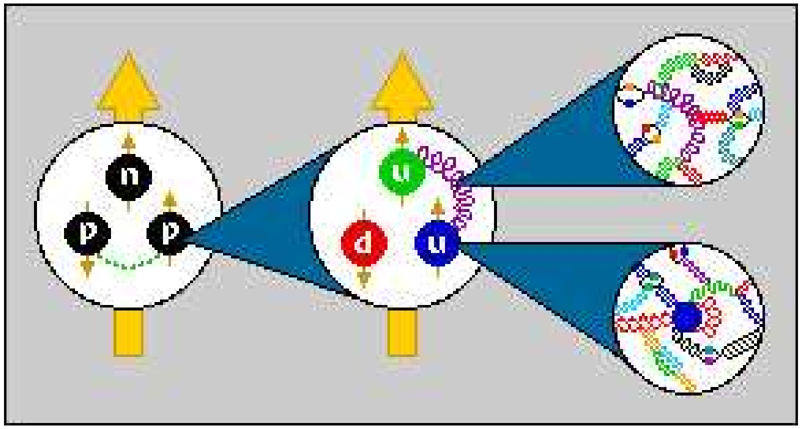
Lecture 19

- The Parton Model
- Bjorken Scaling
- Parton Distribution Functions

Slides from Sobie and Blokland

Extending the Rutherford Experiment

Recall that based on a surprisingly high number of large-angle events in elastic α Au scattering, Rutherford deduced atomic substructure (i.e., the nucleus)



In a similar fashion, one can investigate the angles involved in $e\, p$ scattering, particularly in the *deep inelastic scattering* regime where q^2 is large.

The proton was found to have substructure (SLAC, late 1960s). These constituents came to be known as *partons*. Although we now recognize them as quarks and gluons

Three Levels of Behavior

- A low-energy electron scatters *elastically* off a proton. This is relatively simple to understand in terms of the *elastic form factors* $K_1(q^2)$ and $K_2(q^2)$.
- A medium-energy electron usually scatters *inelastically* off a proton. This behavior is quite complicated and it involves the *inelastic structure functions* $W_1(q^2, x)$ and $W_2(q^2, x)$.
- A high-energy electron scatters *elastically* off partons within the proton. This behavior is simpler to understand than inelastic scattering and it involves *parton distribution functions*.

Elastic and inelastic scattering

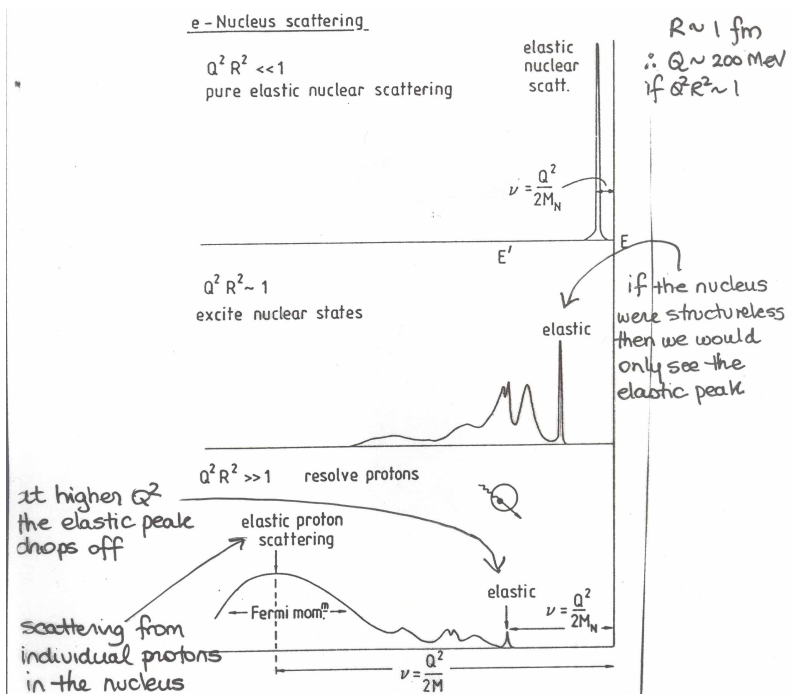


Figure 3. Schematic illustration of the cross-section for electron–nucleus scattering as a function of the energy E' of the scattered electron at three different values of Q^2. The energy lost by the electron in the laboratory frame, $\nu = E - E'$, is given by the distance from the right-hand axis. R is the radius of the nucleus.

Parton scattering

- The cross section $e + p \rightarrow e + X$ should reduce to $e + q \rightarrow e + q$ (which is identical to $e + \mu \rightarrow e + \mu$)

$$\frac{d\sigma}{d\Omega dE_3}(e\mu \rightarrow e\mu) = \frac{4\alpha^2 E_3^2}{q^4}\left[cos^2\frac{\theta}{2} - \frac{q^2}{2m}sin^2\frac{\theta}{2}\right]\delta(\nu + \frac{q^2}{2m})$$

$$\frac{d\sigma}{d\Omega dE_3}(ep \rightarrow eX) = \frac{4\alpha^2 E_3^2}{q^4}\left[W_2 cos^2\frac{\theta}{2} + 2W_1 sin^2\frac{\theta}{2}\right]$$

where $\nu = E_1 - E_3$ (initial and final energies)

- For convience define $Q^2 - -q^2$ (Q^2 is a negative quantity).

- Relating $e\mu \rightarrow e\mu$ and $ep \rightarrow eX$ cross sections gives

$$2W_1^{point} = \frac{Q^2}{2m}\delta(\nu - \frac{Q^2}{2m}) \qquad W_2^{point} = \delta(\nu - \frac{Q^2}{2m})$$

- At large Q^2, inelastic *ep* scattering is viewed as elastic *e*-quark scattering off a free quark inside the proton

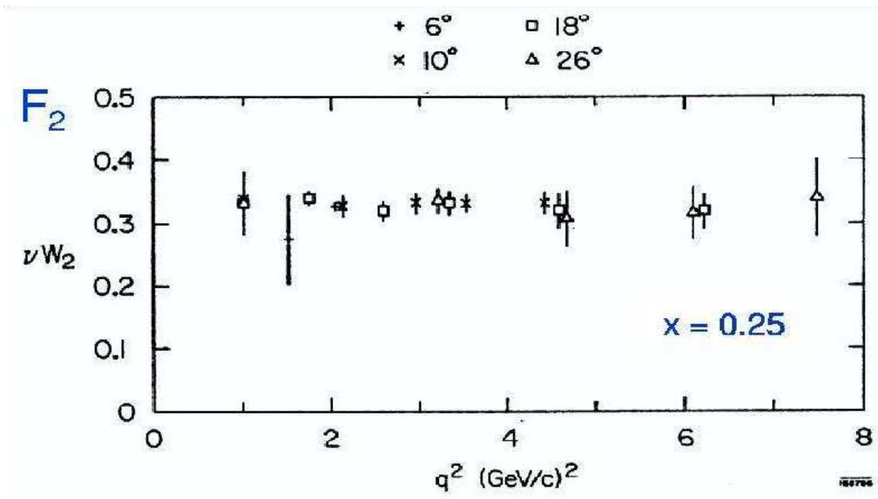
Bjorken Scaling

- Bjorken's hypothesis was that the inelastic structure functions, at high energy and *at fixed* x (recall $x = Q^2/2M\nu$), cease to depend on q^2. More specifically,

$$MW_1(\nu, Q^2) \rightarrow F_1(x) \qquad \nu\, W_2(\nu, Q^2) \rightarrow F_2(x)$$

as $Q^2 \rightarrow \infty$

- This behavior, which was confirmed in the 1970s at SLAC, is known as *scaling*.



J.T.Friedman + H.W.Kendall,
Ann. Rev. Nucl. Sci. **22** (1972) 203

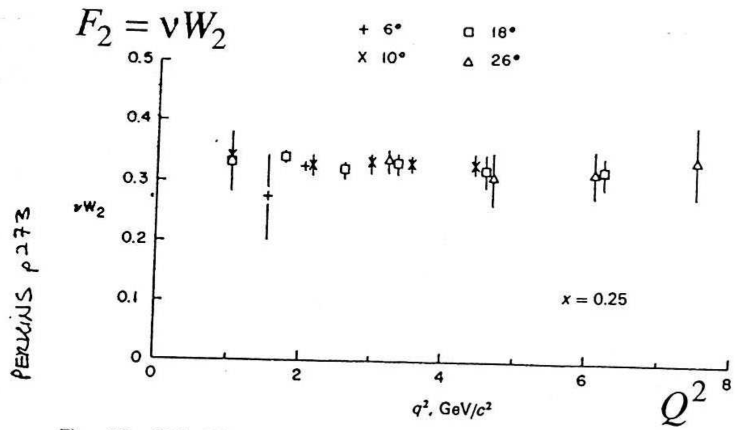
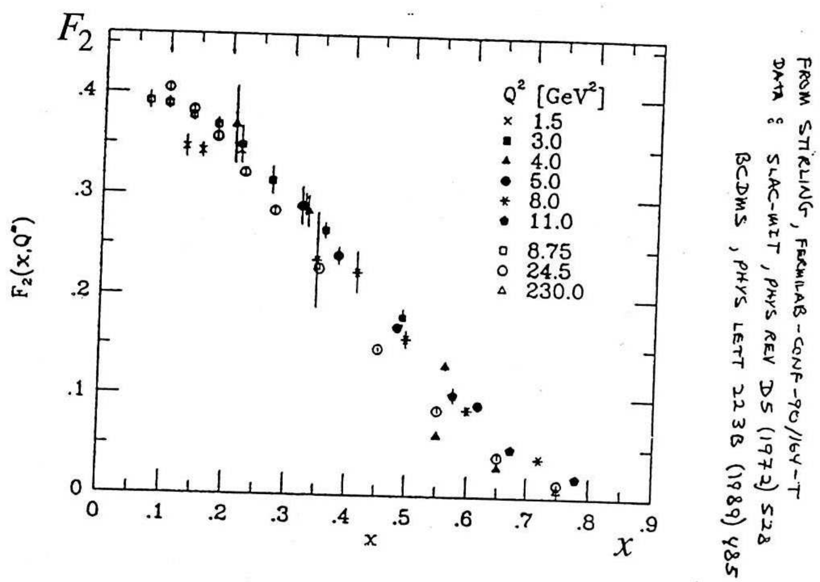


Figure 8.8 νW_2 (or F_2) as a function of q^2 at $x = 0.25$. For this choice of x, there is practically no q^2-dependence, that is, exact "scaling." (After Friedman and Kendall (1972).)



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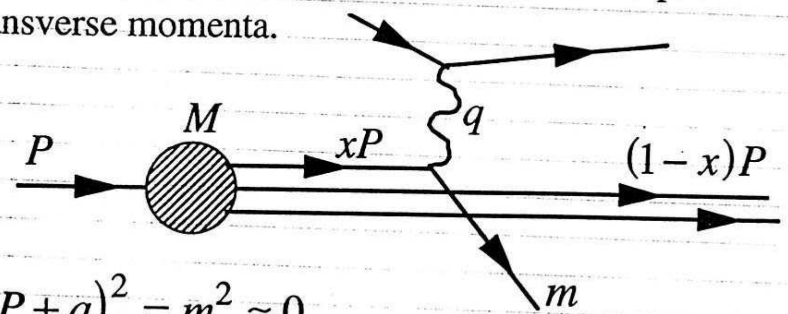
Feynman gave a physical interpretation of scale invariance in the parton model.

Go to the "infinite momentum frame", where the target nucleon has a very large 3-momentum

$$P = P(|\underline{P}|, |\underline{P}|, 0, 0).$$

Since $|\underline{P}|$ is large, we can neglect masses and parton transverse momenta.



$$(xP+q)^2 = m^2 \approx 0$$

$$x^2P^2 + q^2 + 2xP\cdot q \approx 0$$

If $x^2P^2 = x^2M^2 << Q^2 = -q^2$,

then $x = \dfrac{Q^2}{2P\cdot q} = \dfrac{Q^2}{2M\nu}$

Therefore x is the fractional 3-momentum of the parton in the infinite momentum frame.

Callan-Gross relation

- Suppose each parton in the proton carries a fraction x of the proton momentum $p_i^\mu = xp^\mu$ and $m_i = xM_p$
- Comparing the $e\ q \rightarrow e\ q$ and $e\ q \rightarrow e\ X$ cross sections gives

$$2W_1^i(point) = e_i^2 \frac{Q^2}{2M_p^2 x}\delta(\nu - \frac{Q^2}{2M_p x}) \qquad W_2^i(point) = e_i^2\delta(\nu - \frac{Q^2}{2M_p x})$$

- To get the total contributions of all the partons we need to integrate over all x and weight it by the probability $f_i(x)$ for parti i having fraction x of the proton momentum

$$W_2(point) = \sum_i \int_0^1 dx f_i(x) e_i^2 \delta(\nu - \frac{Q^2}{2M_p x})$$

- This gives

$$\nu W_2(point) = F_2(x) = x\sum_i e_i^2 f_i(x)$$

$$M_p W_1(point) = F_1(x) = \sum_i e_i^2 \frac{f_i(x)}{2}$$

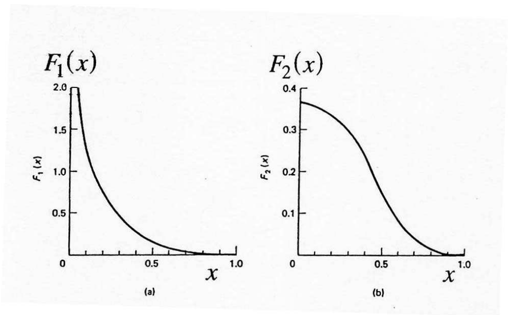
The Callan-Gross Relation

- While Bjorken scaling is a consequence of partons within the proton, it does not restrict the specific type of partons.
- By making an assumption on the spin of an individual parton, Bjorken's scaling functions, $F_1(x)$ and $F_2(x)$ can be related to each other:

$$\text{Spin-0 partons:} \quad \Rightarrow \quad \frac{2xF_1(x)}{F_2(x)} = 0$$

$$\text{Spin-}\tfrac{1}{2}\text{ partons:} \quad \Rightarrow \quad \frac{2xF_1(x)}{F_2(x)} = 1$$

- Experiments indicate the partons have spin-$\frac{1}{2}$. $2xF_1(x) = F_2(x)$ is known as the *Callan-Gross relation*. (Gross is one of the 2004 Nobel Prize winners.)



Parton Distribution Functions

- We have now related Bjorken's scaling functions $F_{1,2}(x)$ to the probability distribution functions (hereafter to be called PDFs) $f_i(x)$.
- To a first approximation, if quarks are truly free within the nucleus for sufficiently high-energy probes, the PDFs will be δ-functions. For the proton then,

$$F_2^p(x) \quad = \quad x\left\{2\left(\frac{2}{3}\right)^2\delta\left(x-\frac{m_u}{M}\right)+\left(-\frac{1}{3}\right)^2\delta\left(x-\frac{m_d}{M}\right)\right\}$$

- More generally, we can incorporate the QCD interactions between quarks by generalizing the PDFs:

$$F_2^p(x) = x\left\{\left(\frac{2}{3}\right)^2 u(x) + \left(\frac{1}{3}\right)^2 d(x)\right\}$$

Constraints on PDFs

- The precise determination of $u(x)$ and $d(x)$ will be left to experiment, but these functions must satisfy certain *sum rules*:

$$\int_0^1 xu(x)\,dx = 2\int_0^1 xd(x)\,dx$$

  (i.e., total momentum carried by u quarks is twice that of d quarks.)

- Experimental surprise: both sides of the above equation are measured to be 0.36, meaning that only 54% of the proton's momentum is accounted for. What happened to the other 46%?

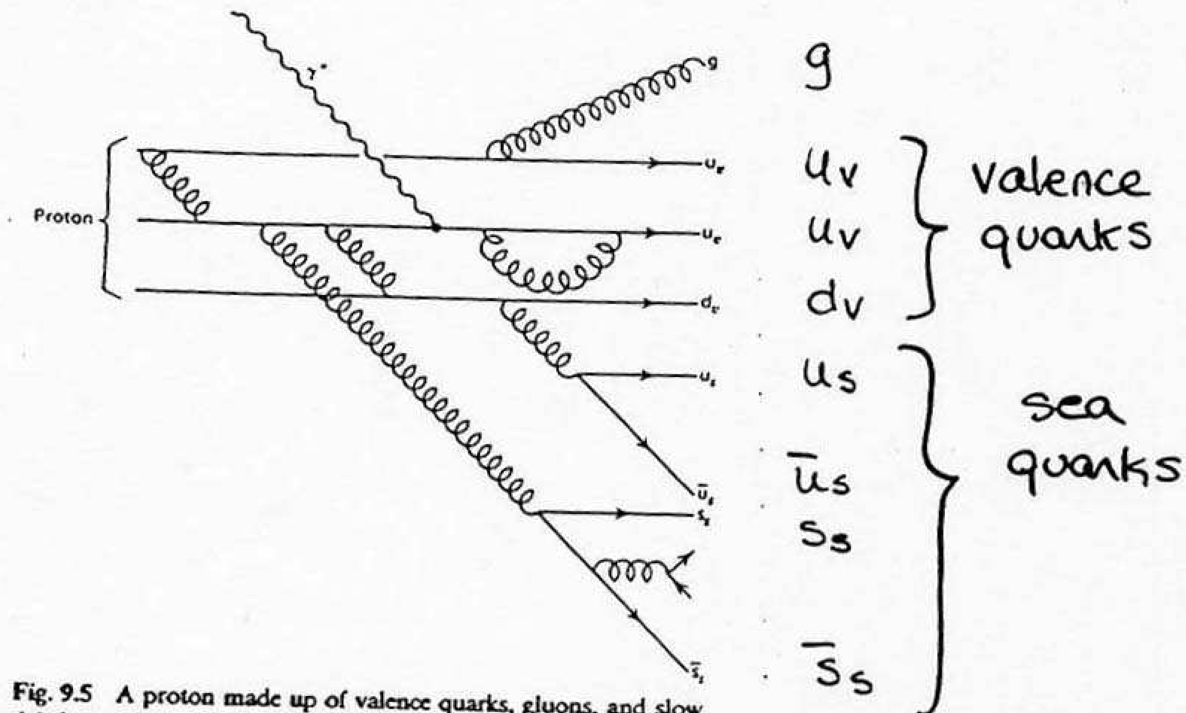


Fig. 9.5 A proton made up of valence quarks, gluons, and slow debris consisting of quark–antiquark pairs.

Gluons

- Since gluons are electrically neutral, they do not contribute to $e\,p$ scattering, but they are evidently hoarding away some of the proton momentum (and spin too).
- This is one way in which QCD adds complexity to the Constituent Quark Model. Another is the presence of additional quarks via $g \to q\,\bar{q}$. This leads to a long list of PDFs that will be required to describe the proton accurately:

$$u(x) \quad d(x) \quad s(x) \quad \ldots \quad \bar{u}(x) \quad \bar{d}(x) \quad \bar{s}(x) \quad \ldots \quad g(x)$$

- This is discouraging. Where we once had just one unknown function $F_2(x)$, we now have 13!

Relating the PDFs

- By distinguishing between *valence* and *sea* quarks, we can clear up most of the clutter. Since the sea quarks are all produced by the same gluon-splitting mechanism,

$$\bar{u}(x) \simeq \bar{d}(x) \simeq \bar{s}(x) \simeq s(x)$$

  The c, b, and t quarks are sufficiently heavy as to be ignored.

- For $u(x)$ and $d(x)$, we separate the valence and sea contributions, so that

$$u(x) = u_v(x) + s(x) \qquad d(x) = d_v(x) + s(x)$$

- The neutron PDFs are related to the proton PDFs by isospin (i.e., $u_v^n(x) = d_v^p(x)$), so we have many different ways to measure the PDFs.

Proton and Neutral PDFs I

Proton structure function (uud)

$$\frac{F_2^p(x)}{x} = \frac{4}{9}\left[u^p + \overline{u}^p\right] + \frac{1}{9}\left[d^p + \overline{d}^p\right] + \frac{1}{9}\left[s^p + \overline{s}^p\right]$$

Similarly the neutron structure function (udd)

$$\frac{F_2^n(x)}{x} = \frac{4}{9}\left[u^n + \overline{u}^n\right] + \frac{1}{9}\left[d^n + \overline{d}^n\right] + \frac{1}{9}\left[s^n + \overline{s}^n\right]$$

Using isospin invariance

$$u^p = d^n = u(x)$$

$$d^p = u^n = d(x)$$

$$s^p = s^n = s(x)$$

We get

$$\frac{F_2^p(x)}{x} = \frac{4}{9}\left[u + \overline{u}\right] + \frac{1}{9}\left[d + \overline{d} + s + \overline{s}\right]$$

$$\frac{F_2^n(x)}{x} = \frac{4}{9}\left[d + \overline{d}\right] + \frac{1}{9}\left[u + \overline{u} + s + \overline{s}\right]$$

Proton and Neutral PDFs II

The proton consists of 3 valence quarks (u_v, u_v, d_v) accompanied by many quark-antiquark pairs

$$u = u_v + u_s \qquad\qquad d = d_v + d_s$$

$$u_s = \overline{u}_s = d_s = \overline{d}_s = s_s = \overline{s}_s = s$$

The quark distributions must give the correct quantum numbers

$$\int [u - \overline{u}]\, dx = 2$$

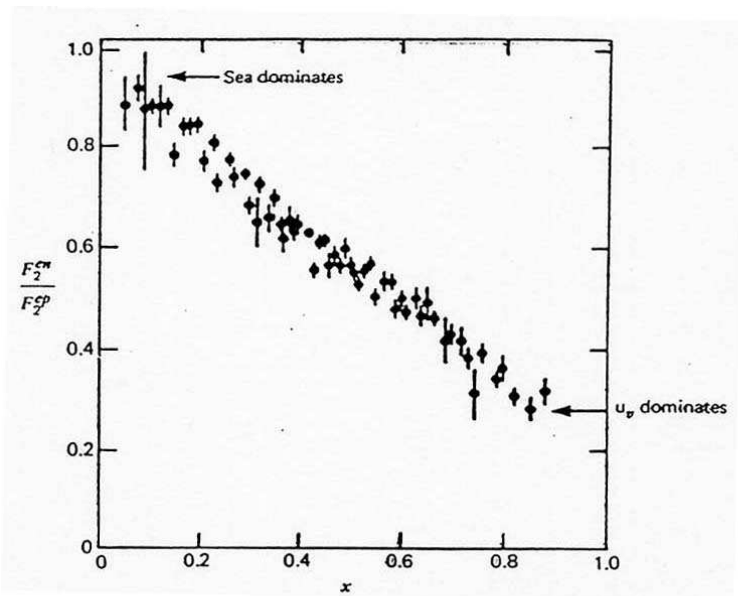
$$\int \left[d - \overline{d}\right] dx = 1$$

$$\int [s - \overline{s}]\, dx = 0$$

So that

$$\frac{F_2^p(x)}{x} = \frac{1}{9}\left[4u_v + d_v\right] + \frac{4}{3}s$$
$$\frac{F_2^n(x)}{x} = \frac{1}{9}\left[u_v + 4d_v\right] + \frac{4}{3}s$$

Comparison with experiment I



$$\frac{F_2^p(x)}{x} = \frac{1}{9}\left[4u_v + d_v\right] + \frac{4}{3}s$$

$$\frac{F_2^n(x)}{x} = \frac{1}{9}\left[u_v + 4d_v\right] + \frac{4}{3}s$$

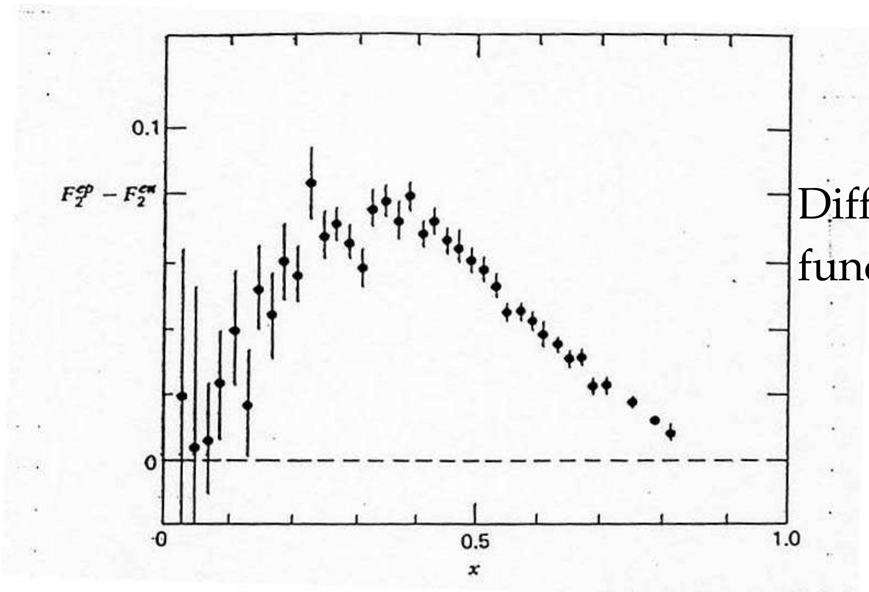
The ratio $\frac{F^n}{F^p}$ tends to

1 if s dominates

4 if d_v dominates

$\frac{1}{4}$ if u_v dominates

Comparison with experiment II



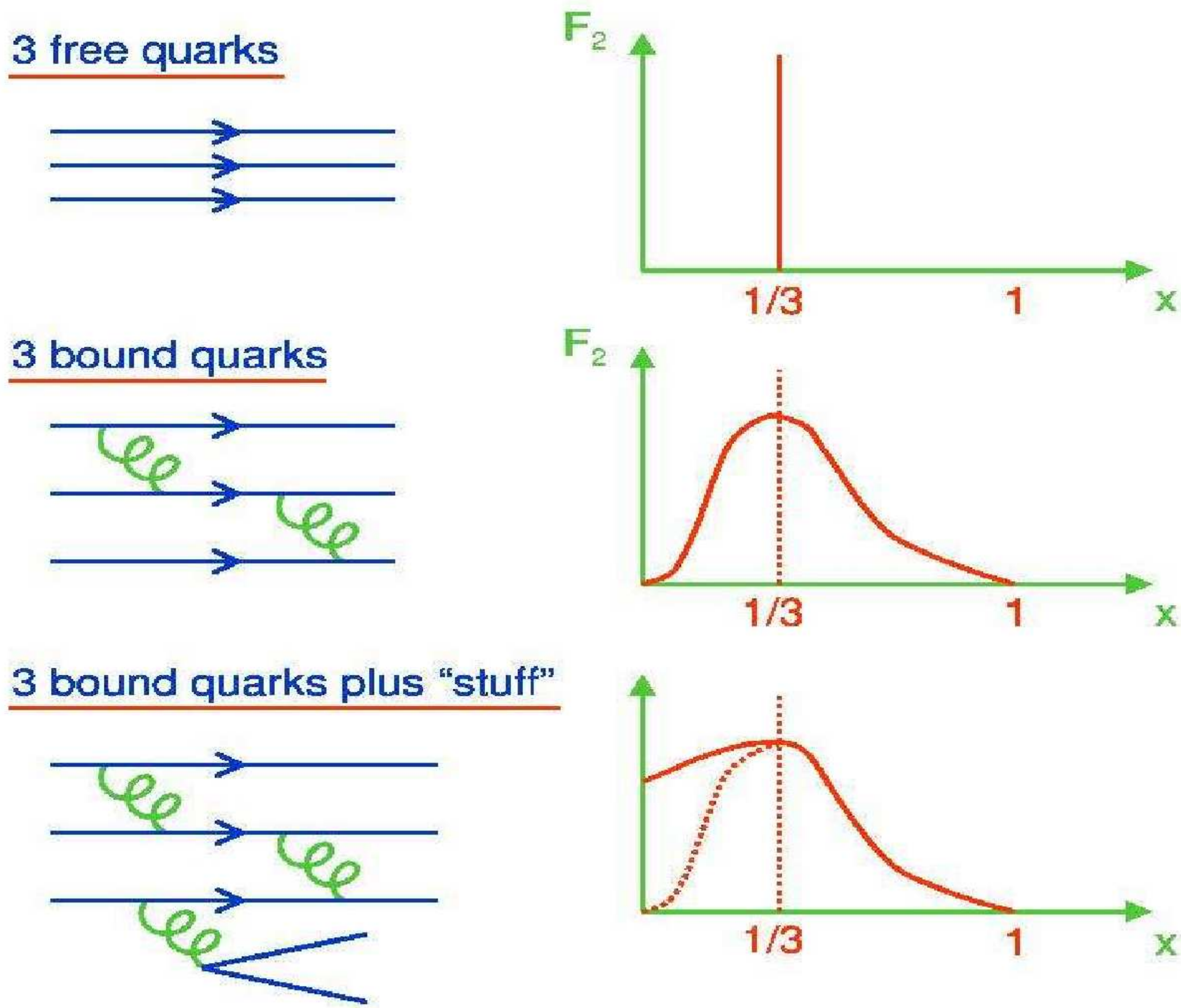
Difference between proton and neutron functions

$$F_2^p(x) - F_2^n(x) = \frac{x}{3}\left[u_v - d_v\right]$$

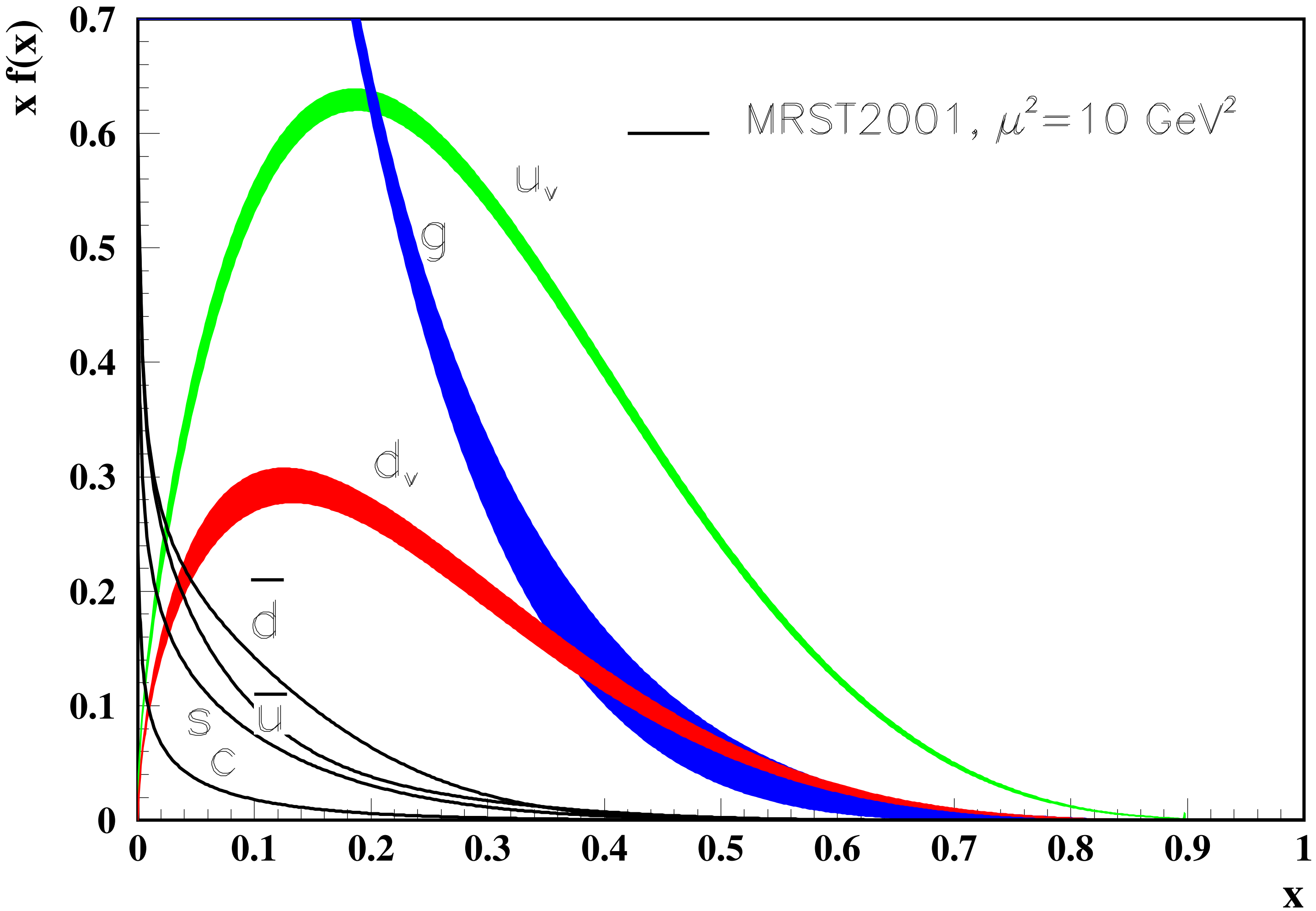
Scaling Violations

- At extremely large values of q^2, it is observed that the PDFs do depend slightly on q^2 ($F_2(x) \to F_2(x, q^2)$), in conflict with Bjorken's scaling hypothesis.
- In particular, as $|q^2|$ increases, the PDFs decrease at large x and increase at small x. In other words, the closer we look, the more *soft* partons we see.
- Quantitatively, these scaling violations fall within the realm of perturbative QCD. The DGLAP (Dokshitzer, Gribov, Lipatov, Altarelli, Parisi) equations describe the evolution of the PDFs with q^2.

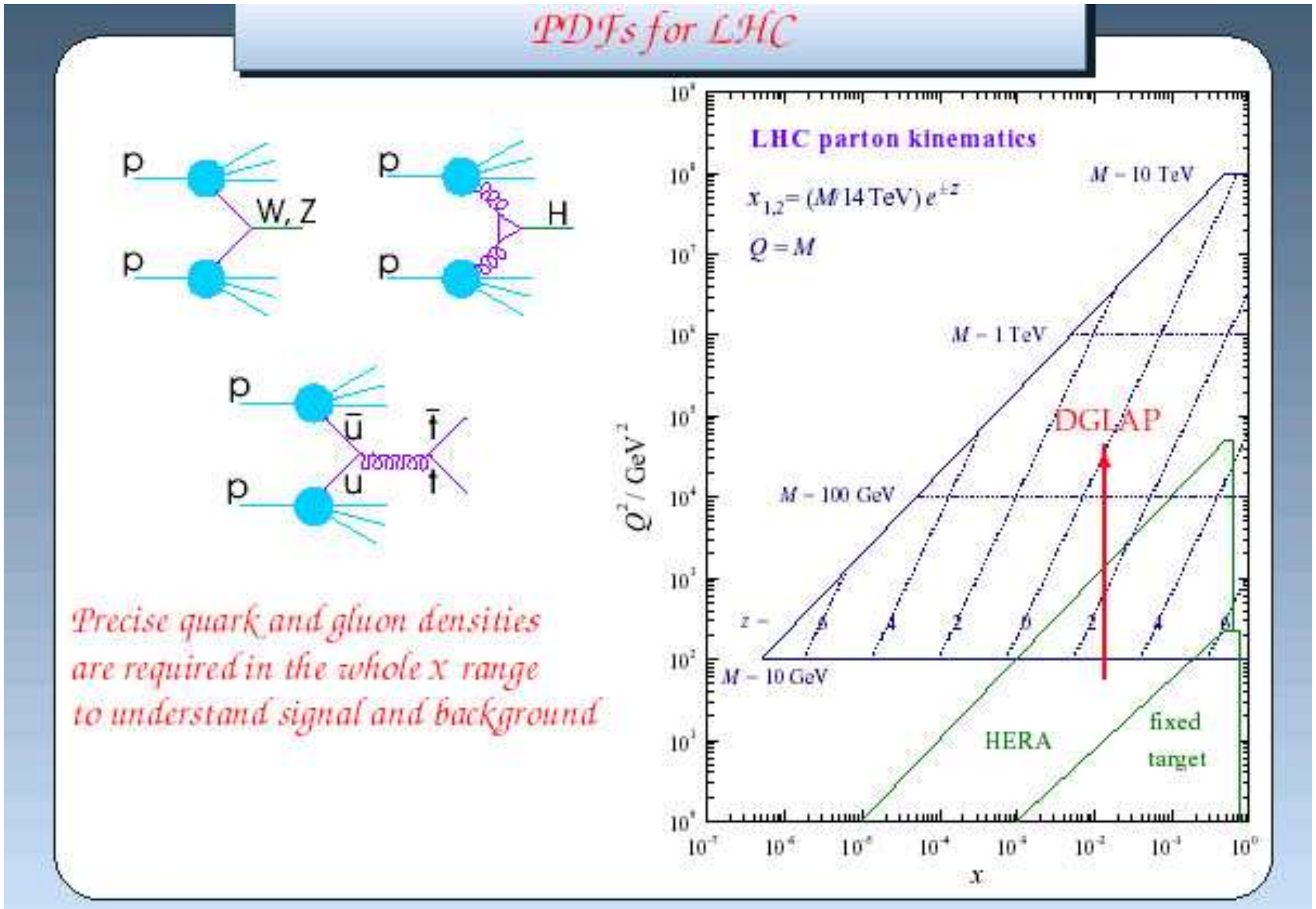
What is $F_2(x)$?


3 free quarks
3 bound quarks
3 bound quarks plus "stuff"
F2
1/3
1
x
F2
1/3
1
x
1/3
1
x

Parton Distribution Functions



Why is this important?



Summary

- Deep Inelastic Scattering provides the best window with which to look into the interior of a proton.
- At high-energies, DIS simplifies considerably, as the electrons begin to scatter elastically off individual partons. This leads to Bjorken scaling.
- The Callan-Gross relation shows that the partons are spin-$\frac{1}{2}$ particles.
- We can describe the structure functions in terms of quark distribution functions. These PDFs display the richness of QCD, as gluon and sea quark contributions cannot always be ignored.