right)^2$$

- G is a symmetry group such that

$$\delta\phi_i = -\eta^a T^a{}_{ij}\phi_j$$

- Since the potential is invariant under G

$$\delta V = \frac{\partial V}{\partial \phi_i}\delta\phi_i = -i\frac{\partial V}{\partial \phi_i}\eta^a (T^a)_{ij}\phi_j = 0$$

- Gauge parameters are arbitrary, giving N_G equations:

$$\frac{\partial V}{\partial \phi_i}(T^a)_{ij}\phi_j = 0 \qquad for\ a = 1 \ldots N_G$$

- Taking the second derivative:

$$\frac{\partial^2 V}{\partial \phi_i \partial \phi_j}(T^a)_{ik}\phi_k + \frac{\partial V}{\partial \phi_i}(T^a)_{ij} = 0 \qquad for\ a = 1 \ldots N_G$$

Vanishes at minimum

More on Goldstone Theorem

- At minimum of potential, $\varphi=\varphi_0$

$$\left.\frac{\partial^2 V}{\partial\phi_i\partial\phi_j}\right|_{\phi=\phi_0} (T^a)_{ik}\phi_{0k}=0 \qquad for\ \ a=1\ldots N_G$$

Scalar mass matrix

- After choosing ground state, a subgroup g of G with dimension n_g, remains symmetry of vacuum. For generators of g

$$(T^a)_{ik}\phi_{k0}=0 \qquad for\ \ a=1\ldots n_g$$

- For (N_G-n_g) generators that break the symmetry

$$(T^a)_{ik}\phi_{k0}\neq 0 \qquad for\ \ a=n_g+1\ldots N_G$$

- (N_G-n_g) zero eigenvalues of the mass matrix: the massless Goldstone bosons

Useful result for general model building

SM Higgs Mechanism

- Standard Model includes complex Higgs SU(2) doublet

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

- With SU(2) x U(1) invariant scalar potential

$$V = \mu^2 \Phi^+ \Phi + \lambda (\Phi^+ \Phi)^2 \qquad \text{Invariant under } \Phi \rightarrow - \Phi$$

- If $\mu^2 < 0$, then spontaneous symmetry breaking
- Minimum of potential at: $\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$
  - Choice of minimum breaks gauge symmetry
  - Why is $\mu^2 < 0$?

Aside: $\mu^2<0$ question is a motivation for mSUGRA

More on SM Higgs Mechanism

- Couple Φ to SU(2) x U(1) gauge bosons (W_i^μ, i=1,2,3; B^μ)

$$L_S = (D^\mu\Phi)^+(D^\mu\Phi) - V(\Phi)$$

$$D_\mu = \partial_\mu + i\frac{g}{2}\sigma^i W^i{}_\mu + i\frac{g'}{2}B_\mu$$

- Gauge boson mass terms from:

$$(D_\mu\phi)^+ D^\mu\phi \rightarrow ... + \frac{1}{8}(0,v)(gW_\mu^a\sigma^a + g'B_\mu)(gW^{b\mu}\sigma^b + g'B^\mu)\begin{pmatrix}0\\v\end{pmatrix} + ...$$

$$\rightarrow ... + \frac{v^2}{8}\left(g^2(W_\mu^1)^2 + g^2(W_\mu^2)^2 + (-gW_\mu^3 + g'B_\mu)^2\right) + ...$$

- With massive gauge bosons:

$W_\mu^\pm = (W_\mu^1 \pm W_\mu^2)/\sqrt{2}$

$Z_\mu^0 = (g\, W_\mu^3 - g' B_\mu)/\sqrt{(g^2+g'^2)}$

$M_W = gv/2$
$M_Z = \sqrt{(g^2+g'^2)}v/2$

- Orthogonal combination to Z is massless photon

$A_\mu^0 = (g'\, W_\mu^3 + gB_\mu)/\sqrt{(g^2+g'^2)}$

$M_W = M_Z \cos\theta_W$

- Weak mixing angle defined

$$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}} \qquad \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$$

SM Higgs Mechanism continued

- Photon corresponds to electric charge $Q_{em} = (T_3 + Y)/2$
- Electric charge of vacuum is zero

$$Q_{em} < \Phi >_0 = \frac{1}{2}\left[\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = 0$$

- SM has special relationship (at tree level) $\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W}$

- In theories with Higgs bosons in general representations:

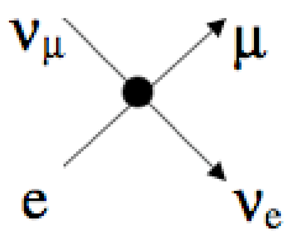
$$\rho = \frac{\sum_i \left[4\tau_i(\tau_i + 1) - (Y_i)^2\right] v_i^2}{2\sum_i (Y_i)^2 v_i^2}$$

  - SU(2) Higgs doublets, τ=1/2, Y=1 $\Rightarrow$ ρ=1
  - SU(2) singlets don't contribute
  - Other representations require tuning: eg triplet Higgs τ=3 and Y=4

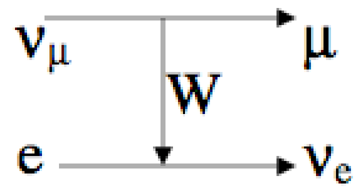
Significant restriction on model building

Higgs VEV fixed

- Consider $\nu_\mu\, e \rightarrow \mu\, \nu_e$



- EW Theory:



$$-i2\sqrt{2}G_\mu g_{\mu\nu}\bar{u}_\mu\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)u_{\nu_\mu}\bar{u}_{\nu_e}\gamma^\nu\left(\frac{1-\gamma_5}{2}\right)u_e$$

$$\frac{ig^2}{2}\frac{1}{k^2-M_W^2}g_{\mu\nu}\bar{u}_\mu\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)u_{\nu_\mu}\bar{u}_{\nu_e}\gamma^\nu\left(\frac{1-\gamma_5}{2}\right)u_e$$

For $|k| \ll M_W$, $2\sqrt{2}G_\mu = g^2/2M_W^2$

For $|k| \gg M_W$, $\sigma \sim 1/E^2$

Adding fermions is more of same…

- Include color triplet quark doublet $Q_L = \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix}$ i=1,2,3
  - Right handed quarks are SU(2) singlets, $u_R=(1+\gamma_5)u$, $d_R=(1+\gamma_5)u$
- With weak hypercharge
  - $Y_{u_R}=4/3$, $Y_{d_R}=-2/3$, $Y_{Q_L}=1/3$

$Q_{em}=(T_3+Y)/2$

- Couplings of charged current to W and Z's take the form:

$$L_{Wqq} = -\frac{g}{2\sqrt{2}}\left(\bar{u}\gamma^\mu(1-\gamma_5)dW_\mu^+ + \bar{d}\gamma^\mu(1-\gamma_5)uW_\mu^-\right)$$

← Left-handed by construction

$$L_{Zqq} = -\frac{g}{4\cos\theta_W}\bar{q}\gamma^\mu\left[L_q(1-\gamma_5)+R_q(1+\gamma_5)\right]qZ_\mu$$

$$L_q = T_3 + 2Q_{em}\sin^2\theta_W$$
$$R_q = 2Q_{em}\sin^2\theta_W$$

- More than one quark doublet implies flavor mixing (see lecture 2)

  - Z decays measured precisely at LEP

Higgs Parameters

- G_μ measured precisely

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8M_W{}^2} = \frac{1}{2v^2}$$

$$v^2 = (\sqrt{2}G_\mu)^{-1} = (246 GeV)^2$$

- Higgs potential has 2 free parameters, μ^2, λ

- Trade μ^2, λ for v^2, M_h^2

$$V = \mu^2 \Phi^+ \Phi + \lambda (\Phi^+ \Phi)^2$$

  - Large $M_h \rightarrow$ strong Higgs self-coupling
  - A priori, Higgs mass can be anything

$$v^2 = -\frac{\mu^2}{2\lambda}$$

$$M_h{}^2 = 2v^2\lambda$$

$$V = \frac{M_h{}^2}{2} h^2 + \frac{M_h{}^2}{2v} h^3 + \frac{M_h{}^2}{8v^2} h^4$$

Parameters of SU(2) x U(1) Sector

- $g, g', v, M_h \Rightarrow$ Trade for:
  - α=1/137.03599911(46) from $(g-2)_e$ and quantum Hall effect
  - G_F=1.16637(1) x 10^{-5} GeV^{-2} from muon lifetime
  - M_Z=91.1876±0.0021 GeV
  - Plus quark and Higgs masses

Everything predicted in terms of α, G_F, M_Z and masses

This is a theory we can test!

Predict M_W, $\sin\theta_W$

Now include Leptons

- Simplest case, include an SU(2) doublet of left-handed leptons

$$\Psi_L = \begin{pmatrix} \nu_L = \frac{1}{2}(1-\gamma_5)\nu \\ e_L = \frac{1}{2}(1-\gamma_5)e \end{pmatrix}$$

- Only right-handed electron, $e_R=(1+\gamma_5)e/2$
  - No right-handed neutrino
- Define weak hypercharge, Y, such that $Q_{em}=(T_3+Y)/2$
  - $Y_L=-1$
  - $Y_R=-2$
- By construction Isospin, T_3, commutes with weak hypercharge $[T_3,Y]=0$
- Couple gauge fields to leptons

$T_3=\pm 1$

$$L_{leptons} = \bar{e}_R i\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}YB_\mu\right)e_R + \overline{\Psi}_L i\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}YB_\mu + i\frac{g}{2}\sigma_i W_i^\mu\right)\Psi_L$$

Review of Higgs Couplings

- Higgs couples to fermion mass
  - Largest coupling is to heaviest fermion

$$L = -\frac{m_f}{v}\bar{f}fh = -\frac{m_f}{v}\left(\bar{f}_L f_R + \bar{f}_R f_L\right)h$$

v=246 GeV

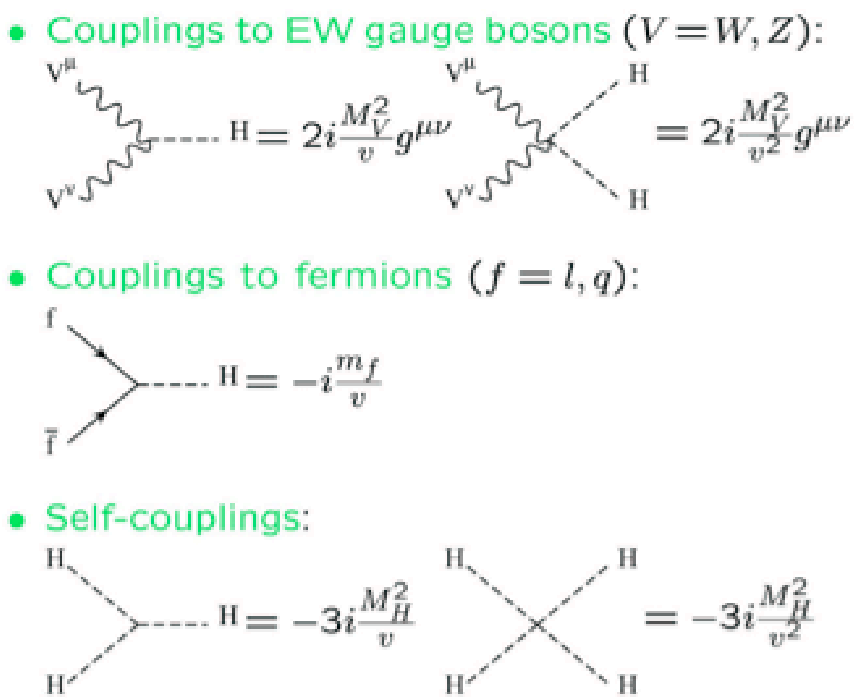
  - Top-Higgs coupling plays special role?
  - No Higgs coupling to neutrinos
- Higgs couples to gauge boson masses

$$L = gM_W W^{+\mu} W_\mu^- h + \frac{gM_Z}{\cos\theta_W} Z^\mu Z_\mu h + ...$$

$$g^2 = \frac{G_F}{\sqrt{2}} 8M_W^2 = \frac{e^2}{\sin^2\theta_W} = \frac{4\pi\alpha}{\sin^2\theta_W}$$

- Only free parameter is Higgs mass!
- Everything is calculable….***testable theory***

Review of Higgs Boson Feynman Rules



- Higgs couples to heavy particles
- ***No tree level coupling to photons (γ) or gluons (g)***
- $M_h^2 = 2v^2\lambda \Rightarrow$ large M_h is strong coupling regime
  - M_h is parameter which separates perturbative/non-perturbative regimes

Spontaneous Symmetry Breakdown

Particle Masses arise through the Higgs mechanism: Spontaneous breakdown of gauge symmetry

$$SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{\rm em} \tag{1}$$

A scalar field, charged under the gauge group, acquires v.e.v.

$$V(H) = m_H^2 H^\dagger H + \frac{\lambda}{2}\left(H^\dagger H\right)^2 \tag{2}$$

Therefore,

$$\langle H^\dagger H \rangle = -\frac{m_H^2}{\lambda} \tag{3}$$

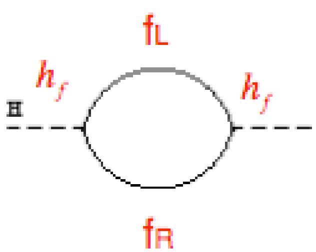
the v.e.v. of the Higgs field is fixed by the value of the negative mass parameter.

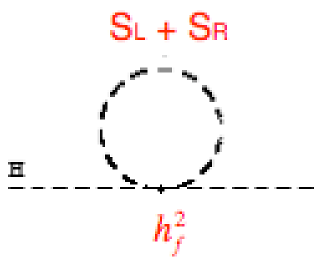
Problem: The mass parameter is unstable under quantum corrections.

Higgs Mass Parameter Corrections

One loop corrections to the Higgs mass parameter cancel if the couplings of scalars and fermions are equal to each other

$$\delta m_H^2 = \frac{N_C h_f^2}{16\pi^2}\left[-2\Lambda^2 + 3m_f^2 \log\left(\frac{\Lambda^2}{m_f^2}\right) + 2\Lambda^2 - 2m_s^2 \log\left(\frac{\Lambda^2}{m_s^2}\right)\right]$$



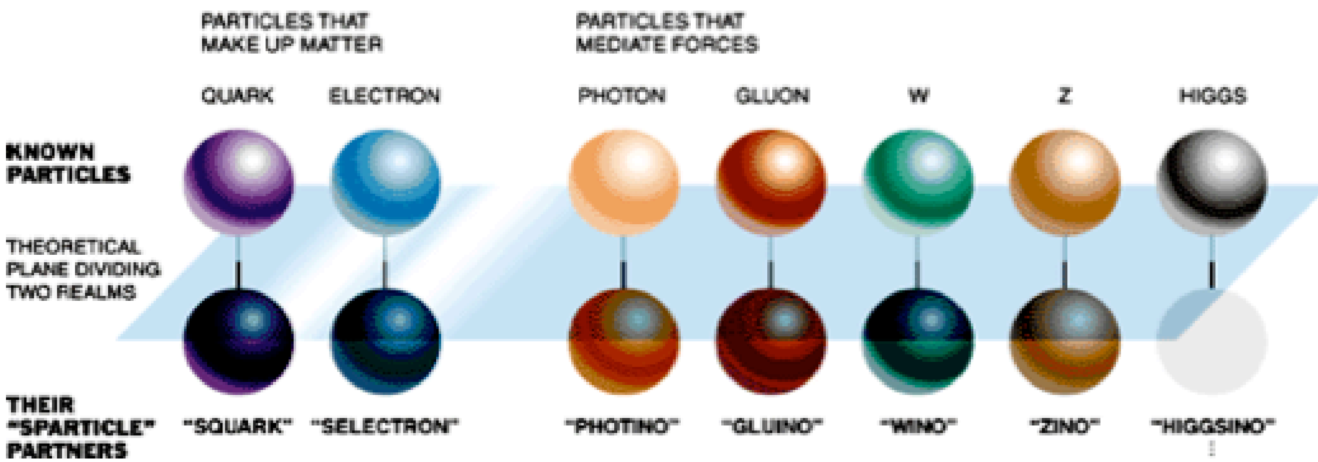


(If the masses proceed from the v.e.v. of H, there is another diagram that ensures also the cancellation of the log term. Observe that the fermion and scalar masses are the same in this case, equal to h_f v.)

Supersymmetry is a symmetry that ensures the equality of these couplings.

supersymmetry

fermions ⟷ **bosons**



Photino, Zino and Neutral Higgsino: Neutralinos

Charged Wino, charged Higgsino: Charginos

No new dimensionless couplings. Couplings of supersymmetric particles equal to couplings of Standard Model ones.
Two Higgs doublets necessary. Ratio of vacuum expectation values denoted by $\tan\beta$

Why Supersymmetry ?

- Helps to stabilize the weak scale—Planck scale hierarchy

- Supersymmetry algebra contains the generator of space-time translations. Necessary ingredient of theory of quantum gravity.

- Minimal supersymmetric extension of the SM : Leads to Unification of gauge couplings

- Starting from positive masses at high energies, electroweak symmetry breaking is induced radiatively

- If discrete symmetry, $P = (-1)^{3B+L+2S}$ is imposed, lightest SUSY particle neutral and stable: Excellent candidate for cold Dark Matter.

Structure of Supersymmetric Gauge Theories

- The Standard Model is based on a Gauge Theory.
- A supersymmetric extension of the Standard Model has then to follow the rules of Supersymmetric Gauge Theories.
- These theories are based on two set of fields:
  - Chiral fields, that contain left handed components of the fermion fields and their superpartners.
  - Vector fields, containing the vector gauge bosons and their superpartners.
- Right-handed fermions are contained on chiral fields by means of their charge conjugate representation

$$(\psi_R)^C = \left(\psi^C\right)_L \tag{4}$$

- Higgs fields are described by chiral fields, with fermion superpartners

Generators of Supersymmetry

- Supersymmetry is a symmetry that relates boson to fermion degrees of freedom, $Q|F>=|B>$, $Q|B>=|F>$.

- The generators of supersymmetry are two component anticommuting spinors, Q_α, $\bar{Q}^{\dot{\alpha}}$, satisfying

$$\{Q_\alpha, Q_\beta\} = 0 \tag{5}$$

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu \tag{6}$$

  where $\sigma^\mu = (I, \vec{\sigma})$, $\bar{\sigma}^\mu = (I, -\vec{\sigma})$, and σ^i are the Pauli matrices. As anticipated, space-time translations are part of the SUSY algebra.

- Two-spinors may be contracted to form Lorentz invariant quantities

$$\psi^\alpha \chi_\alpha = \psi^\alpha \epsilon_{\alpha\beta} \chi^\beta \tag{7}$$

Four-component vs. Two-component fermions

- A Dirac Spinor is a four component object whose components are

$$\psi_D = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}; \qquad \psi_D^C = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} \tag{8}$$

- A Majorana Spinor is a four component object whose components are

$$\psi_M = \begin{pmatrix} \chi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}; \qquad \psi_M^C = \psi_M \tag{9}$$

- Gamma Matrices

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}; \qquad \gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix} \tag{10}$$

- Observe that $\psi_{D,L} = \chi$; $\psi_{D,R} = \bar{\psi}$

- Usual Dirac contractions may be then expressed in terms of two component contractions.

$$\bar{\psi}_D = (\psi^\alpha \quad \bar{\chi}_{\dot{\alpha}}) \tag{11}$$

- For instance,

$$\bar{\psi}_D \, \psi_D = \psi\chi + h.c.; \tag{12}$$

$$\bar{\psi}_D \gamma^\mu \psi_D = \psi \bar{\sigma}^\mu \bar{\psi} + \bar{\chi} \sigma^\mu \chi \quad = \quad -\bar{\psi} \sigma^\mu \psi + \bar{\chi} \sigma^\mu \chi \tag{13}$$

Observe that Majorana particles lead to vanishing vector currents. Therefore, they must be neutral under electromagnetic interactions. Chiral currents don't vanish, $\bar{\psi}_D \gamma^\mu \gamma_5 \psi_D = -\bar{\psi} \sigma^\mu \psi - \bar{\chi} \sigma^\mu \chi$. They may couple to the Z-boson.

- Other relations may be found in the literature.

Superspace

- In order to describe supersymmetric theories, it proves convenient to introduce the concept of superspace.
- Apart from the ordinary coordinates x^μ, one introduces new anticommuting spinor coordinates θ^α and $\bar{\theta}_{\dot{\alpha}}$; $[\theta] = [\bar{\theta}] =$ -1/2.
- This allows to represent fermion and boson fields by the same superfield.
- For instance, a generic chiral field, forming the base for an irreducible representation of SUSY, is given by

$$\Phi(x, \theta, \bar{\theta} = 0) = A(x) + \sqrt{2}\,\theta\,\psi(x) + \theta^2 F(x) \tag{14}$$

$$\Phi(x, \theta, \bar{\theta}) = \exp(-i\partial_\mu \theta \sigma^\mu \bar{\theta})\,\Phi(x, \theta, \bar{\theta} = 0) \tag{15}$$

- A, ψ and F are the scalar, fermion and auxiliary components.

Transformation of chiral field components

- Supersymmetry is a particular translation in superspace, characterized by a Grassman parameter ξ.
- Supersymmetry generators may be given as derivative operators

$$Q_\alpha = i\left[-\partial_\theta - i\sigma^\mu\bar{\theta}\partial_\mu\right] \tag{16}$$

- Under supersymmetric transformations, the components of chiral fields transform like

$$\begin{aligned}\delta A &= \sqrt{2}\xi\psi, \qquad\qquad \delta F = -i\sqrt{2}\bar{\xi}\bar{\sigma}^\mu\partial_\mu\psi \\ \delta\psi &= -i\sqrt{2}\sigma^\mu\bar{\xi}\partial_\mu A + \sqrt{2}\xi F\end{aligned} \tag{17}$$

- Interestingly enough, the F component transforms like a total derivative and it is a good guidance to construct supersymmetric Lagrangians.

Properties of chiral superfields

- The product of two superfields is another superfield.
- For instance, the F-component of the product of two superfields Φ_1 and Φ_2 is obtained by collecting all the terms in θ^2, and is equal to

$$A_1 F_2 + A_2 F_1 + \psi_1 \psi_2 \tag{18}$$

- For a generic Polynomial function of several fields $P(\Phi_i)$, the result is

$$\left(\partial_{A_i} P(A)\right) F_i + \frac{1}{2} \left(\partial^2_{A_i, A_j} P(A)\right) \psi_i \psi_j \tag{19}$$

- Finally, a single chiral field has dimensionality $[A] = [\Phi] = 1$, $[\psi]=$ 3/2 and $[F] = 2$. For $P(A)$, $[P(\Phi)]_F = [P(\Phi)] + 1 \quad ([\theta] = [\bar{\theta}] =$ -1/2).

Vector Superfields

- Vector Superfields are generic hermitian fields. The minimal irreducible representations may be obtained by

$$V(x,\theta,\bar{\theta}) = -\left(\theta\sigma^{\mu}\bar{\theta}\right)V_{\mu} + i\theta^2\bar{\theta}\bar{\lambda} - i\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D \tag{20}$$

- Vector Superfields contain a regular vector field V_{μ}, its fermionic supersymmetric partner λ and an auxiliary scalar field D.
- The D-component of a vector field transform like a total derivative.
- $D = [V] + 2$; $[V_{\mu}] = [V] + 1$; $[\lambda] = [V] + 3/2$. If V_{μ} describes a physical gauge field, then $[\mathrm{V}] = 0$.

Superfield Strength and gauge transformations

- Similarly to $F_{\mu\nu}$ in the regular case, there is a field that contains the field strength. It is a chiral field, derived from V, and it is given by

$$W^\alpha(x,\theta,\bar{\theta}=0) = -i\lambda^\alpha + (\theta\sigma_{\mu\nu})^\alpha F^{\mu\nu} + \theta^\alpha D - \theta^2 \left(\bar{\sigma}^\mu \mathcal{D}_\mu \bar{\lambda}\right)^\alpha \quad (21)$$

- Under gauge transformations, superfields transform like

$$\begin{aligned} \Phi &\to \exp(-ig\Lambda)\Phi, \qquad W_\alpha \to \exp(-ig\Lambda) W_\alpha \exp(ig\Lambda) \\ \exp(gV) &\to \exp(-ig\bar{\Lambda}) \exp(gV) \exp(ig\Lambda) \end{aligned} \quad (22)$$

where Λ is a chiral field of dimension 0.

Towards a Supersymmetric Lagrangian

- The aim is to construct a Lagrangian, invariant under supersymmetry and under gauge transformations.
- One should remember, for that purpose, that both the F-component of a chiral field, as well as the D-component of a vector field transform under SUSY as a total derivative.
- One should also remember that, if renormalizability is imposed, then the dimension of all interaction terms in the Lagrangian

$$[\mathcal{L}_{\rm int}] \leq 4 \tag{23}$$

- On the other hand,

$$[\Phi] = 1, \qquad [W_\alpha] = 3/2, \qquad [V] = 0. \tag{24}$$

and one should remember that $[V]_D = [V] + 2$; $[\Phi]_F = [\Phi] + 1$.

Supersymmetric Lagrangian

- Once the above machinery is introduced, the total Lagrangian takes a particular simple form. The total Lagrangian is given by

$$\begin{aligned} \mathcal{L}_{\mathrm{SUSY}} &= \frac{1}{4g^2}\left(Tr[W^\alpha W_\alpha]_F + h.c.\right) + \sum_i \left(\bar{\Phi}\exp(gV)\Phi\right)_D \\ &+ \left([P(\Phi)]_F + h.c.\right) \end{aligned} \tag{25}$$

  where $P(\Phi)$ is the most generic dimension-three, gauge invariant, polynomial function of the chiral fields Φ, and it is called Superpotential. It has the general expression

$$P(\Phi) = c_i\Phi_i + \frac{m_{ij}}{2}\Phi_i\Phi_j + \frac{\lambda_{ijk}}{3!}\Phi_i\Phi_k\Phi_k \tag{26}$$

- The D-terms of V^a and the F term of Φ_i do not receive any derivative contribution: Auxiliary fields that can be integrated out by equation of motion.

Lagrangian in terms of Component Fields

- The above Lagrangian has the usual kinetic terms for the boson and fermion fields. It also contain generalized Yukawa interactions and contain interactions between the gauginos, the scalar and the fermion components of the chiral superfields.

$$
\begin{aligned}
\mathcal{L}_{\mathrm{SUSY}} &= (\mathcal{D}_\mu A_i)^\dagger \mathcal{D} A_i + \left(\frac{i}{2}\bar{\psi}_i \bar{\sigma}^\mu \mathcal{D}_\mu \psi_i + \mathrm{h.c.}\right) \\
&- \frac{1}{4}\left(G^a_{\mu\nu}\right)^2 + \left(\frac{i}{2}\bar{\lambda}^a \bar{\sigma}^\mu \mathcal{D}_\mu \lambda^a + \mathrm{h.c.}\right) \\
&- \left(\frac{1}{2}\frac{\partial^2 P(A)}{\partial A_i \partial A_j}\psi_i \psi_j - i\sqrt{2} g A_i^* T_a \psi_i \lambda^a + h.c.\right) \\
&- V(F_i, F_i^*, D^a)
\end{aligned} \tag{27}
$$

- The last term is a potential term that depend only on the auxiliary fields

Notation Refreshment

- All standard matter fermion fields are described by their left-handed components (using the charge conjugates for right-handed fields) ψ_i
- All standard matter fermion superpartners are described the scalar fields A_i. There is one for each chiral fermion.
- Gauge bosons are inside covariant derivatives and in the $G_{\mu\nu}$ terms.
- Gauginos, the superpartners of the gauge bosons are described by the fermion fields λ_a. There is one Weyl fermion for each massless gauge boson.
- Higgs bosons and their superpartners are described as regular chiral fields. Their only distinction is that their scalar components acquire a v.e.v. and, as we will see, they are the only scalars with positive R-Parity.

Scalar Potential

$$V(F_i, F_i^*, D^a) = \sum_i F_i^* F_i + \frac{1}{2} \sum_a (D^a)^2 \tag{28}$$

where the auxiliary fields may be obtained from their equation of motion, as a function of the scalar components of the chiral fields:

$$F_i^* = -\frac{\partial P(A)}{\partial A_i}, \qquad D^a = -g \sum_i (A_i^* T^a A_i) \tag{29}$$

Observe that the quartic couplings are governed by the gauge couplings and that scalar potential is positive definite ! The latter is not a surprise. From the supersymmetry algebra, one obtains,

$$H = \frac{1}{4} \sum_{\alpha=1}^{2} (Q_\alpha^\dagger Q_\alpha + Q_\alpha Q_\alpha^\dagger) \tag{30}$$

- If for a physical state the energy is zero, this is the ground state.
- Supersymmetry is broken if the vacuum energy is non-zero !

Couplings

- The Yukawa couplings between scalar and fermion fields,

$$\frac{1}{2}\frac{\partial^2 P(A)}{\partial A_i \partial A_j}\psi_i\psi_j + h.c. \tag{31}$$

  are governed by the same couplings as the scalar interactions coming from

$$\left(\frac{\partial P(A)}{\partial A_i}\right)^2 \tag{32}$$

- Similarly, the gaugino-scalar-fermion interactions, coming from

$$-i\sqrt{2}gA_i^* T_a \psi_i \lambda^a + h.c. \tag{33}$$

  are governed by the gauge couplings.

- No new couplings ! Same couplings are obtained by replacing particles by their superpartners and changing the spinorial structure.

Trilinear coupling

- A most useful example of the relation between couplings is provided by trilinear (Yukawa) couplings. To avoid complications, let's treat the abelian case:

$$P[\Phi] = h_t HUQ \tag{34}$$

where H is a Higgs superfield.

- Fermion Yukawa:

$$h_t H\psi_U\psi_Q + h.c. \qquad\qquad h_t\left(H\bar{\psi}_R\psi_L + h.c.\right) \tag{35}$$

- Scalar Yukawas

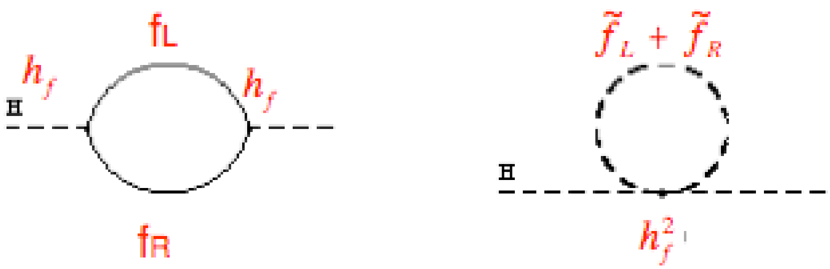
$$|h_t|^2|H|^2\left(|Q|^2 + |U|^2\right) \tag{36}$$

- As anticipated, same couplings of the Higgs field to fermions and to scalar fields.

Higgs Mass Parameter Corrections in SUSY

One loop corrections to the Higgs mass parameter cancel if the couplings of scalars and fermions are equal to each other

$$\delta m_H^2 = \frac{N_C h_f^2}{16\pi^2}\left[-2\Lambda^2 + 3m_f^2 \log\left(\frac{\Lambda^2}{m_f^2}\right) + 2\Lambda^2 - 2m_{\tilde{f}}^2 \log\left(\frac{\Lambda^2}{m_{\tilde{f}}^2}\right)\right]$$



If supersymmetry is exact, there is always an additional, logarithmically divergent diagram, induced by the presence of Higgs-scalar trilinear couplings, that ensure the cancellation of the logarithmic term.

Properties of supersymmetric theories

- To each complex scalar A_i (two degrees of freedom) there is a Weyl fermion ψ_i (two degrees of freedom)
- To each gague boson V_μ^a, there is a gauge fermion (gaugino) λ^a.
- The mass eigenvalues of fermions and bosons are the same !
- Theory has only logarithmic divergences in the ultraviolet associated with wave-function and gauge-coupling constant renormalizations.
- Couplings in superpotential $P[\Phi]$ have no counterterms associated with them.
- The equality of fermion and boson couplings are essential for the cancellation of all quadratic divergences, at all oders in perturbation theory.

Supersymmetric Extension of the Standard Model

- Apart from the superpotential $P[\Phi]$, all other properties are directly determined by the gauge interactions of the theory.
- To construct the superpotential, one should remember that chiral fields contain only left-handed fields, and right-handed fields should be represented by their charge conjugates.
- SM right-handed fields are singlet under $SU(2)$. Their complex conjugates have opposite hypercharge to the standard one.
- There is one chiral superfield for each chiral fermion of the Standard Model.
- In total, there are 15 chiral fields per generation, including the six left-handed quarks, the six right-handed quarks, the two left-handed leptons and the right-handed charged leptons.

Minimal Supersymmetric Standard Model

SM particle	SUSY partner	G_{SM}
(S = 1/2)	(S = 0)	
$Q = (t, b)_L$	$(\tilde{t}, \tilde{b})_L$	(3,2,1/6)
$L = (\nu, l)_L$	$(\tilde{\nu}, \tilde{l})_L$	(1,2,-1/2)
$U = \left(t^C\right)_L$	$\tilde{t}^*_R$	($\bar{3}$,1,-2/3)
$D = \left(b^C\right)_L$	$\tilde{b}^*_R$	($\bar{3}$,1,1/3)
$E = \left(l^C\right)_L$	$\tilde{l}^*_R$	(1,1,1)
(S = 1)	(S = 1/2)	
B_μ	$\tilde{B}$	(1,1,0)
W_μ	$\tilde{W}$	(1,3,0)
g_μ	$\tilde{g}$	(8,1,0)

The Higgs problem

- Problem: What to do with the Higgs field ?
- In the Standard Model masses for the up and down (and lepton) fields are obtained with Yukawa couplings involving H and $H^\dagger$ respectively.
- Impossible to recover this from the Yukawas derived from $P[\Phi]$, since no dependence on $\bar{\Phi}$ is admitted.
- Another problem: In the SM all anomalies cancel,

$$\begin{aligned} \sum_{quarks} Y_i &= 0; \qquad \sum_{left} Y_i = 0; \\ \sum_i Y_i^3 &= 0; \qquad \sum_i Y_i = 0 \end{aligned} \tag{37}$$

- In all these sums, whenever a right-handed field appear, its charge conjugate is considered.
- A Higgsino doublet spoils anomaly cancellation !

Solution to the problem

- Solution: Add a second doublet with opposite hypercharge.
- Anomalies cancel automatically, since the fermions of the second Higgs superfield act as the vector mirrors of the ones of the first one.
- Use the second Higgs doublet to construct masses for the down quarks and leptons.

$$P[\Phi] = h_u QUH_2 + h_d QDH_1 + h_l LEH_1 \tag{38}$$

- Once these two Higgs doublets are introduced, a mass term may be written

$$\delta P[\Phi] = \mu H_1 H_2 \tag{39}$$

- μ is only renormalized by wave functions of H_1 and H_2.

Higgs Fields

- Two Higgs fields with opposite hypercharge.

(S = 0)	(S = 1/2)	
H_1	$\tilde{H}_1$	(1,2,-1/2)
H_2	$\tilde{H}_2$	(1,2,1/2)

- It is important to observe that the quantum numbers of H_1 are exactly the same as the ones of the lepton superfield L.

- This means that one can extend the superpotential $P[\Phi]$ to contain terms that replace H_1 by L.

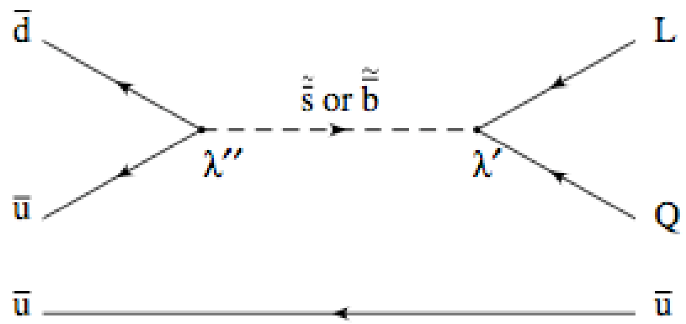
Baryon and Lepton Number Violation

- General superpotential contains, apart from the Yukawa couplings of the Higgs to lepton and quark fields, new couplings:

$$P[\Phi]_{\rm new} = \lambda' \, LQD + \lambda \, LLE + \lambda'' \, UDD \tag{40}$$

- Assigning every lepton chiral (antichiral) superfield lepton number 1 (-1) and every quark chiral (antichiral) superfield baryon number 1/3 (-1/3) one obtains :
  - Interactions in $P[\Phi]$ conserve baryon and lepton number.
  - Interactions in $P[\Phi]_{\rm new}$ violate either baryon or lepton number.
- One of the most dangerous consequences of these new interaction is to induce proton decay, unless couplings are very small and/or sfermions are very heavy.

Proton Decay



- Both lepton and baryon number violating couplings involved.
- Proton: Lightest baryon. Lighter fermions: Leptons

R-Parity

- A solution to the proton decay problem is to introduce a discrete symmetry, called R-Parity. In the language of component fields,

$$R_P = (-1)^{3B+2S+L} \tag{41}$$

- All Standard Model particles have $R_P = 1$.
- All supersymmetric partners have $R_P = -1$.
- All interactions with odd number of supersymmetric particles, like the Yukawa couplings induced by $P[\Phi]_{\rm new}$ are forbidden.
- Supersymmetric particles should be produced in pairs.
- The lightest supersymmetric particle is stable.
- Good dark matter candidate. Missing energy at colliders.