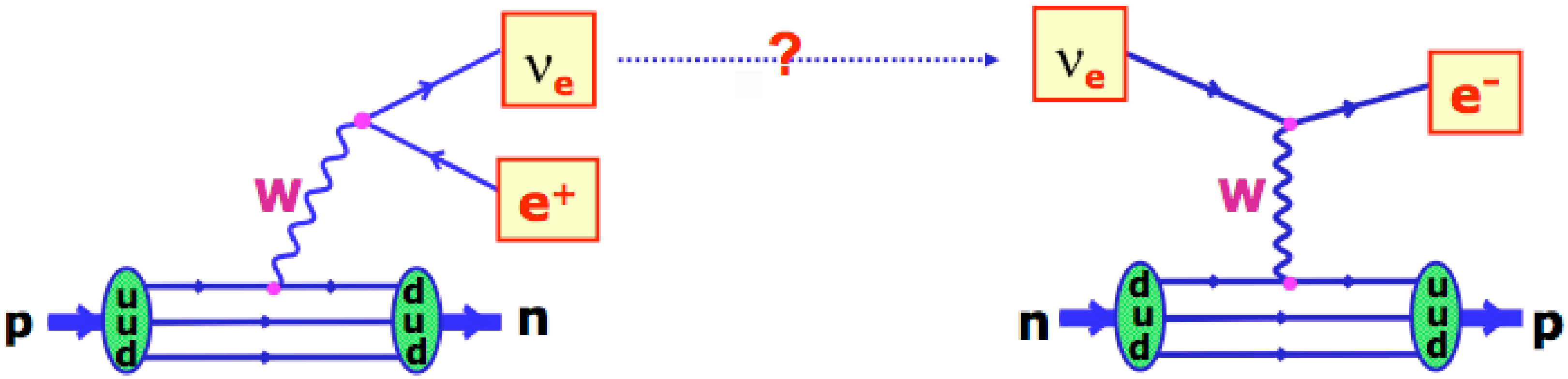
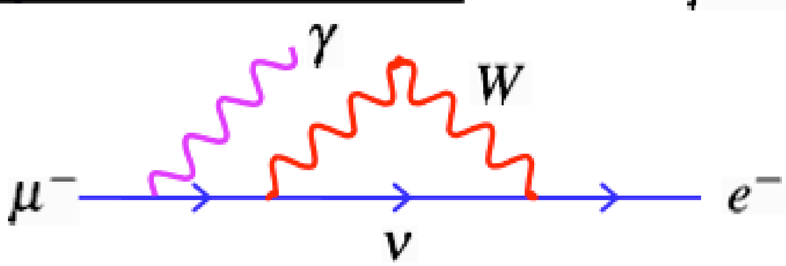
★ Never directly observe neutrinos – can only detect them by their weak interactions. Hence by definition $\nu_e$ is the neutrino state produced along with an electron. Similarly, charged current weak interactions of the state $\nu_e$ produce an electron

$\nu_e, \nu_\mu, \nu_\tau$ = weak eigenstates

- For many years, assumed that $\nu_e, \nu_\mu, \nu_\tau$ were massless fundamental particles
- <u>Experimental evidence:</u> neutrinos produced along with an electron always produced an electron in CC Weak interactions, etc.




- <u>Experimental evidence:</u> absence $\mu^- \rightarrow e^-\gamma$  $\mathrm{BR}(\mu^- \rightarrow e^-\gamma) < 10^{-11}$




Suggests that $\nu_e$ and $\nu_\mu$ are distinct particles

# Neutrino Oscillations

★ Over past 10 years a mass of evidence that $\nu_e, \nu_\mu, \nu_\tau$ are not distinct fundamental particle states, when neutrinos propagate over large distances observe neutrino oscillations:

- Atmospheric neutrinos $\nu_\mu \rightarrow \nu_\tau$
- Solar neutrinos $\nu_e \rightarrow \nu_\mu$ $\nu_e \rightarrow \nu_\tau$
- Reactor neutrinos $\nu_e \rightarrow \nu_\mu$ $\nu_e \rightarrow \nu_\tau$
- Particle beam neutrinos $\nu_\mu \rightarrow \nu_\tau$

★ Here:

- Brief historical overview
- Introduce basic idea in simple two-flavour oscillations
- Extend to the full three flavour treatment
- Discuss ideas of Charge Conjugation, Parity and Time Reversal symmetries
- Describe main recent experimental results

# Neutrino Cross sections

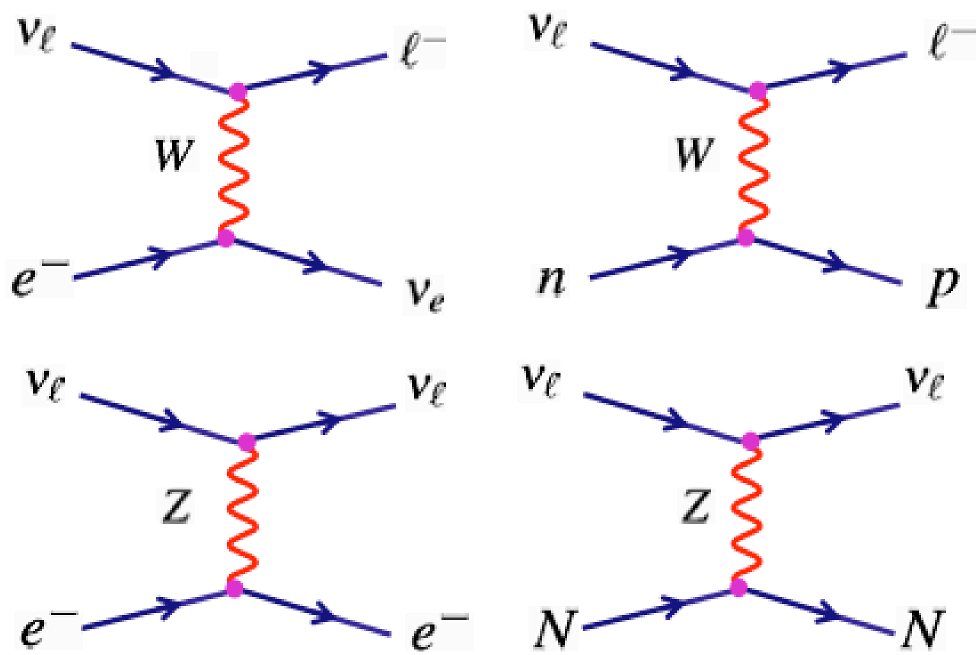
- Before discussing current experimental data, need to consider how neutrinos interact in matter (i.e. our detectors)

  <u>Two processes:</u>
  - Charged current (CC) interactions (via a W-boson) ⇨ charged lepton
  - Neutral current (NC) interactions (via a Z-boson)

  <u>Two possible targets:</u>
  - atomic electrons
  - nucleons within the nucleus

CHARGED CURRENT

NEUTRAL CURRENT



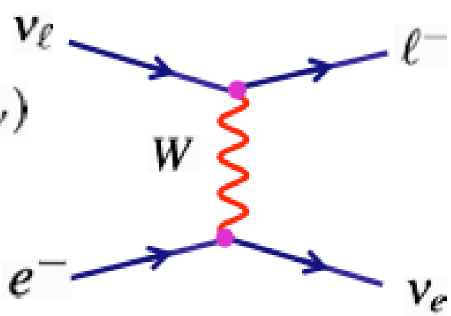
# Neutrino Interaction Thresholds

- Neutrinos from the sun and nuclear reactions have $E_\nu \sim 1\,\text{MeV}$
- Atmospheric neutrinos have $E_\nu \sim 1\,\text{GeV}$

★ These energies are relatively low and not all interactions can occur. Require sufficient energy in the centre-of-mass frame to produce the final state particles

e.g. charged current interactions on atomic electrons (in lab. frame)

$$p_\nu = (E_\nu, 0, 0, E_\nu)$$
$$p_e = (m_e, 0, 0, 0)$$




$$s = (p_\nu + p_e)^2 = (E_\nu + m_e)^2 - E_\nu^2$$

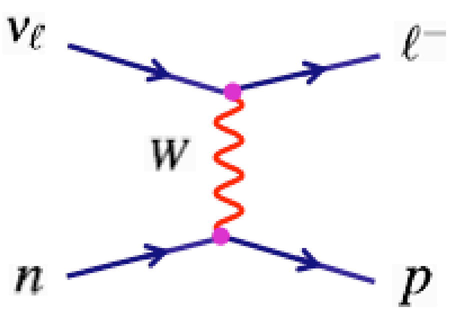
Require: $s > m_\ell^2$

$$\Rightarrow \quad E_\nu > \left[\left(\frac{m_\ell}{m_e}\right)^2 - 1\right]\frac{m_e}{2}$$

- For CC interactions from atomic electrons require

$$E_{\nu_e} > 0 \qquad E_{\nu_\mu} > 11\,\text{GeV} \qquad E_{\nu_\tau} > 3090\,\text{GeV}$$

High energy thresholds compared to typical energies considered here e.g. charged current interactions on nucleons (in lab. frame)




$$s = (p_\nu + p_n)^2 = (E_\nu + m_n)^2 - E_\nu^2$$

Require: $s > (m_\ell^2 + m_p)^2$

$$\Rightarrow \quad E_\nu > \frac{(m_p^2 - m_n^2) + m_\ell^2 + 2m_p m_\ell}{2m_n}$$

- For CC interactions from neutrons require

$E_{\nu_e} > 0 \qquad E_{\nu_\mu} > 110\,\mathrm{MeV} \qquad E_{\nu_\tau} > 3.5\,\mathrm{GeV}$

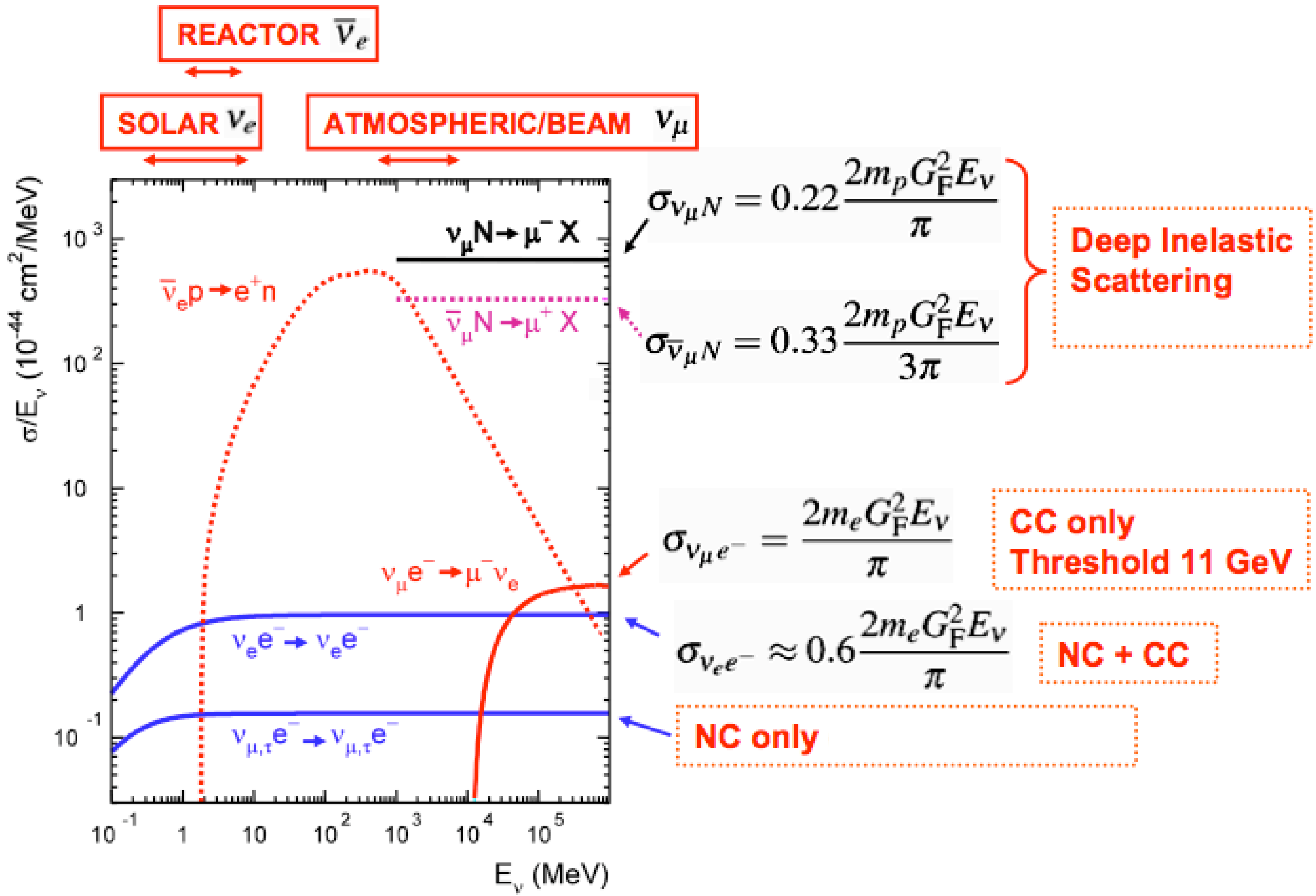
★ Electron neutrinos from the sun and nuclear reactors $E_\nu \sim 1\,\mathrm{MeV}$ which oscillate into muon or tau neutrinos cannot interact via charged current interactions – "they effectively disappear"

★ Atmospheric muon neutrinos $E_\nu \sim 1\,\mathrm{GeV}$ which oscillate into tau neutrinos cannot interact via charged current interactions – "disappear"

- To date, most experimental signatures for neutrino oscillation are a deficit of neutrino interactions (with the exception of SNO)

# Neutrino Detection

★ The detector technology/interaction process depends on type of neutrino and energy



**e.g. Solar Neutrinos** $\nu_e\ :\ E_\nu < 20\,\text{MeV}$

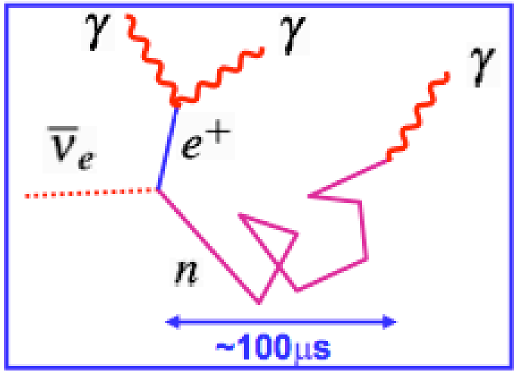
❶ **Water Čerenkov:** e.g. Super Kamiokande

- Detect Čerenkov light from electron produced in $\nu_e + e^- \rightarrow \nu_e + e^-$
- Because of background from natural radioactivity limited to $E_\nu > 5\,\text{MeV}$
- Because Oxygen is a doubly magic nucleus don't get $\nu_e + n \rightarrow e^- + p$

❷ **Radio-Chemical:** e.g. Homestake, SAGE, GALLEX

- Use inverse beta decay process, e.g. $\nu_e + {}^{71}\text{Ga} \rightarrow e^- + {}^{71}\text{Ge}$
- Chemically extract produced isotope and count decays (only gives a rate)

**e.g. Reactor Neutrinos** $\overline{\nu}_e\ :\ E_{\overline{\nu}} < 5\,\text{MeV}$




❶ **Liquid Scintillator:** e.g. KamLAND

- Low energies → large radioactive background
- Dominant interaction: $\overline{\nu}_e + p \rightarrow e^+ + n$
- Prompt positron annihilation signal + delayed signal from *n* (space/time correlation reduces background)
- electrons produced by photons excite scintillator which produces light

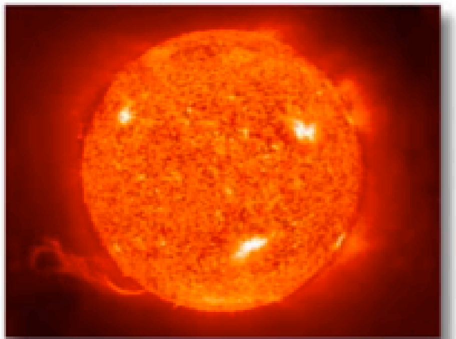
**e.g. Atmospheric/Beam Neutrinos** $\nu_e, \nu_\mu, \overline{\nu}_e, \overline{\nu}_\mu : E_\nu > 1\,\text{GeV}$

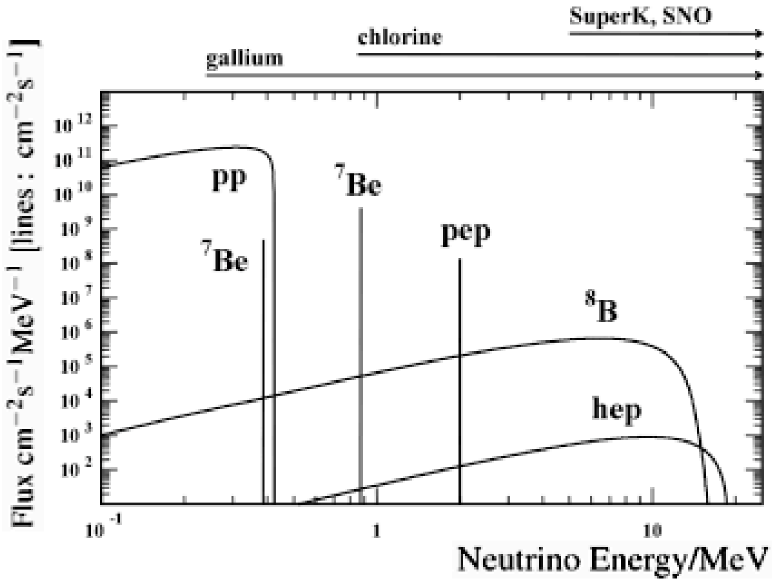
❶ **Water Čerenkov:** e.g. Super Kamiokande

❷ **Iron Calorimeters:** e.g. MINOS, CDHS

- Produce high energy charged lepton – relatively easy to detect

# Solar Neutrinos






- The Sun is powered by the weak interaction – producing a very large flux of electron neutrinos

$$2 \times 10^{38}\, \nu_e\, s^{-1}$$

- Several different nuclear reactions in the sun ⇨ complex neutrino energy spectrum

$$p + p \rightarrow d + e^+ + \nu_e \qquad E_\nu < 0.5\,\text{MeV}$$

$$^8B \rightarrow {}^8Be^* + e^+ + \nu_e \qquad E_\nu \sim 5\text{MeV}$$

$$p + e^- + p \rightarrow d + \nu_e$$

$$^7Be + e^- \rightarrow {}^7Li + \nu_e$$

$$^3He + p \rightarrow {}^4He + e^+ + \nu_e$$

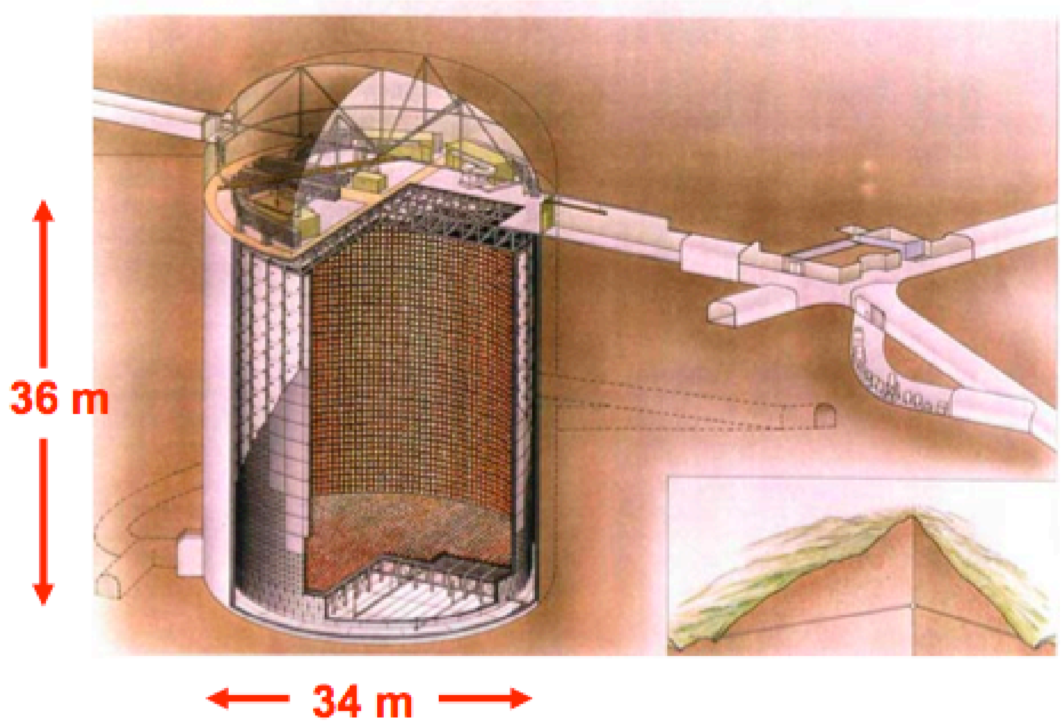
- All experiments saw a deficit of electron neutrinos compared to experimental prediction – the SOLAR NEUTRINO PROBLEM

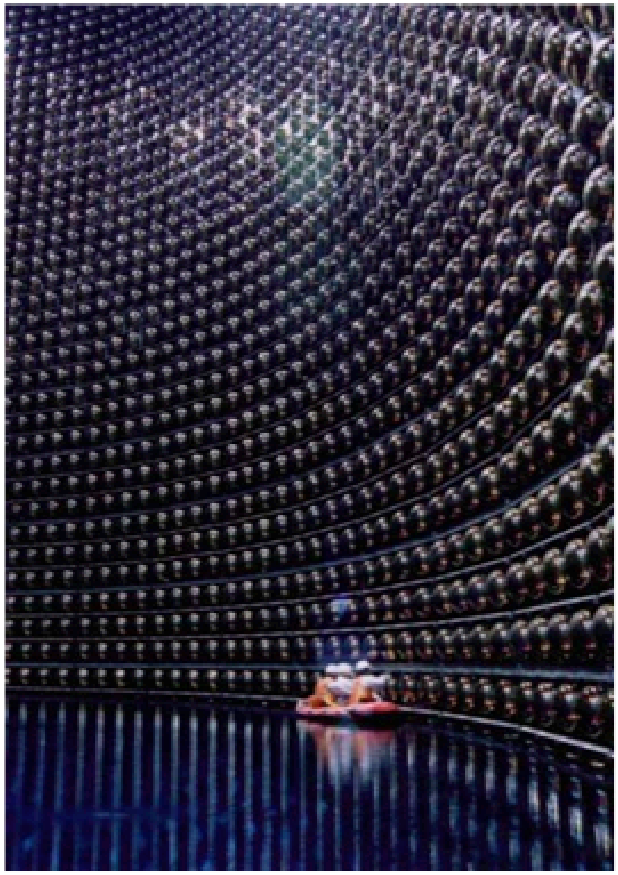
- e.g. Super Kamiokande

# Solar Neutrinos in Super Kamiokande

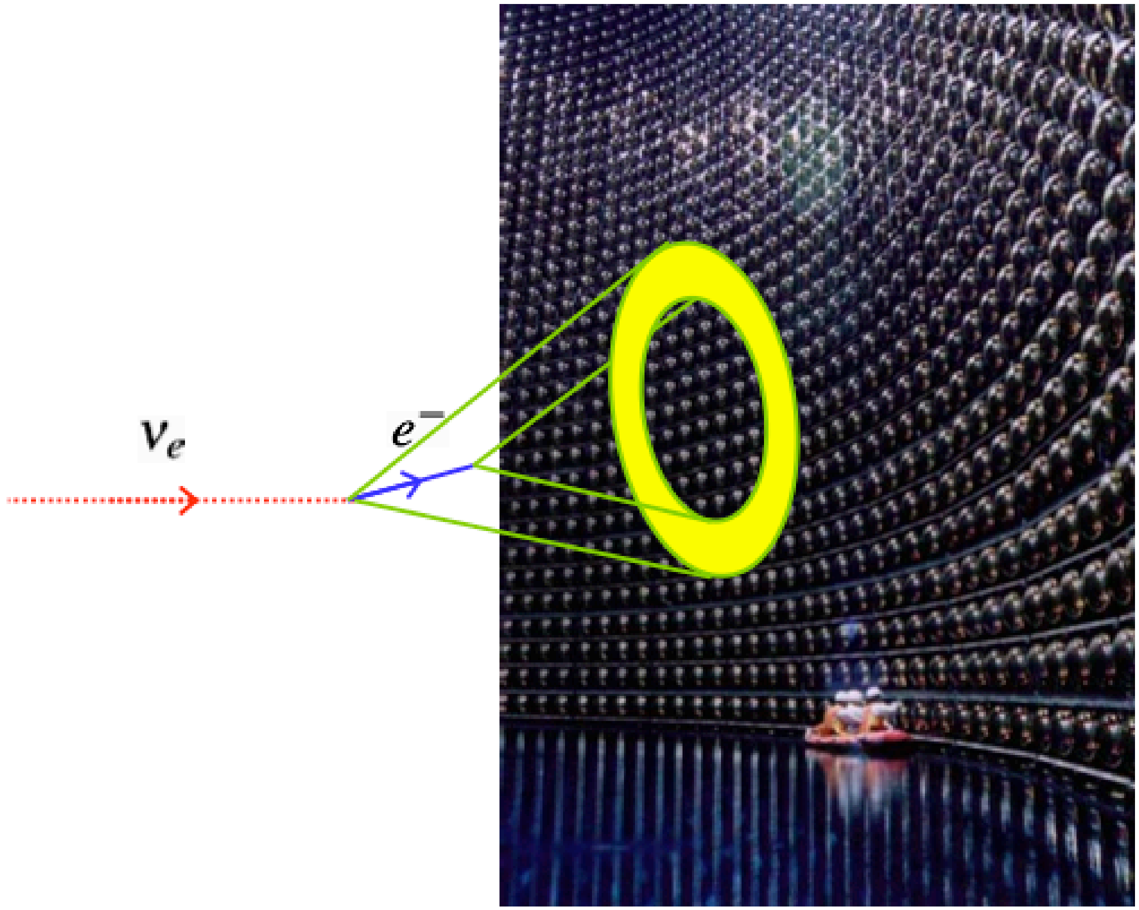
**Mt. Ikenoyama, Japan**

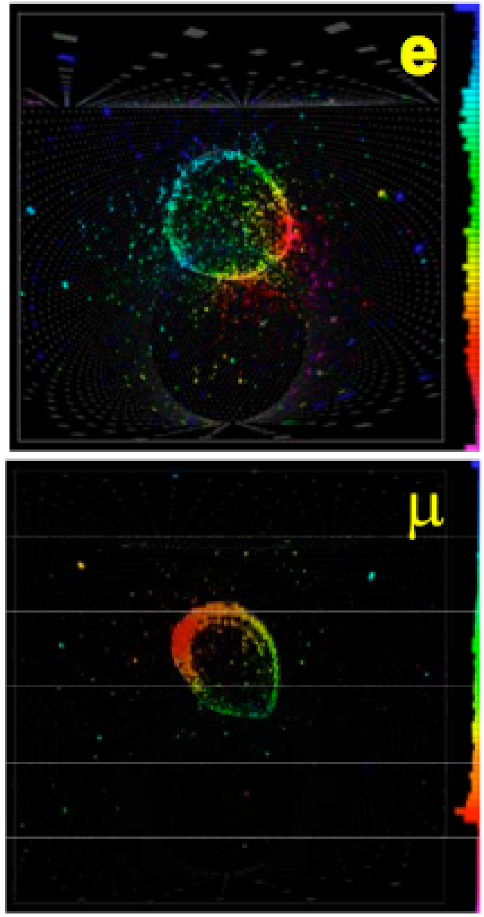
- 50000 ton water Čerenkov detector
- Water viewed by 11146 Photo-multiplier tubes
- Deep underground to filter out cosmic rays otherwise difficult to detect rare neutrino interactions



- Detect neutrinos by observing Čerenkov radiation from charged particles which travel faster than speed of light in water *c/n*



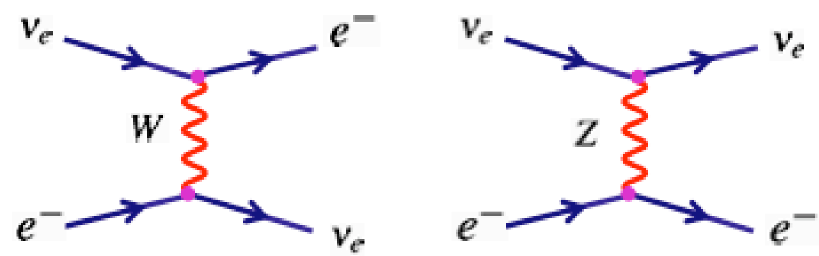




- Can distinguish electrons from muons from pattern of light – muons produce clean rings whereas electrons produce more diffuse "fuzzy" rings

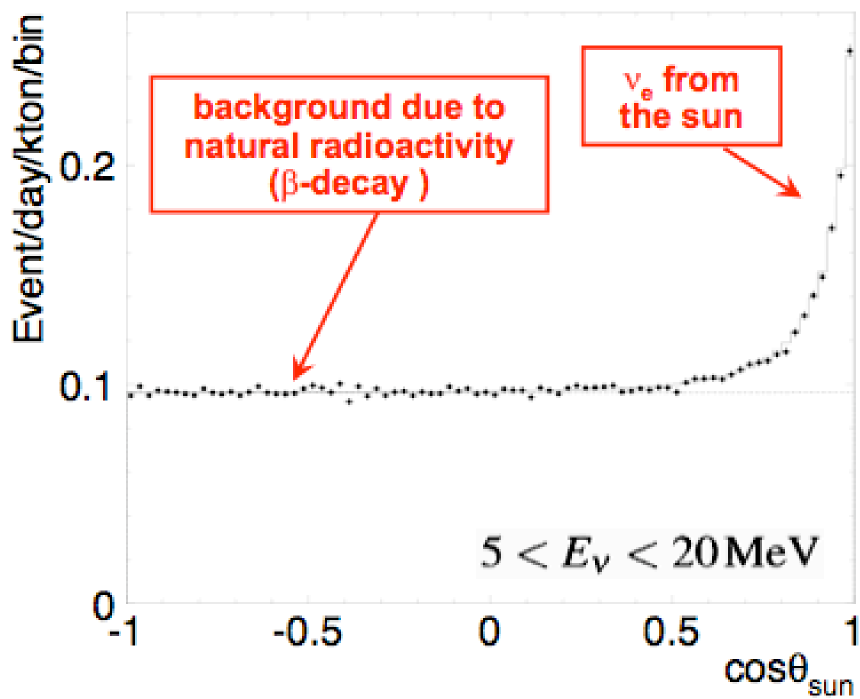
- Sensitive to solar neutrinos with $E_\nu > 5\,\mathrm{MeV}$
- For lower energies too much background from natural radioactivity (β-decays)
- Hence detect mostly neutrinos from $^8B \rightarrow ^8Be^* + e^+ + \nu_e$
- Detect electron Čerenkov rings from

$$\nu_e + e^- \rightarrow \nu_e + e^-$$

- In LAB frame the electron is produced preferentially along the $\nu_e$ direction




S.Fukada et al., Phys. Rev. Lett. 86 5651-5655, 2001




**Results:**

- Clear signal of neutrinos from the sun
- However too few neutrinos

**DATA/SSM = 0.45±0.02**

SSM = "Standard Solar Model" Prediction

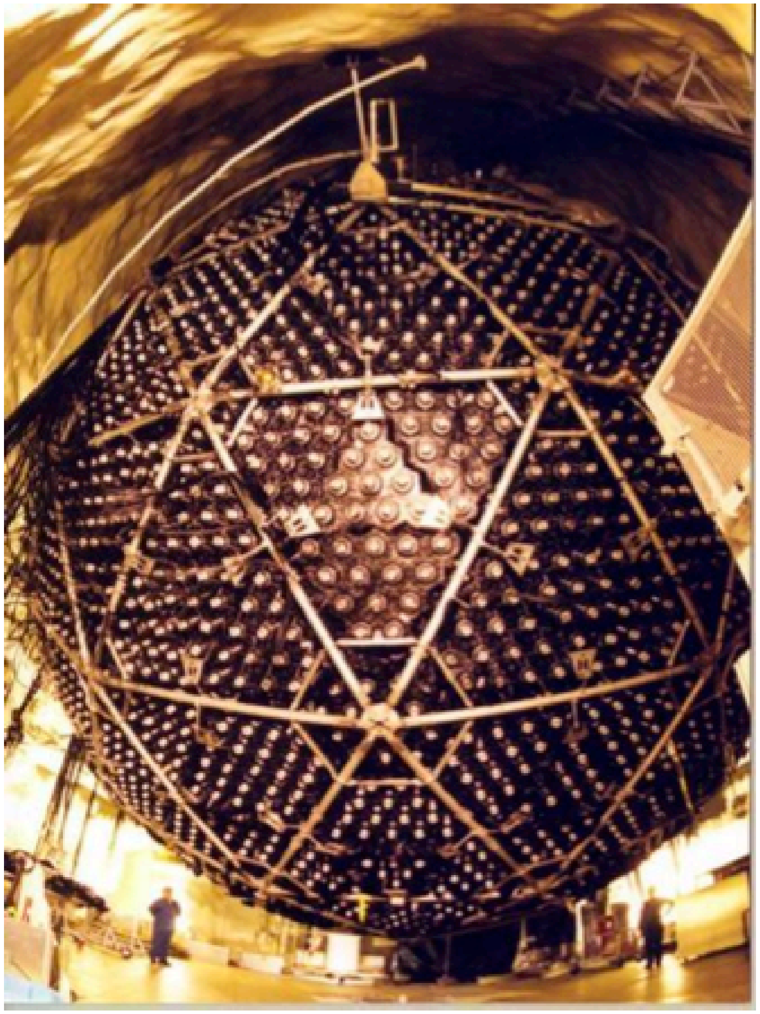
**Explanation:**

- Neutrino oscillations

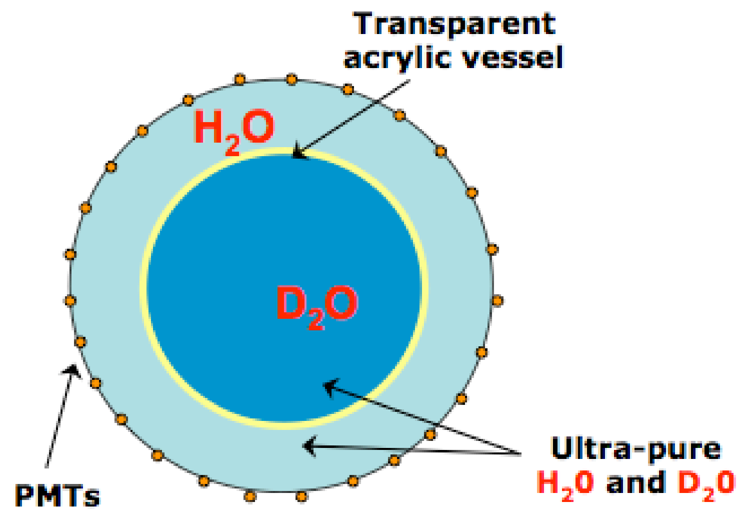
$$\nu_e \rightarrow \nu_\mu \;\; / \;\; \nu_e \rightarrow \nu_\tau$$

# Solar Neutrinos II : SNO

•Sudbury Neutrino Observatory located in a deep mine in Ontario



- 1000 ton heavy water ($D_2O$) Čerenkov detector
- $D_2O$ inside a 12m diameter acrylic vessel
- Surrounded by 3000 tons of normal water
- Main experimental challenge is the need for very low background from radioactivity
- Ultra-pure $H_2O$ and $D_2O$
- Surrounded by 9546 PMTs

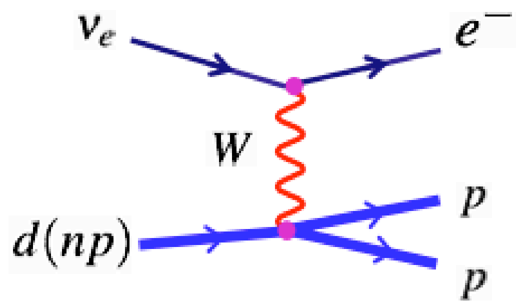


★ **Detect Čerenkov light from three different reactions:**

## CHARGE CURRENT

- Detect Čerenkov light from electron
- Only sensitive to $\nu_e$
- Gives a measure of $\nu_e$ flux

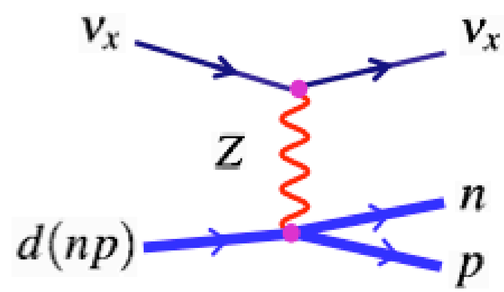
$$\text{CC Rate} \propto \phi(\nu_e)$$




## NEUTRAL CURRENT

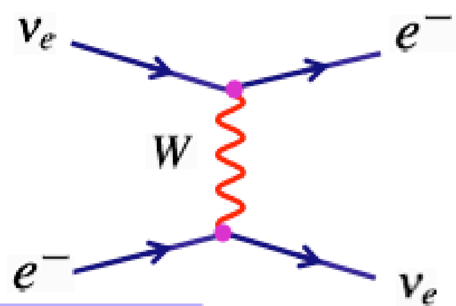
- Neutron capture on a deuteron gives **6.25 MeV**
- Detect Čerenkov light from electrons scattered by $\gamma$
- Measures total neutrino flux

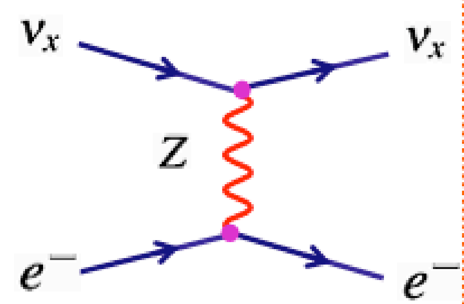
$$\text{NC Rate} \propto \phi(\nu_e) + \phi(\nu_\mu) + \phi(\nu_\tau)$$




## ELASTIC SCATTERING

- Detect Čerenkov light from electron
- Sensitive to all neutrinos – but larger cross section for $\nu_e$



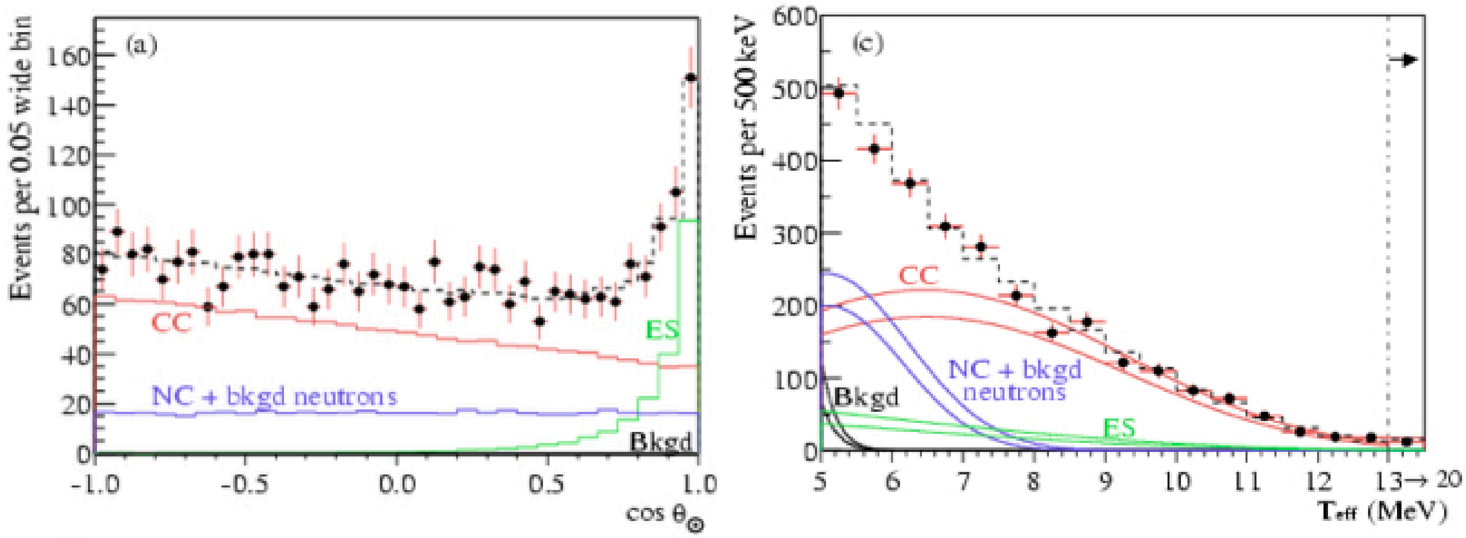




$$\text{ES Rate} \propto \phi(\nu_e) + 0.154(\phi(\nu_\mu) + \phi(\nu_\tau))$$

★ **Experimentally can determine rates for different interactions from:**

- **angle with respect to sun: electrons from ES point back to sun**
- **energy: NC events have lower energy – 6.25 MeV photon from neutron capture**
- **radius from centre of detector: gives a measure of background from neutrons**

**SNO Collaboration, Q.R. Ahmad et al., Phys. Rev. Lett. 89:011301, 2002**




★ **Using different distributions obtain a measure of numbers of events of each type:**

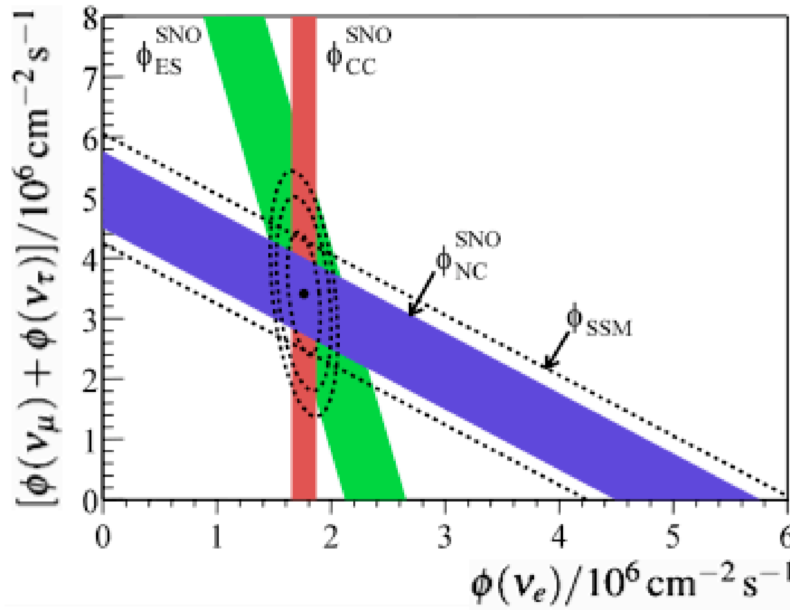
| | |
|---|---|
| **CC : 1968 ± 61** | $\propto \phi(\nu_e)$ |
| **ES : 264 ± 26** | $\propto \phi(\nu_e) + 0.154[\phi(\nu_\mu) + \phi(\nu_\tau)]$ |
| **NC : 576 ± 50** | $\propto \phi(\nu_e) + \phi(\nu_\mu) + \phi(\nu_\tau)$ |

→ **Measure of electron neutrino flux + total flux !**

★ Using known cross sections can convert observed numbers of events into fluxes

★ The different processes impose different constraints

★ Where constraints meet gives separate measurements of $\nu_e$ and $\nu_\mu + \nu_\tau$ fluxes




SNO Collaboration, Q.R. Ahmad et al., Phys. Rev. Lett. 89:011301, 2002

## SNO Results:

$$\phi(\nu_e) = (1.8 \pm 0.1) \times 10^{-6}\,cm^{-2}\,s^{-1}$$

$$\phi(\nu_\mu) + \phi(\nu_\tau) = (3.4 \pm 0.6) \times 10^{-6}\,cm^{-2}\,s^{-1}$$

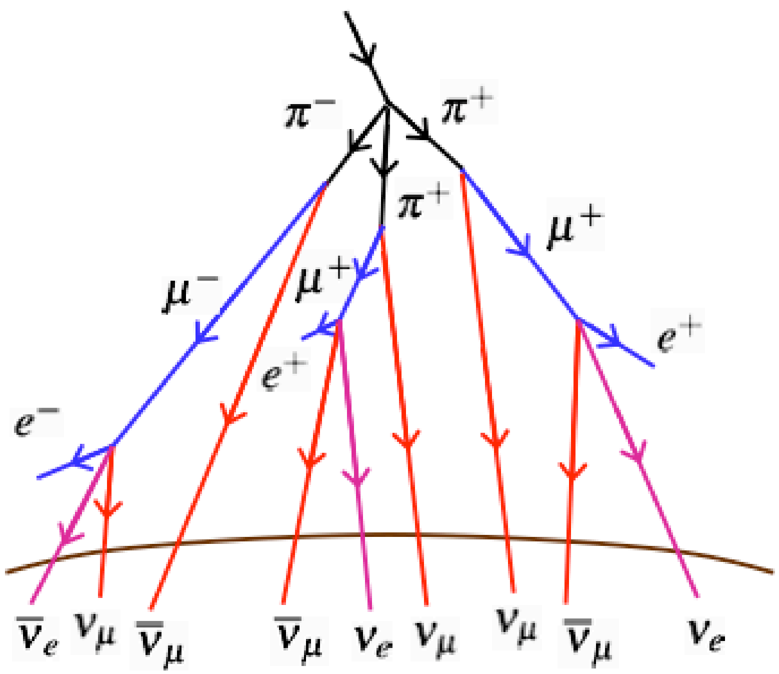
## SSM Prediction:

$$\phi(\nu_e) = 5.1 \times 10^{-6}\,cm^{-2}\,s^{-1}$$

- Clear evidence for a flux of $\nu_\mu$ and/or $\nu_\tau$ from the sun
- Total neutrino flux is consistent with expectation from SSM
- Clear evidence of $\nu_e \rightarrow \nu_\mu$ and/or $\nu_e \rightarrow \nu_\tau$ neutrino oscillations

# Atmospheric Neutrinos

- **High energy cosmic rays** (up to $10^{20}$ eV) interact in the upper part of the Earth's atmosphere
- The cosmic rays (~86% protons, 11% He Nuclei, ~1% heavier nuclei, 2% electrons ) mostly interact hadronically giving showers of mesons (mainly pions)



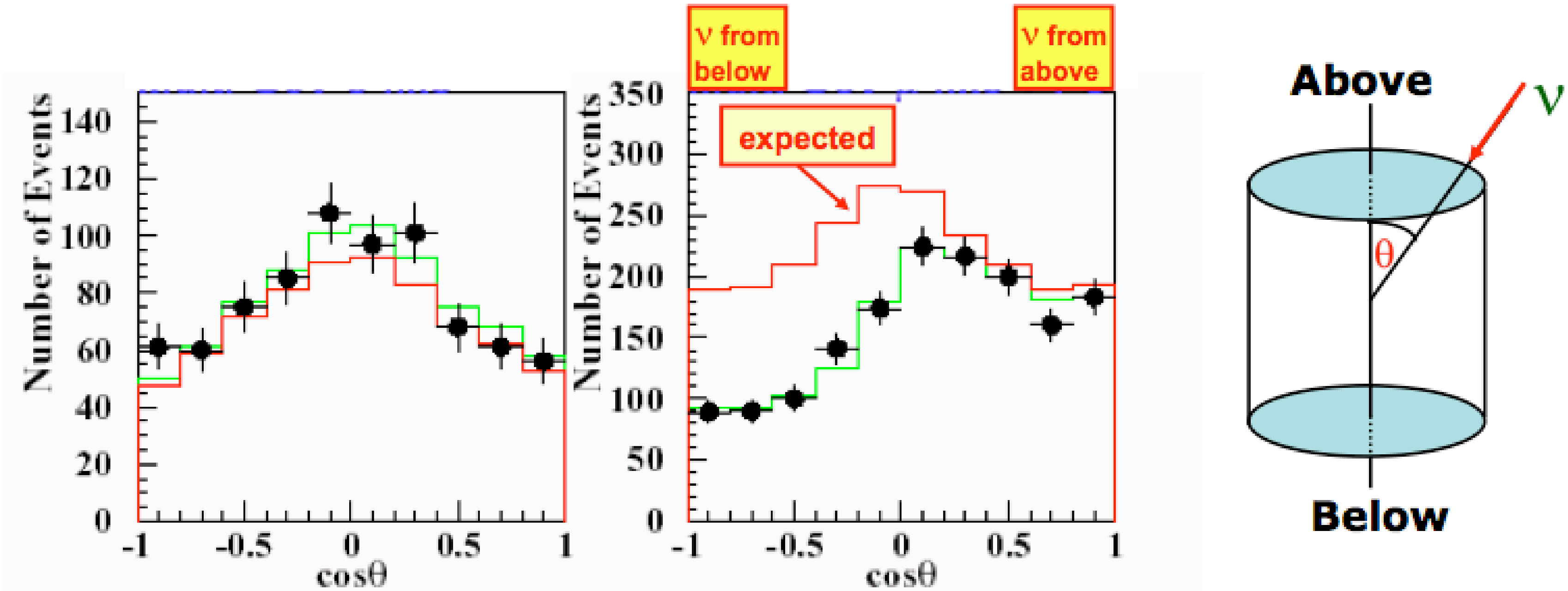

- Neutrinos produced by:

$$\pi^+ \to \mu^+ \nu_\mu \qquad \pi^- \to \mu^- \overline{\nu}_\mu$$
$$\hookrightarrow e^+ \nu_e \overline{\nu}_\mu \qquad \hookrightarrow e^- \overline{\nu}_e \nu_\mu$$

- Flux $\sim 1\,\mathrm{cm^{-2}sr^{-1}s^{-1}}$
- Typical energy : $\mathrm{E}_\nu \sim 1\,\mathrm{GeV}$
- Expect $$\frac{N(\nu_\mu + \overline{\nu}_\mu)}{N(\nu_e + \overline{\nu}_e)} \approx 2$$
- Observe a lower ratio with deficit of $\nu_\mu / \overline{\nu}_\mu$ coming from below the horizon, i.e. large distance from production point on other side of the Earth

# Super Kamiokande Atmospheric Results

- Typical energy: $E_\nu \sim 1\,\mathrm{GeV}$ (much greater than solar neutrinos – no confusion)
- Identify $\nu_e$ and $\nu_\mu$ interactions from nature of Čerenkov rings
- Measure rate as a function of angle with respect to local vertical
- Neutrinos coming from above travel ~20 km
- Neutrinos coming from below (i.e. other side of the Earth) travel ~12800 km




★ Prediction for $\nu_e$ rate agrees with data

★ Strong evidence for disappearance of $\nu_\mu$ for large distances

★ Consistent with $\nu_\mu \rightarrow \nu_\tau$ oscillations

★ Don't detect the oscillated $\nu_\tau$ as typically below interaction threshold of 3.5 GeV

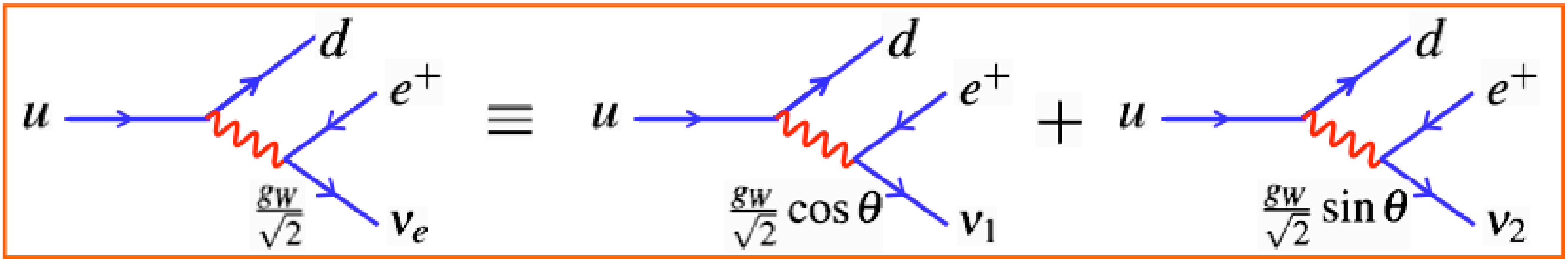
# Neutrino Oscillations for Two Flavours

- ★ Solar/Atmospheric neutrino data understood in terms of NEUTRINO OSCILLATIONS
- ★ Neutrinos are produced and interact as weak eigenstates. $\nu_e, \nu_\mu$
- ★ The weak eigenstates are coherent linear combinations of the fundamental "mass eigenstates" $\nu_1, \nu_2$
- ★ The mass eigenstates are the free particle solutions to the wave-equation and will be taken to propagate as plane waves

$$|\nu_1(t)\rangle = |\nu_1\rangle e^{i\vec{p}_1.\vec{x}-iE_1t} \qquad |\nu_2(t)\rangle = |\nu_2\rangle e^{i\vec{p}_2.\vec{x}-iE_2t}$$

- ★ The weak and mass eigenstates are related by the unitary 2x2 matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \qquad (1)$$




- ★ Equation (1) can be inverted to give

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \qquad (2)$$

- Suppose at time $t=0$ a neutrino is produced in a pure $\nu_e$ state, e.g. in a decay $u \rightarrow d e^+ \nu_e$

$$|\psi(0)\rangle = |\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

- Take the z-axis to be along the neutrino direction
- The wave-function evolves according to the time-evolution of the **mass eigenstates** (free particle solutions to the wave equation)

$$|\psi(t)\rangle = \cos\theta|\nu_1\rangle e^{-ip_1.x} + \sin\theta|\nu_2\rangle e^{-ip_2.x}$$

where $p_i.x = E_i t - \vec{p}_i.\vec{x} = E_i t - |\vec{p}_i|z$

- Suppose make an observation at a distance z from the production point. Making the (very good) approximation that $v_i \approx c$

$$p_i.x = E_i t - |\vec{p}_i|z = (E_i - |\vec{p}_i|)z \qquad (z = ct = t)$$

$$|\psi(z)\rangle = \cos\theta|\nu_1\rangle e^{-i(E_1-|\vec{p}_1|)z} + \sin\theta|\nu_2\rangle e^{-i(E_2-|\vec{p}_2|)z}$$

- For $(m_i \ll E_i)$

$$|\vec{p}_i|^2 = E_i^2 - m_i^2 = E_i^2\left(1 - \frac{m_i^2}{E_i^2}\right) \Rightarrow |\vec{p}_i| = E_i^2\left(1 - \frac{m_i^2}{E_i^2}\right)^{\frac{1}{2}} \approx E_i - \frac{m_i^2}{2E_i}$$

giving $$(E_i - |\vec{p}_i|)z = E_i z - \left(E_i - \frac{m_i}{2E_i}\right)z = \frac{m_i^2}{2E_i}z$$

$$\rightarrow \quad |\psi(z)\rangle = \cos\theta|\nu_1\rangle e^{-i\phi_1} + \sin\theta|\nu_2\rangle e^{-i\phi_2}$$

with $\phi_i = \frac{m_i^2}{2E_i^2}L$ which is the phase of the wave for mass eigenstate $i$ at a distance $L$ from the point of production

★ Expressing the mass eigenstates, $|\nu_1\rangle, |\nu_2\rangle$ , in terms of weak eigenstates (eq 2):

$$|\psi(z=L)\rangle = \cos\theta(\cos\theta|\nu_e\rangle - \sin\theta|\nu_\mu\rangle)e^{-i\phi_1} + \sin\theta(\sin\theta|\nu_e\rangle + \cos\theta|\nu_\mu\rangle)e^{-i\phi_2}$$

$$\boxed{|\psi(L)\rangle = |\nu_e\rangle(\cos^2\theta e^{-i\phi_1} + \sin^2\theta e^{-i\phi_2}) + |\nu_\mu\rangle \sin\theta\cos\theta(-e^{-i\phi_1} + e^{-i\phi_2})}$$

★ If the masses of $|\nu_1\rangle, |\nu_2\rangle$ are the same, the mass eigenstates remain in phase, $\phi_1 = \phi_2$ , and the state remains the linear combination corresponding to $|\nu_e\rangle$

★ If the masses differ, the wave-function no longer remains a pure $|\nu_e\rangle$

$$\begin{aligned} P(\nu_e \rightarrow \nu_\mu) &= |\langle\nu_\mu|\psi(L)\rangle|^2 \\ &= \cos^2\theta\sin^2\theta(-e^{-i\phi_1} + e^{-i\phi_2})(-e^{+i\phi_1} + e^{+i\phi_2}) \\ &= \frac{1}{4}\sin^2 2\theta(2 - 2\cos(\phi_1 - \phi_2)) \\ &= \sin^2 2\theta \sin^2\left(\frac{\phi_1 - \phi_2}{2}\right) \end{aligned}$$

with $\frac{\phi_2 - \phi_1}{2} = \frac{m_2^2 L}{4E_2} - \frac{m_1^2 L}{4E_1} \approx \frac{(m_2^2 - m_1^2)L}{4E}$

★ **Hence the two-flavour oscillation probability is:**

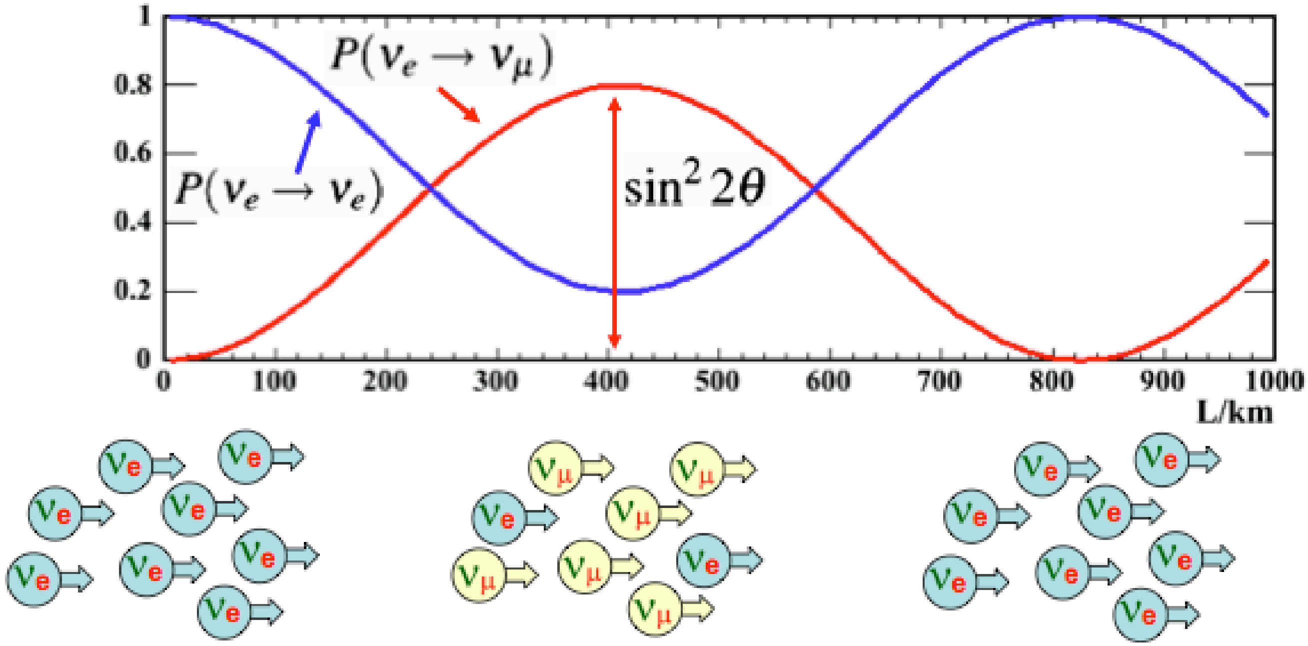
$$P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

**with** $\Delta m_{21}^2 = m_2^2 - m_1^2$

★ **The corresponding two-flavour survival probability is:**

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

•**e.g.** $\Delta m^2 = 0.003\,\mathrm{eV}^2, \qquad \sin^2 2\theta = 0.8, \qquad E_\nu = 1\,\mathrm{GeV}$

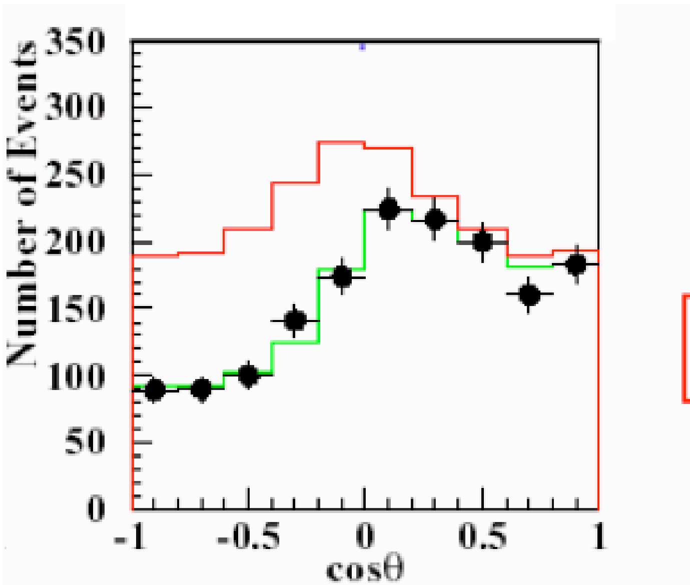


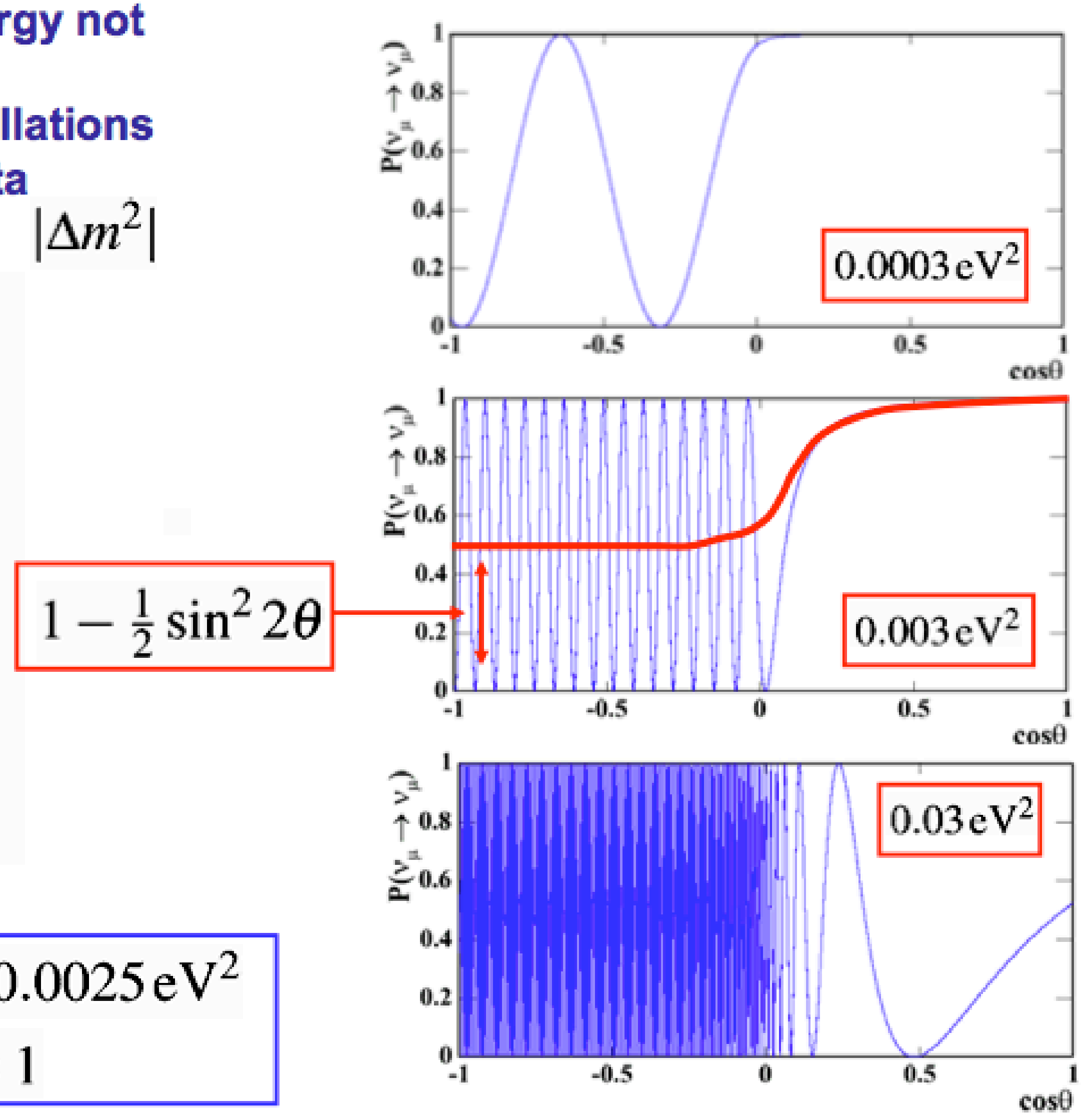

•**wavelength**

$$\lambda_{osc} = \frac{4\pi E}{\Delta m^2}$$

# Interpretation of Atmospheric Neutrino Data

- Measure muon direction and energy not neutrino direction/energy
- Don't have resolution to see oscillations
- Oscillations "smeared" out in data
- Compare data to predictions for $|\Delta m^2|$




★ **Data consistent with:**

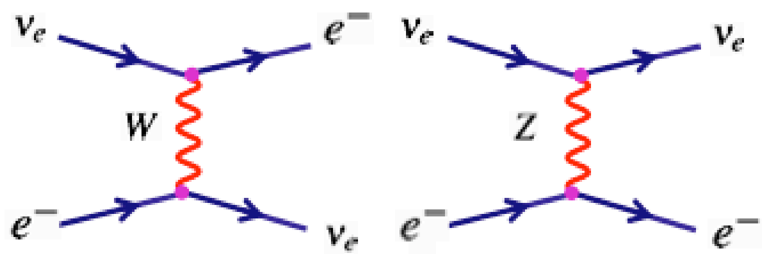
$$|\Delta m^2_{\mathrm{atmos}}| \approx 0.0025\,\mathrm{eV}^2$$

$$\sin^2 2\theta_{\mathrm{atmos}} \approx 1$$

# Interpretation of Solar Neutrino Data

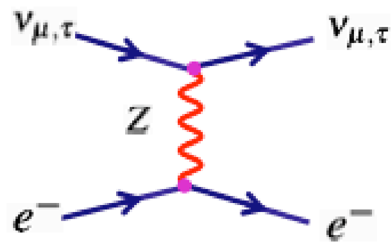
★ The interpretation of the solar neutrino data is complicated by **MATTER EFFECTS**

- The quantitative treatment is non-trivial and is not given here
- Basic idea is that as a neutrino leaves the sun it crosses a region of high electron density
- The coherent forward scattering process ( $\nu_e \rightarrow \nu_e$) for an electron neutrino




CC + NC

- Is different to that for a muon or tau neutrino




NC

- Can enhance oscillations – "MSW effect"

★ A combined analysis of all solar neutrino data gives:

$$\Delta m^2_{\mathrm{solar}} \approx 8 \times 10^{-5}\,\mathrm{eV}^2,\ \sin^2 2\theta_{\mathrm{solar}} \approx 0.85$$

★ **Neutrino masses:**

- Neutrino oscillations require non-zero neutrino masses
- But only determine mass-squared differences – not the masses themselves
- No direct measure of neutrino mass – only mass limits:

$$m_\nu(e) < 2\,\mathrm{eV}; \quad m_\nu(\mu) < 0.17\,\mathrm{MeV}; \quad m_\nu(\tau) < 18.2\,\mathrm{MeV}$$

Note the $e, \mu, \tau$ refer to charged lepton flavour in the experiment, e.g. $m_\nu(e) < 2\,\mathrm{eV}$ refers to the limit from tritium beta-decay

★ The interpretation of solar and atmospheric neutrino data using the two flavour neutrino oscillations formula

SOLAR NEUTRINOS

$$\Delta m^2_{\mathrm{solar}} \approx 8 \times 10^{-5}\,\mathrm{eV}^2, \ \sin^2 2\theta_{\mathrm{solar}} \approx 0.85$$

$$\lambda_{\mathrm{osc}} = \frac{4\pi E}{\Delta m^2}$$

ATMOSPHERIC NEUTRINOS

$$\Delta m^2_{\mathrm{atmos}} \approx 2.5 \times 10^{-3}\,\mathrm{eV}^2, \ \sin^2 2\theta_{\mathrm{atmos}} > 0.92$$

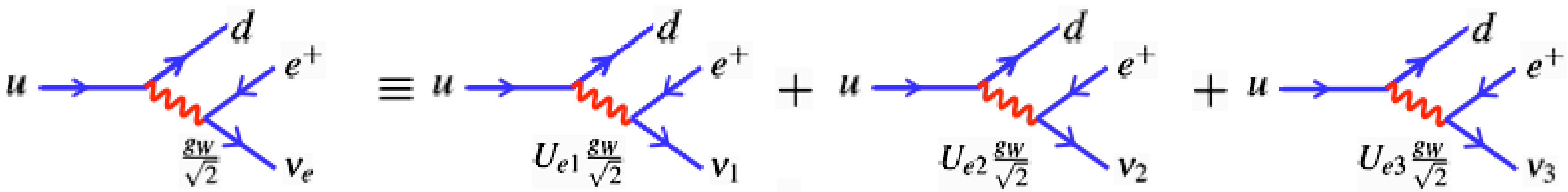
Note: for a given neutrino energy the wavelength for "solar" neutrino oscillations is 30 times that of the atmospheric neutrino oscillations

★ To fully understand current neutrino data need to extend the analysis to neutrino oscillations of three flavours….

# Neutrino Oscillations for Three Flavours

★ In the case of three generations we have:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$




★ The 3x3 Unitary matrix $U$ is known as the Pontecorvo-Maki-Nakagawa-Sakata matrix, usually abbreviated **PMNS**

•Using $U^\dagger U = I \quad \Rightarrow \quad U^{-1} = U^\dagger = (U^*)^T$

gives

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U^*_{e1} & U^*_{\mu 1} & U^*_{\tau 1} \\ U^*_{e2} & U^*_{\mu 2} & U^*_{\tau 2} \\ U^*_{e3} & U^*_{\mu 3} & U^*_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

# Unitarity Relations

★The Unitary of the PMNS matrix gives several useful relations: $UU^\dagger = I \quad \Rightarrow$

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} U_{e1}^* & U_{\mu 1}^* & U_{\tau 1}^* \\ U_{e2}^* & U_{\mu 2}^* & U_{\tau 2}^* \\ U_{e3}^* & U_{\mu 3}^* & U_{\tau 3}^* \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

gives:

$$U_{e1}U_{e1}^* + U_{e2}U_{e2}^* + U_{e3}U_{e3}^* = 1 \quad \text{(U1)}$$

$$U_{\mu 1}U_{\mu 1}^* + U_{\mu 2}U_{\mu 2}^* + U_{\mu 3}U_{\mu 3}^* = 1 \quad \text{(U2)}$$

$$U_{\tau 1}U_{\tau 1}^* + U_{\tau 2}U_{\tau 2}^* + U_{\tau 3}U_{\tau 3}^* = 1 \quad \text{(U3)}$$

$$U_{e1}U_{\mu 1}^* + U_{e2}U_{\mu 2}^* + U_{e3}U_{\mu 3}^* = 0 \quad \text{(U4)}$$

$$U_{e1}U_{\tau 1}^* + U_{e2}U_{\tau 2}^* + U_{e3}U_{\tau 3}^* = 0 \quad \text{(U5)}$$

$$U_{\mu 1}U_{\tau 1}^* + U_{\mu 2}U_{\tau 2}^* + U_{\mu 3}U_{\tau 3}^* = 0 \quad \text{(U6)}$$

•Now consider a state which is produced at $t = 0$ as a $|\nu_e\rangle$

$$|\psi(t=0)\rangle = |\nu_e\rangle = U_{e1}|\nu_1\rangle + U_{e2}|\nu_2\rangle + U_{e3}|\nu_3\rangle$$

- The wave-function evolves as:

$$|\psi(t)\rangle = U_{e1}|\nu_1\rangle e^{-ip_1.x} + U_{e2}|\nu_2\rangle e^{-ip_2.x} + U_{e3}|\nu_3\rangle e^{-ip_3.x}$$

where $p_i.x = E_i t - \vec{p}_i.\vec{x} = E_i t - |\vec{p}|z$

z axis in direction of propagation

- After a travelling a distance $L$

$$|\psi(L)\rangle = U_{e1}|\nu_1\rangle e^{-i\phi_1} + U_{e2}|\nu_2\rangle e^{-i\phi_2} + U_{e3}|\nu_3\rangle e^{-i\phi_3}$$

where $\phi_i = p_i.x = E_i t - |\vec{p}|L = (E_i - |\vec{p}_i|)L$

- As before we can approximate $\phi_i \approx \frac{m_i^2}{2E_i}L$

- Expressing the mass eigenstates in terms of the weak eigenstates

$$\begin{aligned} |\psi(L)\rangle &= U_{e1}(U_{e1}^*|\nu_e\rangle + U_{\mu 1}^*|\nu_\mu\rangle + U_{\tau 1}^*|\nu_\tau\rangle)e^{-i\phi_1} \\ &+ U_{e2}(U_{e2}^*|\nu_e\rangle + U_{\mu 2}^*|\nu_\mu\rangle + U_{\tau 2}^*|\nu_\tau\rangle)e^{-i\phi_2} \\ &+ U_{e3}(U_{e3}^*|\nu_e\rangle + U_{\mu 3}^*|\nu_\mu\rangle + U_{\tau 3}^*|\nu_\tau\rangle)e^{-i\phi_3} \end{aligned}$$

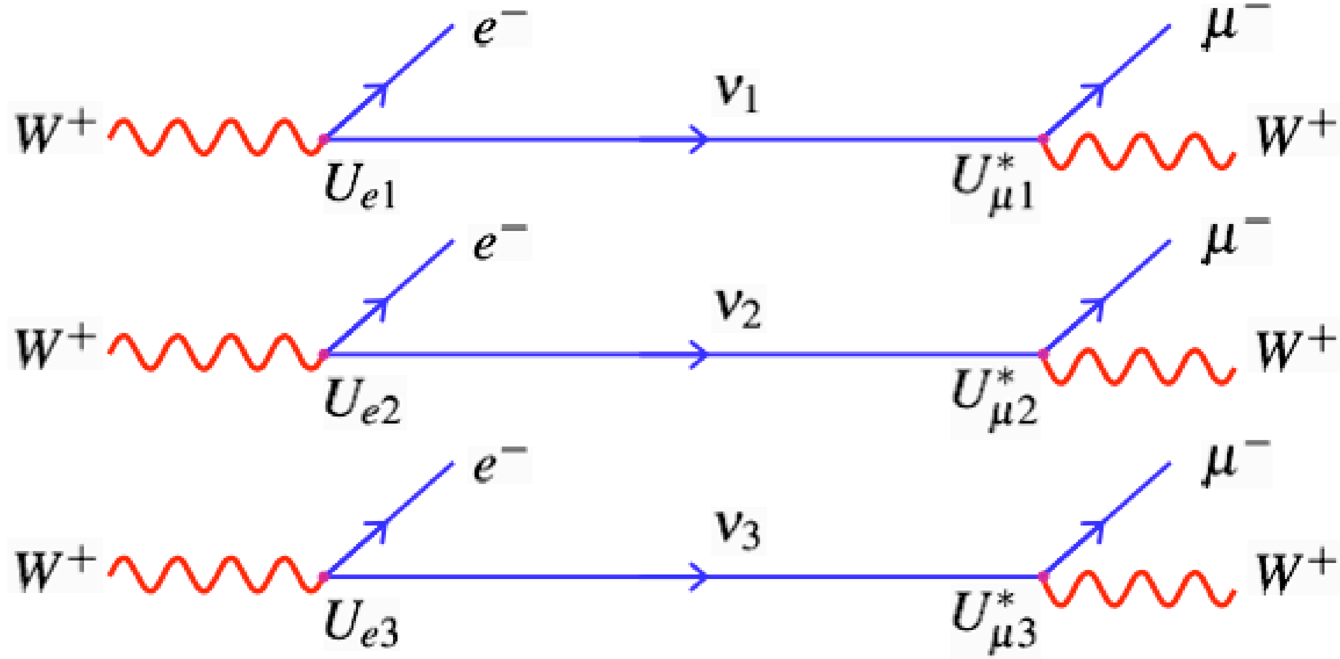
- Which can be rearranged to give

$$\begin{aligned} |\psi(L)\rangle &= (U_{e1}U_{e1}^* e^{-i\phi_1} + U_{e2}U_{e2}^* e^{-i\phi_2} + U_{e3}U_{e3}^* e^{-i\phi_3})|\nu_e\rangle \\ &+ (U_{e1}U_{\mu 1}^* e^{-i\phi_1} + U_{e2}U_{\mu 2}^* e^{-i\phi_2} + U_{e3}U_{\mu 3}^* e^{-i\phi_3})|\nu_\mu\rangle \\ &+ (U_{e1}U_{\tau 1}^* e^{-i\phi_1} + U_{e2}U_{\tau 2}^* e^{-i\phi_2} + U_{e3}U_{\tau 3}^* e^{-i\phi_3})|\nu_\tau\rangle \end{aligned} \qquad (3)$$

•From which

$$P(\nu_e \to \nu_\mu) = |\langle \nu_\mu | \psi(L) \rangle|^2$$
$$= |U_{e1}U^*_{\mu 1}e^{-i\phi_1} + U_{e2}U^*_{\mu 2}e^{-i\phi_2} + U_{e3}U^*_{\mu 3}e^{-i\phi_3}|^2$$

•The terms in this expression can be represented as:




•However, unless the phases of the different components are different, the sum of these three diagrams is zero as the unitarity of the PMNS matrix gives (U4):

$$U_{e1}U^*_{\mu 1} + U_{e2}U^*_{\mu 2} + U_{e3}U^*_{\mu 3} = 0$$

i.e. the components cancel.

•Evaluate

$$P(\nu_e \to \nu_\mu) = |U_{e1}U^*_{\mu 1}e^{-i\phi_1} + U_{e2}U^*_{\mu 2}e^{-i\phi_2} + U_{e3}U^*_{\mu 3}e^{-i\phi_3}|^2$$

using $|z_1 + z_2 + z_3|^2 \equiv |z_1|^2 + |z_2|^2 + |z_3|^2 + 2\Re(z_1 z_2^* + z_1 z_3^* + z_2 z_3^*)$ **(4)**

which gives:

$$P(\nu_e \to \nu_\mu) = |U_{e1}U^*_{\mu 1}|^2 + |U_{e2}U^*_{\mu 2}|^2 + |U_{e3}U^*_{\mu 3}|^2 + \quad (5)$$

$$2\Re(U_{e1}U^*_{\mu 1}U^*_{e2}U_{\mu 2}e^{-i(\phi_1-\phi_2)} + U_{e1}U^*_{\mu 1}U^*_{e3}U_{\mu 3}e^{-i(\phi_1-\phi_3)} + U_{e2}U^*_{\mu 2}U^*_{e3}U_{\mu 3}e^{-i(\phi_2-\phi_3)})$$

•This can be simplified by applying identity (4) to $|(U4)|^2$

$$|U_{e1}U^*_{\mu 1} + U_{e2}U^*_{\mu 2} + U_{e3}U^*_{\mu 3}|^2 = 0$$

$$\Rightarrow \quad |U_{e1}U^*_{\mu 1}|^2 + |U_{e2}U^*_{\mu 2}|^2 + |U_{e3}U^*_{\mu 3}|^2 = -2\Re(U_{e1}U^*_{\mu 1}U^*_{e2}U_{\mu 2} + U_{e1}U^*_{\mu 1}U^*_{e3}U_{\mu 3} + U_{e2}U^*_{\mu 2}U^*_{e3}U_{\mu 3})$$

•Substituting into equation (5) gives

$$\begin{aligned} P(\nu_e \to \nu_\mu) &= 2\Re\{U_{e1}U^*_{\mu 1}U^*_{e2}U_{\mu 2}[e^{-i(\phi_1-\phi_2)} - 1]\} \\ &+ 2\Re\{U_{e1}U^*_{\mu 1}U^*_{e3}U_{\mu 3}[e^{-i(\phi_1-\phi_3)} - 1]\} \\ &+ 2\Re\{U_{e2}U^*_{\mu 2}U^*_{e3}U_{\mu 3}[e^{-i(\phi_2-\phi_3)} - 1]\} \end{aligned} \quad (6)$$

★ This expression for the electron survival probability is obtained from the coefficient for $|\nu_e\rangle$ in eqn. (3):

$$P(\nu_e \to \nu_e) = |\langle \nu_e|\psi(L)\rangle|^2 = |U_{e1}U_{e1}^*e^{-i\phi_1} + U_{e2}U_{e2}^*e^{-i\phi_2} + U_{e3}U_{e3}^*e^{-i\phi_3}|^2$$

which using the unitarity relation (U1)

$$|U_{e1}U_{e1}^* + U_{e2}U_{e2}^* + U_{e3}U_{e3}^*|^2 = 1$$

can be written

$$\begin{aligned} P(\nu_e \to \nu_e) = 1 &+ 2|U_{e1}|^2|U_{e2}|^2\Re\{[e^{-i(\phi_1-\phi_2)} - 1]\} \\ &+ 2|U_{e1}|^2|U_{e3}|^2\Re\{[e^{-i(\phi_1-\phi_3)} - 1]\} \\ &+ 2|U_{e2}|^2|U_{e3}|^2\Re\{[e^{-i(\phi_2-\phi_3)} - 1]\} \end{aligned} \quad (7)$$

★ This expression can simplified using

$$\begin{aligned} \Re\{e^{-i(\phi_1-\phi_2)} - 1\} &= \cos(\phi_2 - \phi_1) - 1 \\ &= -2\sin^2\left(\frac{\phi_2 - \phi_1}{2}\right) \\ &= -2\sin^2\left(\frac{(m_2^2 - m_1^2)L}{4E}\right) \end{aligned}$$

with $\phi_i \approx \frac{m_i^2}{2E}L$

Phase of mass eigenstate $i$ at $z = L$

•Define: $$\Delta_{21} = \frac{(m_2^2 - m_1^2)L}{4E} = \frac{\Delta m_{21}^2 L}{4E}$$ with $$\Delta m_{21}^2 = m_2^2 - m_1^2$$

**NOTE:** $\Delta_{21} = (\phi_2 - \phi_1)/2$ **is a phase difference (i.e. dimensionless)**

•Which gives the electron neutrino survival probability

$$P(\nu_e \to \nu_e) = 1 - 4|U_{e1}|^2|U_{e2}|^2 \sin^2\Delta_{21} - 4|U_{e1}|^2|U_{e3}|^2 \sin^2\Delta_{31} - 4|U_{e2}|^2|U_{e3}|^2 \sin^2\Delta_{32}$$

•Similar expressions can be obtained for the muon and tau neutrino survival probabilities for muon and tau neutrinos.

★ Note that since we only have three neutrino generations there are only two independent mass-squared differences, i.e.

$$m_3^2 - m_1^2 = (m_3^2 - m_2^2) + (m_2^2 - m_1^2)$$

and in the above equation only two of the $\Delta_{ij}$ are independent

★All expressions are in Natural Units, conversion to more useful units here gives:

$$\Delta_{21} = 1.27\frac{\Delta m_{21}^2(\mathrm{eV}^2)L(\mathrm{km})}{E(\mathrm{GeV})}$$ and $$\lambda_{\mathrm{osc}}(\mathrm{km}) = 2.47\frac{E(\mathrm{GeV})}{\Delta m^2(\mathrm{eV}^2)}$$

# Neutrino Mass Hierarchy

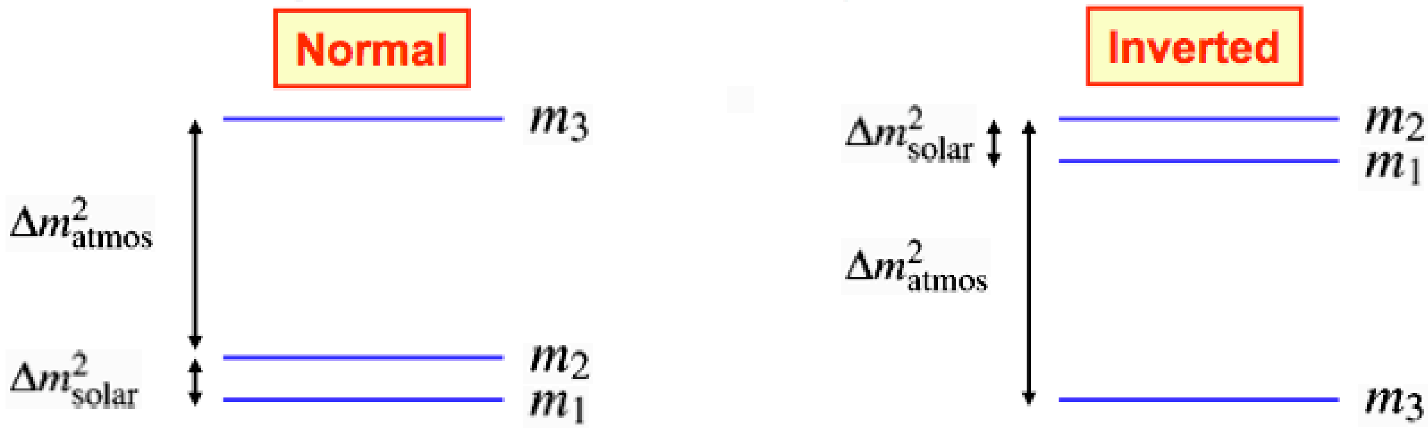
★ To date, results on neutrino oscillations only determine

$$|\Delta m^2_{ji}| = |m^2_j - m^2_i|$$

★ Two distinct and very different mass scales:

- Atmospheric neutrino oscillations : $|\Delta m^2|_{\text{atmos}} \sim 2.5 \times 10^{-3}\,\text{eV}^2$
- Solar neutrino oscillations: $|\Delta m^2|_{\text{solar}} \sim 8 \times 10^{-5}\,\text{eV}^2$

•Two possible assignments of mass hierarchy:




•In both cases: $\Delta m^2_{21} \sim 8 \times 10^{-5}\,\text{eV}^2$

$$|\Delta m^2_{31}| \approx |\Delta m^2_{32}| \sim 2.5 \times 10^{-3}\,\text{eV}^2$$

•Hence we can approximate $\Delta_{31} \approx \Delta_{32}$

# CP and T Violation in Neutrino Oscillations

- Previously derived the oscillation probability for $\nu_e \rightarrow \nu_\mu$

$$
\begin{aligned}
P(\nu_e \rightarrow \nu_\mu) &= 2\Re\{U_{e1}U^*_{\mu 1}U^*_{e2}U_{\mu 2}[e^{-i(\phi_1-\phi_2)}-1]\} \\
&+ 2\Re\{U_{e1}U^*_{\mu 1}U^*_{e3}U_{\mu 3}[e^{-i(\phi_1-\phi_3)}-1]\} \\
&+ 2\Re\{U_{e2}U^*_{\mu 2}U^*_{e3}U_{\mu 3}[e^{-i(\phi_2-\phi_3)}-1]\}
\end{aligned}
$$

- The oscillation probability for $\nu_\mu \rightarrow \nu_e$ can be obtained in the same manner or by simply exchanging the labels $(e) \leftrightarrow (\mu)$

$$
\begin{aligned}
P(\nu_\mu \rightarrow \nu_e) &= 2\Re\{U_{\mu 1}U^*_{e1}U^*_{\mu 2}U_{e2}[e^{-i(\phi_1-\phi_2)}-1]\} \\
&+ 2\Re\{U_{\mu 1}U^*_{e1}U^*_{\mu 3}U_{e3}[e^{-i(\phi_1-\phi_3)}-1]\} \\
&+ 2\Re\{U_{\mu 2}U^*_{e2}U^*_{\mu 3}U_{e3}[e^{-i(\phi_2-\phi_3)}-1]\}
\end{aligned}
\qquad (8)
$$

★ Unless the elements of the PMNS matrix are real (see note below)

$$P(\nu_e \rightarrow \nu_\mu) \neq P(\nu_\mu \rightarrow \nu_e) \qquad (9)$$

- If any of the elements of the PMNS matrix are complex, neutrino oscillations are not invariant under time reversal $t \rightarrow -t$

**NOTE:** can multiply entire PMNS matrix by a complex phase without changing the oscillation prob. T is violated if one of the elements has a different complex phase than the others

# Neutrinos and *C*, *P*, *T*

- Consider the effects of T, CP and CPT on neutrino oscillations

| | | | |
|---|---|---|---|
| T | $\nu_e \to \nu_\mu$ | $\xrightarrow{\hat{T}}$ | $\nu_\mu \to \nu_e$ |
| CP | $\nu_e \to \nu_\mu$ | $\xrightarrow{\hat{C}\hat{P}}$ | $\overline{\nu}_e \to \overline{\nu}_\mu$ |
| CPT | $\nu_e \to \nu_\mu$ | $\xrightarrow{\hat{C}\hat{P}\hat{T}}$ | $\overline{\nu}_\mu \to \overline{\nu}_e$ |

Note C alone is not sufficient as it transforms LH neutrinos into LH anti-neutrinos (not involved in Weak Interaction)

- If the weak interactions is invariant under CPT

$$P(\nu_e \to \nu_\mu) = P(\overline{\nu}_\mu \to \overline{\nu}_e)$$

and similarly

$$P(\nu_\mu \to \nu_e) = P(\overline{\nu}_e \to \overline{\nu}_\mu) \qquad (10)$$

- If the PMNS matrix is not purely real, then (9)

$$P(\nu_e \to \nu_\mu) \neq P(\nu_\mu \to \nu_e)$$

and from (10)

$$P(\nu_e \to \nu_\mu) \neq P(\overline{\nu}_e \to \overline{\nu}_\mu)$$

★Hence unless the PMNS matrix is real, **CP is violated** in neutrino oscillations!

Future experiments, e.g. "a neutrino factory", are being considered as a way to investigate CP violation in neutrino oscillations. However, CP violating effects are well below the current experimental sensitivity. In the following discussion we will take the PMNS matrix to be real.

# Neglecting CP Violation

- Neglecting CP violation considerably simplifies the algebra. Taking the PMNS matrix, equation (6) becomes:

$$P(\nu_e \to \nu_\mu) = -4U_{e1}U_{\mu 1}U_{e2}U_{\mu 2}\sin^2\Delta_{21} - 4U_{e1}U_{\mu 1}U_{e3}U_{\mu 3}\sin^2\Delta_{31} - 4U_{e2}U_{\mu 2}U_{e3}U_{\mu 3}\sin^2\Delta_{32}$$

with $\Delta_{ji} = \frac{(m_j^2 - m_i^2)L}{4E} = \frac{\Delta m_{ji}^2 L}{4E}$

- Using: $\Delta_{31} \approx \Delta_{32}$

$$P(\nu_e \to \nu_\mu) \approx -4U_{e1}U_{\mu 1}U_{e2}U_{\mu 2}\sin^2\Delta_{21} - 4(U_{e1}U_{\mu 1} + U_{e2}U_{\mu 2})U_{e3}U_{\mu 3}\sin^2\Delta_{32}$$

- Which can be simplified using (U4) $U_{e1}U_{\mu 1}^* + U_{e2}U_{\mu 2}^* + U_{e3}U_{\mu 3}^* = 0$

$$\Rightarrow \boxed{P(\nu_e \to \nu_\mu) \approx -4U_{e1}U_{\mu 1}U_{e2}U_{\mu 2}\sin^2\Delta_{21} + 4U_{e3}^2U_{\mu 3}^2\sin^2\Delta_{32}}$$

- Can apply $\Delta_{31} \approx \Delta_{32}$ to the expression for electron neutrino survival probability

$$P(\nu_e \to \nu_e) = 1 - 4U_{e1}^2U_{e2}^2\sin^2\Delta_{21} - 4U_{e1}^2U_{e3}^2\sin^2\Delta_{31} - 4U_{e2}^2U_{e3}^2\sin^2\Delta_{32}$$

$$\approx 1 - 4U_{e1}^2U_{e2}^2\sin^2\Delta_{21} - 4(U_{e1}^2 + U_{e2}^2)U_{e3}^2\sin^2\Delta_{32}$$

- Which can be simplified using (U1) $U_{e1}^2 + U_{e2}^2 + U_{e3}^2 = 1$

$$\Rightarrow \boxed{P(\nu_e \to \nu_e) \approx 1 - 4U_{e1}^2U_{e2}^2\sin^2\Delta_{21} - 4(1 - U_{e3}^2)U_{e3}^2\sin^2\Delta_{32}}$$

# Summary of Neutrino Oscillation Theory

★ Neglecting CP violation (i.e. taking the PMNS matrix to be real) and making the approximation that $|\Delta m_{31}^2| \approx |\Delta m_{32}^2|$ obtain the following expressions for neutrino oscillation probabilities:

$$P(\nu_e \to \nu_e) \approx 1 - 4U_{e1}^2 U_{e2}^2 \sin^2 \Delta_{21} - 4(1 - U_{e3}^2) U_{e3}^2 \sin^2 \Delta_{32} \quad (11)$$

$$P(\nu_\mu \to \nu_\mu) \approx 1 - 4U_{\mu 1}^2 U_{\mu 2}^2 \sin^2 \Delta_{21} - 4(1 - U_{\mu 3}^2) U_{\mu 3}^2 \sin^2 \Delta_{32} \quad (12)$$

$$P(\nu_\tau \to \nu_\tau) \approx 1 - 4U_{\tau 1}^2 U_{\tau 2}^2 \sin^2 \Delta_{21} - 4(1 - U_{\tau 3}^2) U_{\tau 3}^2 \sin^2 \Delta_{32} \quad (13)$$

$$P(\nu_e \to \nu_\mu) = P(\nu_\mu \to \nu_e) \approx -4U_{e1} U_{\mu 1} U_{e2} U_{\mu 2} \sin^2 \Delta_{21} + 4U_{e3}^2 U_{\mu 3}^2 \sin^2 \Delta_{32} \quad (14)$$

$$P(\nu_e \to \nu_\tau) = P(\nu_\tau \to \nu_e) \approx -4U_{e1} U_{\tau 1} U_{e2} U_{\tau 2} \sin^2 \Delta_{21} + 4U_{e3}^2 U_{\tau 3}^2 \sin^2 \Delta_{32} \quad (15)$$

$$P(\nu_\mu \to \nu_\tau) = P(\nu_\tau \to \nu_\mu) \approx -4U_{\mu 1} U_{\tau 1} U_{\mu 2} U_{\tau 2} \sin^2 \Delta_{21} + 4U_{\mu 3}^2 U_{\tau 3}^2 \sin^2 \Delta_{32} \quad (16)$$

★ The wavelengths associated with $\sin^2 \Delta_{21}$ and $\sin^2 \Delta_{32}$ are:

"SOLAR" $\lambda_{21} = \frac{4\pi E}{\Delta m_{21}^2}$ "Long"-Wavelength

and

$\lambda_{32} = \frac{4\pi E}{\Delta m_{32}^2}$ "ATMOSPHERIC" "Short"-Wavelength

# PMNS Matrix

★ The PMNS matrix is usually expressed in terms of 3 rotation angles $\theta_{12}, \theta_{23}, \theta_{13}$ and a complex phase $\delta$, using the notation $s_{ij} = \sin\theta_{ij}, \quad c_{ij} = \cos\theta_{ij}$

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{"Atmospheric"}} \times \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \times \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{"Solar"}}$$

**Dominates:** "Atmospheric" "Solar"

- Writing this out in full:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

★ There are six SM parameters that can be measured in ν oscillation experiments

| | | |
|---|---|---|
| $\lvert\Delta m_{21}\rvert^2 = \lvert m_2^2 - m_1^2\rvert$ | $\theta_{12}$ | Solar and reactor neutrino experiments |
| $\lvert\Delta m_{32}\rvert^2 = \lvert m_3^2 - m_2^2\rvert$ | $\theta_{23}$ | Atmospheric and beam neutrino experiments |
| | $\theta_{13}$ | Reactor neutrino experiments + future beam |
| | $\delta$ | Future beam experiments |

# Reactor Experiments

- To explain reactor neutrino experiments we need the full three neutrino expression for the electron neutrino survival probability (11) which depends on $U_{e1}, U_{e2}, U_{e3}$
- Substituting these PMNS matrix elements in Equation (11):

$$
\begin{aligned}
P(\nu_e \to \nu_e) &\approx 1 - 4U_{e1}^2 U_{e2}^2 \sin^2\Delta_{21} - 4(1 - U_{e3}^2)U_{e3}^2 \sin^2\Delta_{32} \\
&= 1 - 4(c_{12}c_{13})^2(s_{12}c_{13})^2 \sin^2\Delta_{21} - 4(1 - s_{13}^2)s_{13}^2 \sin^2\Delta_{32} \\
&= 1 - c_{13}^4(2s_{12}c_{12})^2 \sin^2\Delta_{21} - (2c_{13}s_{13})^2 \sin^2\Delta_{32} \\
&= 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{32}
\end{aligned}
$$

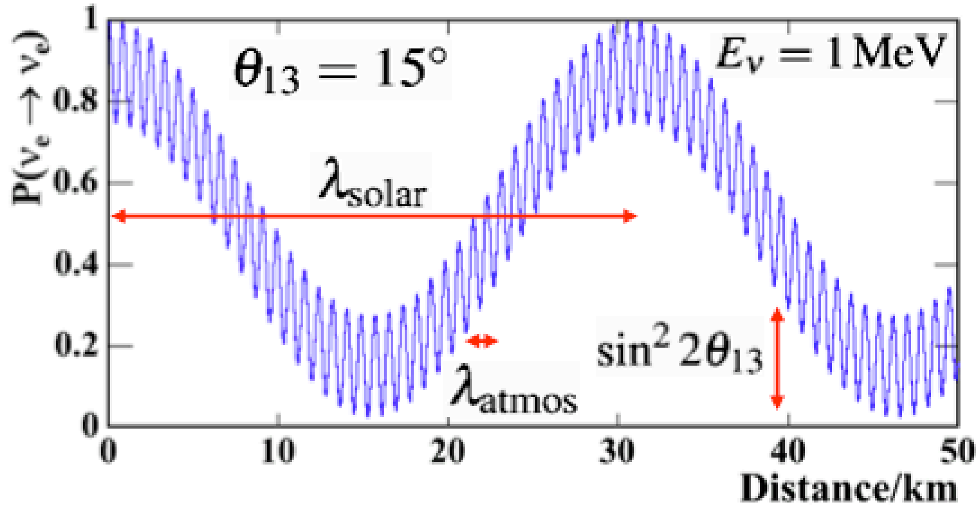
- Contributions with short wavelength (atmospheric) and long wavelength (solar)
- For a 1 MeV neutrino

$$\lambda_{osc}(\text{km}) = 2.47\frac{E(\text{GeV})}{\Delta m^2(\text{eV}^2)}$$

⇨ $\lambda_{21} = 30.0\,\text{km}$

$\lambda_{32} = 0.8\,\text{km}$

- Amplitude of short wavelength oscillations given by $\sin^2 2\theta_{13}$



# Reactor Experiments I : CHOOZ

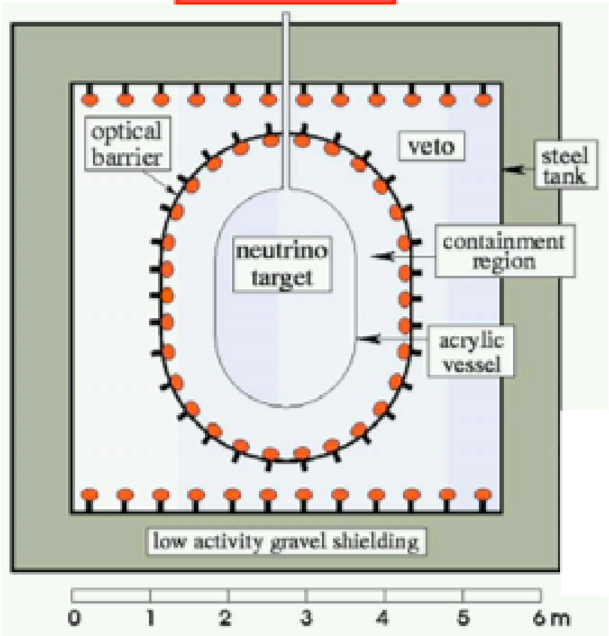
- Two nuclear reactors, each producing 4.2 GW
- Place detector 1km from reactor cores
- Reactors produce intense flux of $\overline{\nu}_e$

reactors

Detector
150m underground

France

Detector




- Anti-neutrinos interact via inverse beta decay $\overline{\nu}_e + p \rightarrow e^+ + n$
- Detector is liquid scintillator loaded with Gadolinium (large n capture cross section)
- Detect photons from positron annihilation and a delayed signal from photons from neutron capture on Gadolinium

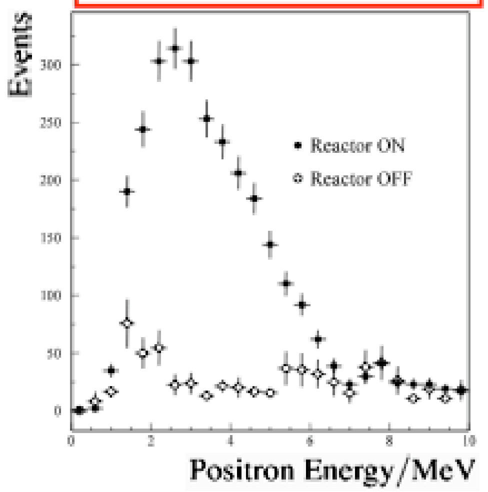
$$e^+ + e^- \rightarrow \gamma + \gamma$$

$$n + \mathrm{Gd} \rightarrow \mathrm{Gd}^* \rightarrow \mathrm{Gd} + \gamma + \gamma + \ldots$$

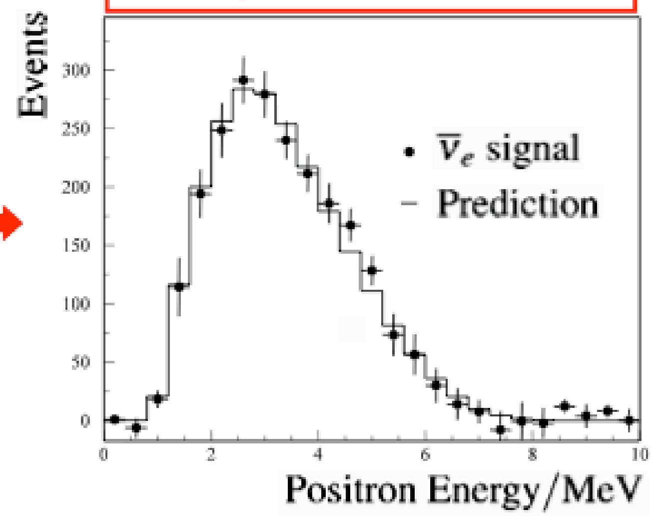
• At 1km and energies > 1 MeV, only the short wavelength component matters

$$P(\nu_e \to \nu_e) = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{32}$$
$$\approx 1 - \sin^2 2\theta_{13}\sin^2\Delta_{32}$$

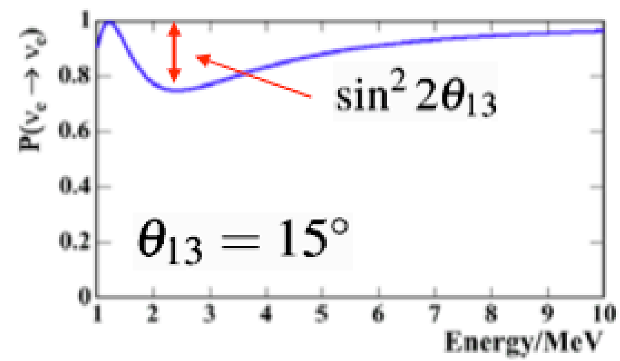
**CHOOZ Raw Data**




**Background subtracted**




**Compare to effect of oscillations**




★ Data agree with unoscillated prediction both in terms of rate and energy spectrum

$$N_{data}/N_{expect} = 1.01 \pm 0.04$$

CHOOZ Collaboration,
M.Apollonio et al.,
Phys. Lett. B420, 397-404, 1998

★ Hence $\sin^2 2\theta_{13}$ must be small !

⇨ $\sin^2 2\theta_{13} < 0.12 - 0.2$

Exact limit depends on $|\Delta m^2_{32}|$

★ But is $\theta_{13} \sim 0$ or $\theta_{13} \sim \frac{\pi}{2}$ ? Need to consider atmospheric neutrinos…

# Atmospheric Neutrinos : Revisited

★ The energies of the detected atmospheric neutrinos are of order 1 GeV

★ The wavelength of oscillations associated with $|\Delta m^2_{21}| = 8 \times 10^{-5}\,\mathrm{eV}^2$ is

$$\lambda_{21} = 31000\,\mathrm{km}$$

$$\lambda_{\mathrm{osc}}(\mathrm{km}) = 2.47\,\frac{E(\mathrm{GeV})}{\Delta m^2(\mathrm{eV}^2)}$$

- If we neglect the corresponding term in the expression for $P(\nu_\mu \to \nu_\tau)$ - equation (16)

$$\begin{aligned} P(\nu_\mu \to \nu_\tau) &\approx -4U_{\mu 1}U_{\tau 1}U_{\mu 2}U_{\tau 2}\sin^2\Delta_{21} + 4U^2_{\mu 3}U^2_{\tau 3}\sin^2\Delta_{32} \\ &\approx 4U^2_{\mu 3}U^2_{\tau 3}\sin^2\Delta_{32} \\ &= 4\sin^2\theta_{23}\cos^2\theta_{23}\cos^4\theta_{13}\sin^2\Delta_{32} \\ &= \cos^4\theta_{13}\sin^2 2\theta_{23}\sin^2\Delta_{32} \end{aligned}$$

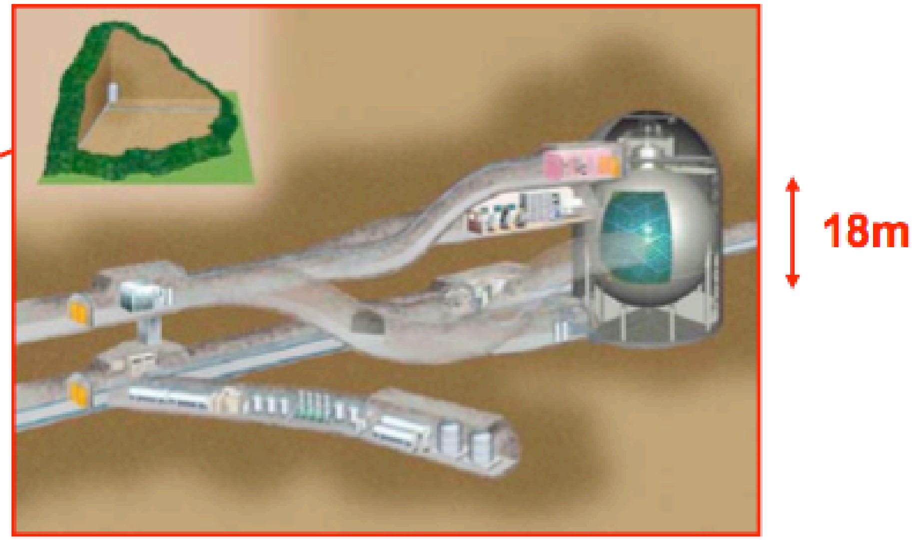
- The Super-Kamiokande data are consistent with $\nu_\mu \to \nu_\tau$ which excludes the possibility of $\cos^4\theta_{13}$ being small

- Hence the CHOOZ limit: $\sin^2 2\theta_{13} < 0.2$ can be interpreted as $\sin^2\theta_{13} < 0.05$

**NOTE:** the three flavour treatment of atmospheric neutrinos is discussed in the appendix. The oscillation parameters in nature conspire in such a manner that the two flavour treatment provides a good approximation of the observable effects of atmospheric neutrino oscillations

# Reactor Experiments II : KamLAND


- Detector located in same mine as Super Kamiokande



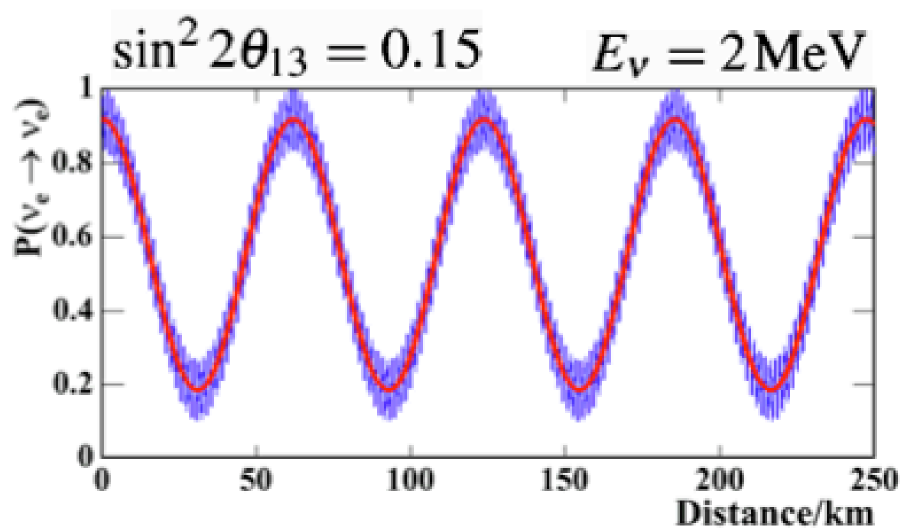

- 70 GW from nuclear power (7% of World total) from reactors within 130-240 km
- Liquid scintillator detector, 1789 PMTs
- Detection via inverse beta decay: $\nu_e + p \rightarrow e^+ + n$

  Followed by $e^+ + e^- \rightarrow \gamma + \gamma$ prompt

  $n + p \rightarrow d + \gamma(2.2\,\mathrm{MeV})$ delayed

- For MeV neutrinos at a distance of 130-240 km oscillations due to $\Delta m^2_{32}$ are very rapid
- Experimentally, only see average effect

$$\langle \sin^2 \Delta_{32} \rangle = 0.5$$




★ **Here:**

$$\begin{aligned} P(\nu_e \to \nu_e) &= 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{32} \\ &\approx 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \frac{1}{2}\sin^2 2\theta_{13} \\ &= \cos^4\theta_{13} + \sin^4\theta_{13} - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} \\ &\approx \cos^4\theta_{13}(1 - \sin^2 2\theta_{12}\sin^2\Delta_{21}) \end{aligned}$$

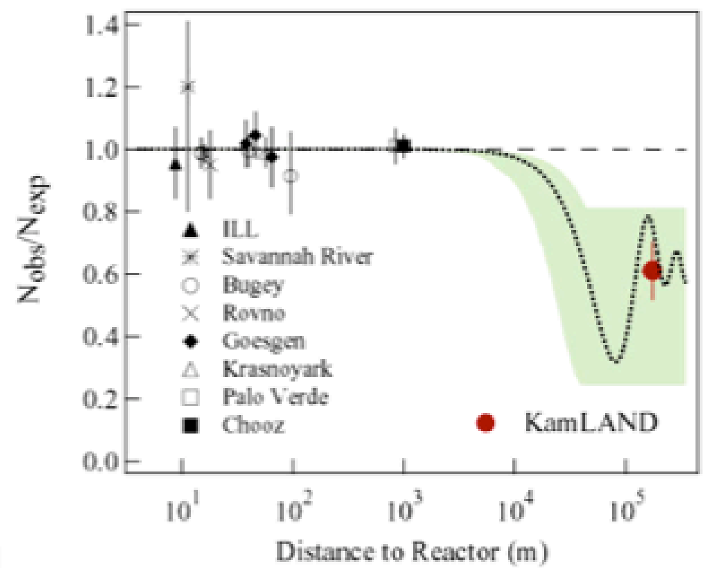
neglect $\sin^4\theta_{13}$

- Obtain two-flavour oscillation formula multiplied by $\cos^4\theta_{13}$
- From CHOOZ $\cos^4\theta_{13} > 0.9$

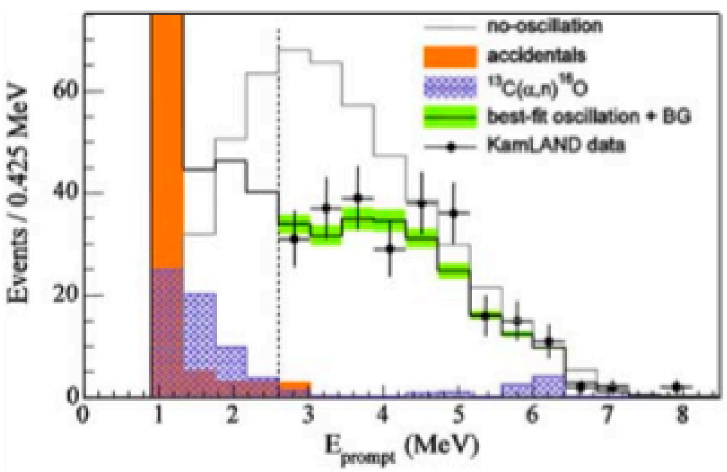
## KamLAND RESULTS:

**Observe:** 258 events
**Expect:** 365±24 events

$$\langle P(\overline{\nu}_e \rightarrow \overline{\nu}_e) \rangle = 0.66 \pm 0.07$$




+ observe distortion of spectrum

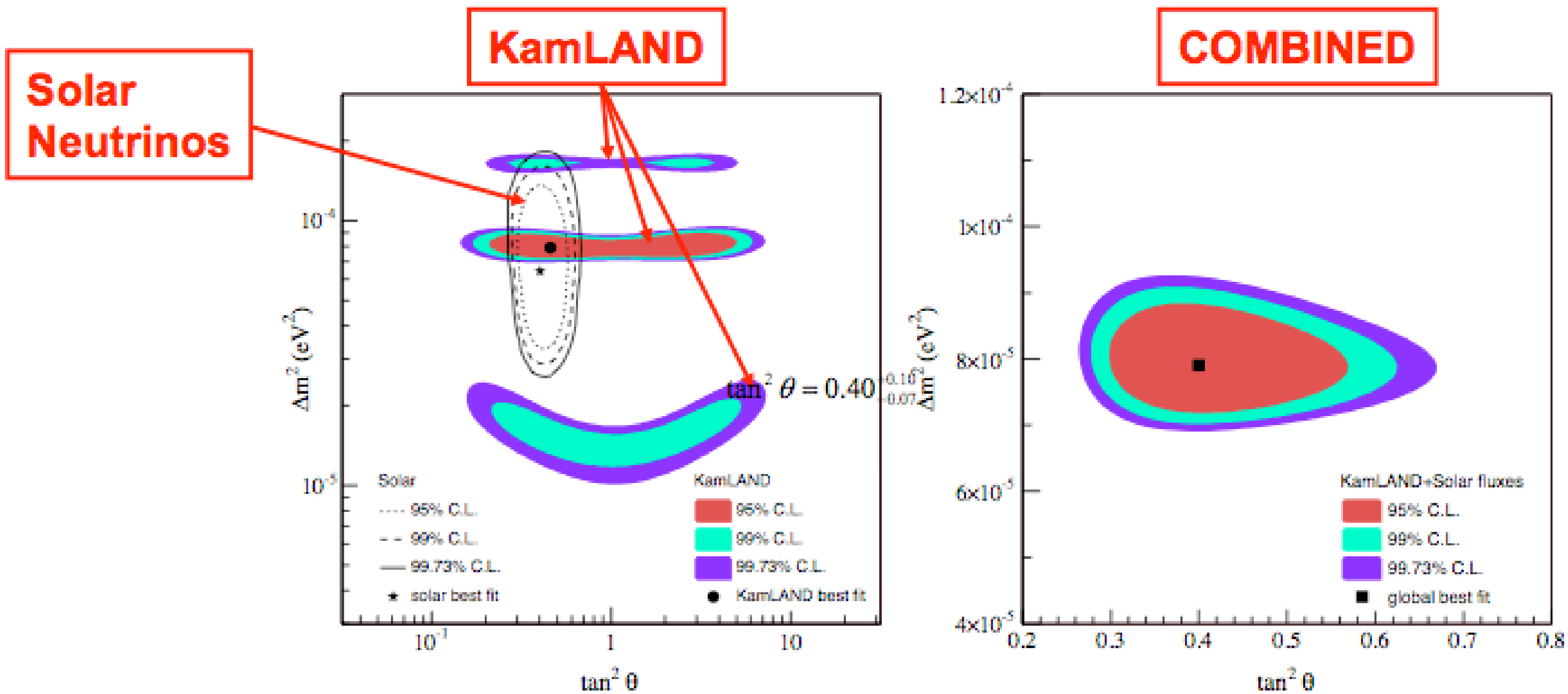



KamLAND, T. Arake et al., Phys.Rev.Lett.94:081801,2005

★ **Clear evidence of electron anti-neutrino oscillations consistent with the results from solar neutrinos**

# Combined Solar Neutrino and KamLAND Results

- ★ Solar neutrino data (especially SNO) provides a strong constraint on $\theta_{12}$
- ★ KamLAND data provides strong constraints on $|\Delta m^2_{32}|$




$$|\Delta m^2_{21}| = 7.9^{+0.6}_{-0.5} \times 10^{-5}\,\mathrm{eV}^2$$

$$\tan^2\theta_{12} = 0.40^{+0.1}_{-0.07}$$

# Long Baseline Neutrino Experiments

- From studies of atmospheric and solar neutrinos we have learnt a great deal.
- In future, emphasis of neutrino research will shift to neutrino beam experiments
- Allows the physicist to take control – design experiment with specific goals
- In the last few years, long baseline neutrino oscillation experiments have started taking data: K2K, MINOS, CNGS
- Opening up a new era in precision neutrino physics

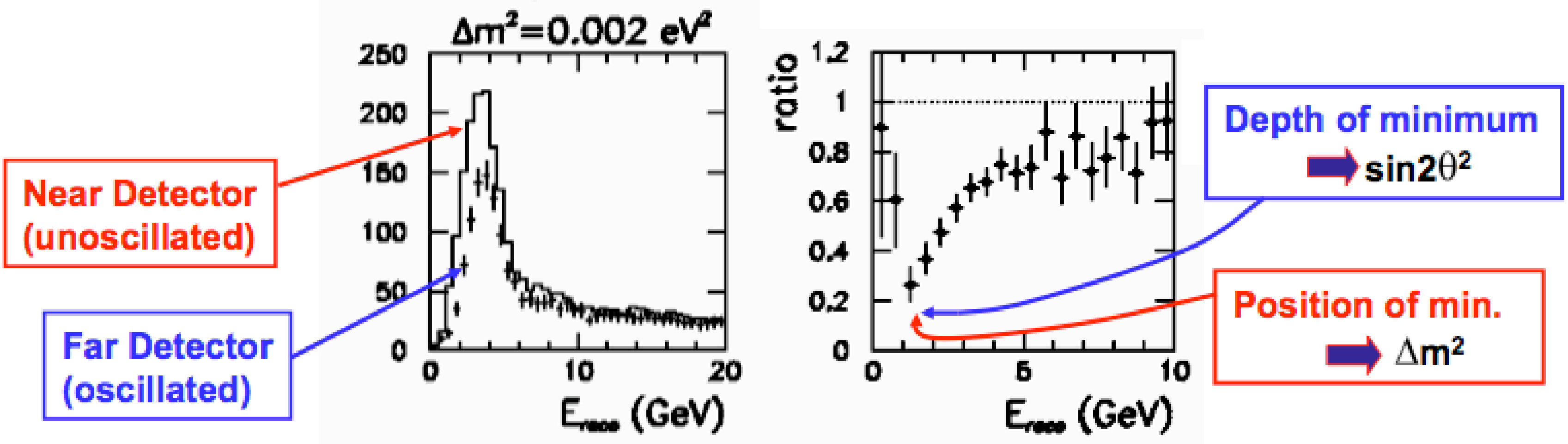
**Basic Idea:**

★ Intense neutrino beam

★ Two detectors: one close to beam the other hundreds of km away

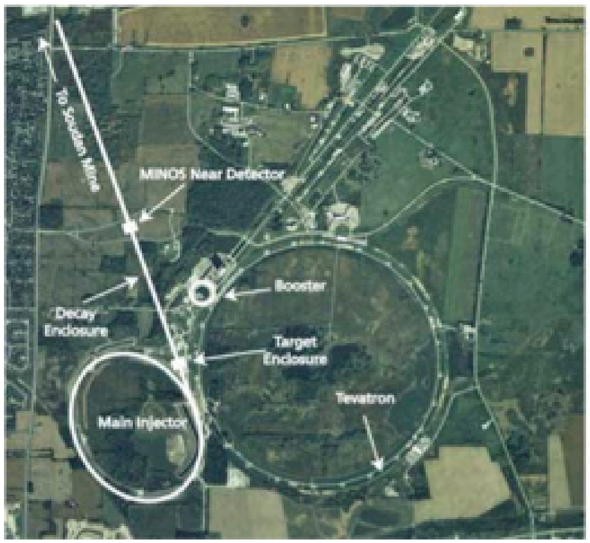
★ Measure ratio of the neutrino energy spectrum in far detector (oscillated) to that in the near detector (unoscillated)

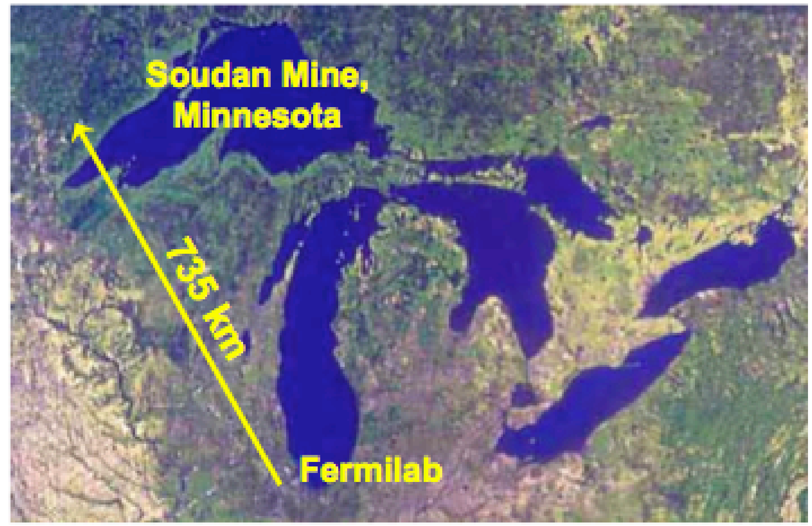
★ Partial cancellation of systematic biases



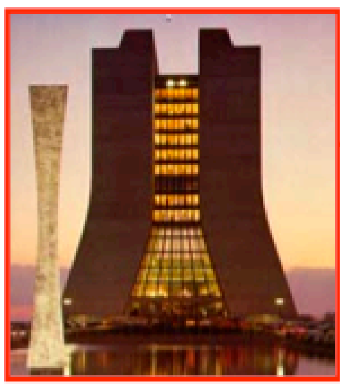
# The MINOS Experiment

- 120 GeV protons extracted from the MAIN INJECTOR at Fermilab (see p. 267)
- $2.5 \times 10^{13}$ protons per pulse hit target → very intense beam - 0.3 MW on target




**<u>Two detectors:</u>**

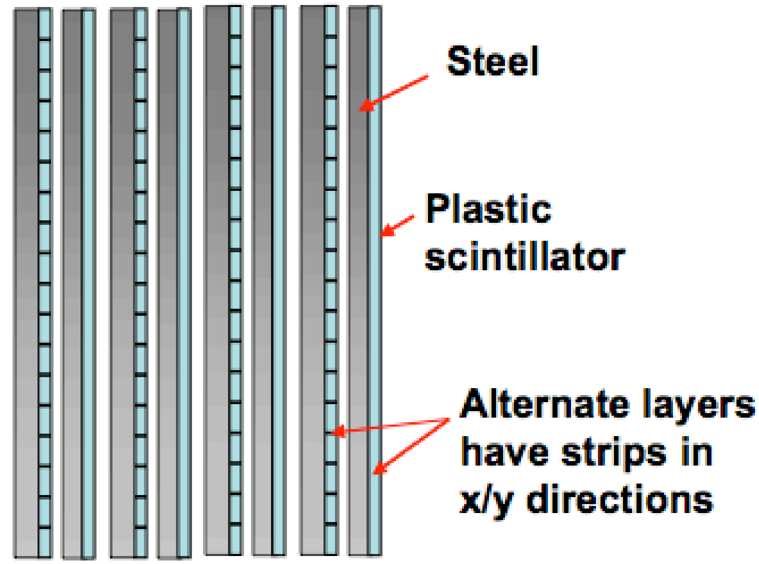


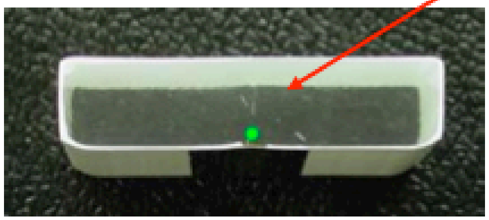
★ 1000 ton, NEAR Detector at Fermilab : 1 km from beam

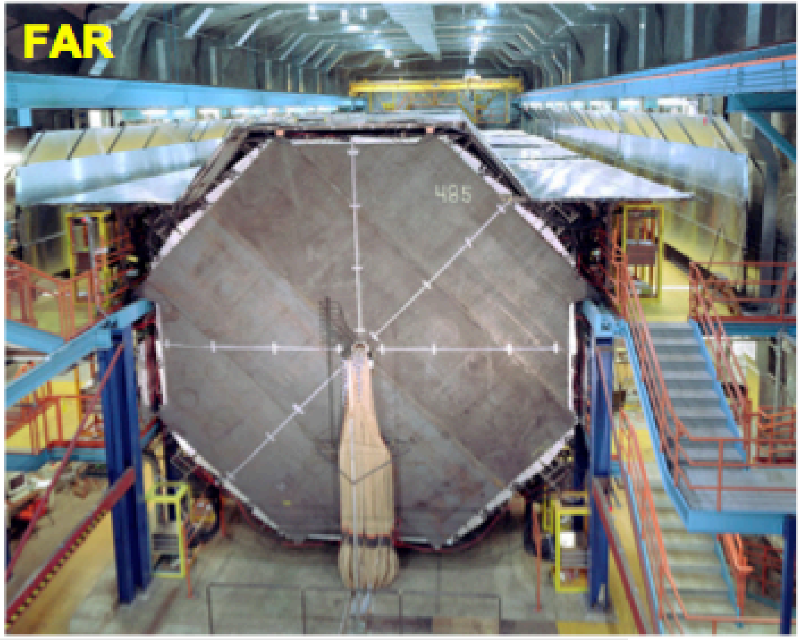
★ 5400 ton FAR Detector, 720m underground in Soudan mine, N. Minnesota: 735 km from beam

## The MINOS Detectors:

- Dealing with high energy neutrinos $E_\nu > 1\,\mathrm{GeV}$
- The muons produced by $\nu_\mu$ interactions travel several metres
- Steel-Scintillator sampling calorimeter
- Each plane: 2.54 cm steel +1 cm scintillator
- A charged particle crossing the scintillator produces light – detect with PMTs

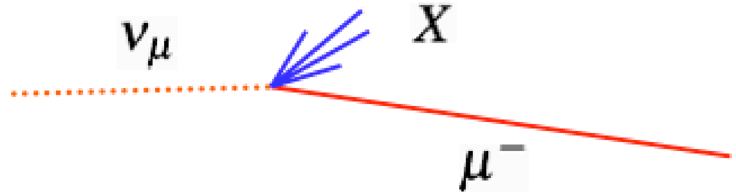


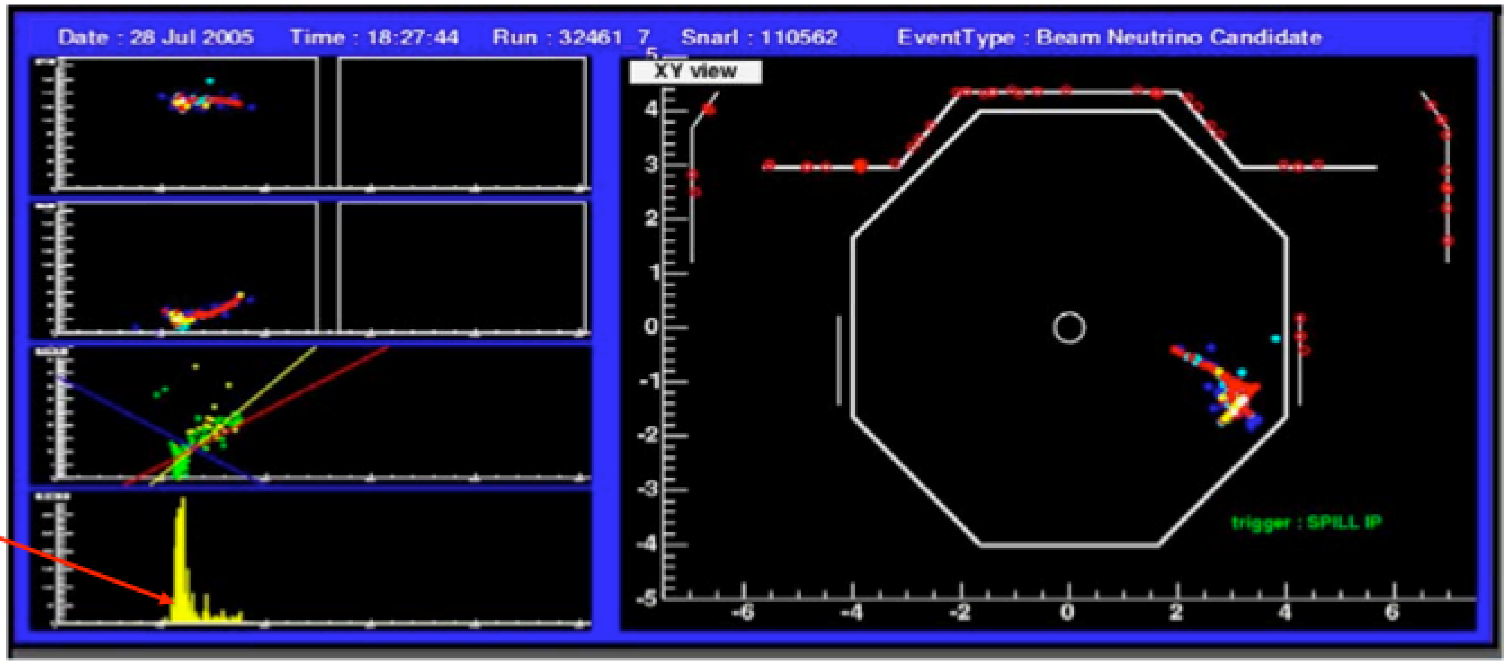



- Neutrino detection via CC interactions on nucleon

$$\nu_\mu + N \rightarrow \mu^- + X$$




**Example event:**




Signal from hadronic shower

- The main feature of the MINOS detector is the very good neutrino energy resolution

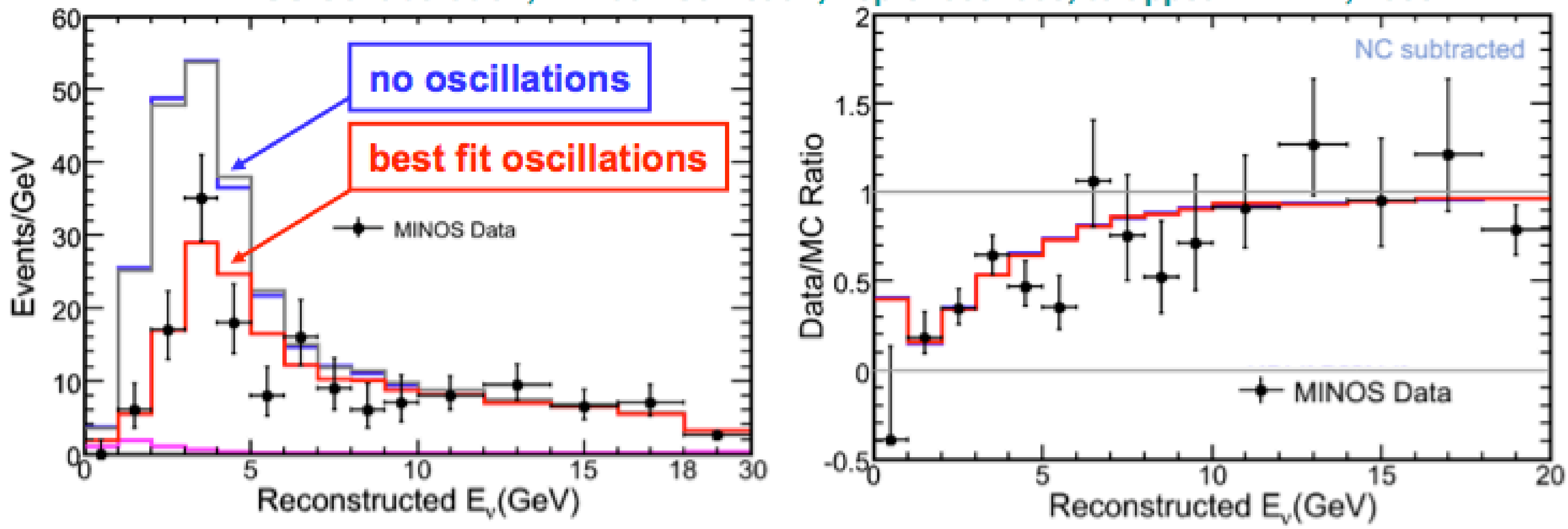
$$E_\nu = E_\mu + E_X$$

- Muon energy from range/curvature in B-field
- Hadronic energy from amount of light observed

# MINOS Results

- For the MINOS experiment L is fixed and observe oscillations as function of $E_\nu$
- For $|\Delta m^2_{32}| \sim 2.5 \times 10^{-3}\,\mathrm{eV}^2$ first oscillation minimum at $E_\nu = 1.5\,\mathrm{GeV}$
- To a very good approximation can use two flavour formula as oscillations corresponding to $|\Delta m^2_{21}| \sim 8 \times 10^{-5}\,\mathrm{eV}^2$ occur at $E_\nu = 50\,\mathrm{MeV}$, beam contains very few neutrinos at this energy + well below detection threshold
- First results (Summer 2006) – relatively small amount of data

MINOS Collaboration, P. Adamson et al., hep-ex0607088, to appear in PRL, 2006




$$|\Delta m^2_{32}| = 2.74^{+0.44}_{-0.26} \times 10^{-3}\,\mathrm{eV}^2$$

# Summary of Current Knowledge

**SOLAR Neutrinos/KamLAND**

KamLAND + Solar: $|\Delta m^2_{21}| \approx (7.9 \pm 0.5) \times 10^{-5}\, \mathrm{eV}^2$

SNO + KamLAND + Solar: $\tan^2\theta_{12} \approx 0.40 \pm 0.08$

➡ $\sin\theta_{12} \approx 0.53; \quad \cos\theta_{12} \approx 0.85$

**Atmospheric Neutrinos/Long Baseline experiments**

MINOS/Super-Kamiokande: $|\Delta m^2_{32}| \approx (2.7 \pm 0.3) \times 10^{-3}\, \mathrm{eV}^2$

Super Kamiokande: $\sin^2 2\theta_{23} > 0.92$

$$\cos\theta_{23} \approx \sin\theta_{23} \approx \frac{1}{\sqrt{2}}$$

**CHOOZ + (atmospheric)**

$$\sin^2\theta_{13} < 0.06$$

★ Currently no knowledge about CP violating phase $\delta$

•In the limit $\theta_{13} \approx 0$

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \approx \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}c_{23} & c_{12}c_{23} & s_{23} \\ s_{12}s_{23} & -c_{12}s_{23} & c_{23} \end{pmatrix}$$

•For the approximate values of the mixing angles on the previous page obtain:

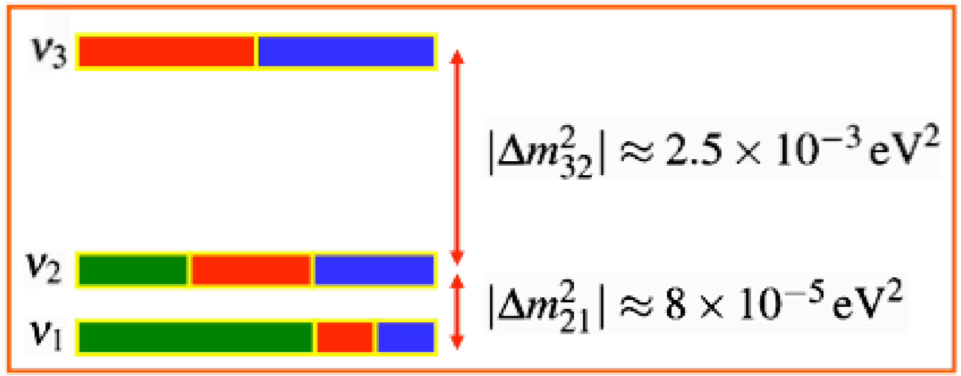
$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \approx \begin{pmatrix} 0.85 & 0.53 & 0 \\ -0.37 & 0.60 & 0.71 \\ 0.37 & -0.60 & 0.71 \end{pmatrix}$$

★ Have approximate expressions for mass eigenstates in terms of weak eigenstates:

$$|\nu_3\rangle \approx \frac{1}{\sqrt{2}}(|\nu_\mu\rangle + |\nu_\tau\rangle)$$

$$|\nu_2\rangle \approx 0.53|\nu_e\rangle + 0.60(|\nu_\mu\rangle - |\nu_\tau\rangle)$$

$$|\nu_1\rangle \approx 0.85|\nu_e\rangle - 0.37(|\nu_\mu\rangle - |\nu_\tau\rangle)$$




★ 10 years ago – assumed massless neutrinos + hints that neutrinos might oscillate !

★ Now, know a great deal about massive neutrinos

★ But many unknowns: $\theta_{13}, \delta$ , mass hierarchy, absolute values of neutrino masses

★ Focus of current/next generation of experiments