Neutral Weak Interactions

- The Z^0 Boson:
- Feynman Rules
- The Weak Mixing Angle
- Resonance in $e^+ \, e^-$ Scattering

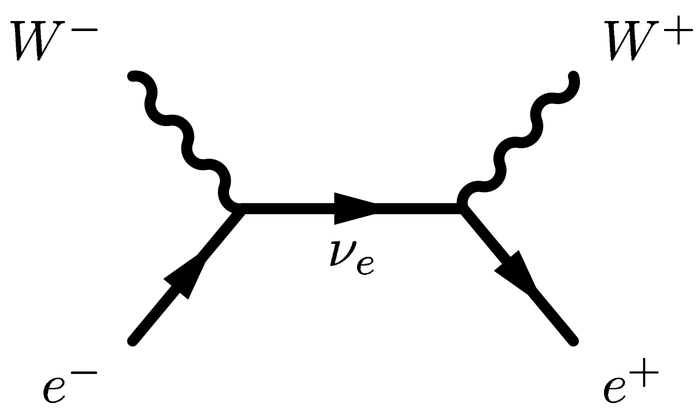
Slides from Sobie and Blokland

Who Needs the Z^0?

- In the 1960s, there was no compelling experimental evidence for neutral weak currents.
- Theoretically, Fermi's four-fermion theory of the weak interaction suggested charged weak currents, but there was no neutral current analogue.
- Why, then, would we want to invent a particle without any experimental or theoretical justification?
- It turns out there was a subtle theoretical justification based on considering what happens at very high energies

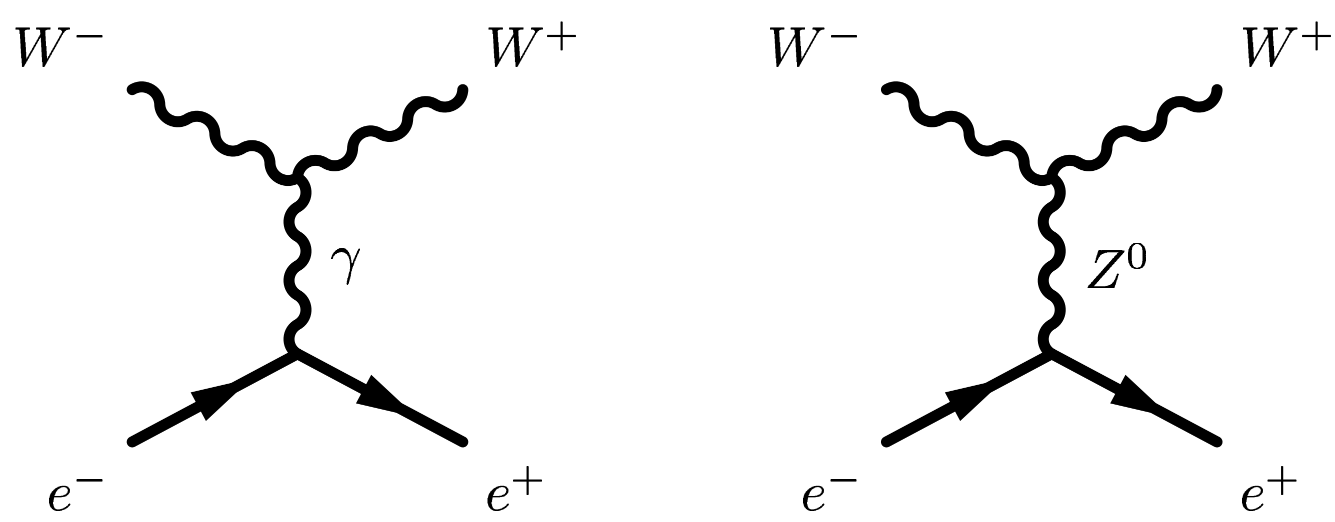
Why do we need a Z^0?

- Violation of a unitarity bound, i.e. a scattering cross-section which exceeds its maximum theoretical value, is encountered in the process $e^+ \, e^- \to W^+ \, W^-$ assuming that it proceeds by the Feynman diagram:



Restoring Unitarity

- In order to make the weak interaction self-consistent, we require two additional contributions to the $e^+ e^- \to W^+ W^-$ scattering process:



The Weak Mixing Angle

- As we'll see next lecture, many of the parameters of the electroweak interaction are related to each other. For starters,

$$M_W = M_Z \cos\theta_w$$

where θ_w is the *weak mixing angle*, also known as the *Weinberg angle*.

- Experimentally,

$$\sin^2\theta_w(M_Z) = 0.23120(15)$$

Relations Between Coupling Constants

- The vertex factor for interactions with the Z^0 will involve a coupling constant g_z. Just as the Z and W masses are related by the Weinberg angle, so are the coupling constants:

$$g_z = g_w \cos\theta_w$$

- It gets better. Both g_w and g_z are related to the QED coupling constant g_e:

$$g_w = \frac{g_e}{\sin\theta_w} \qquad g_z = \frac{g_e}{\sin\theta_w \cos\theta_w}$$

  This is why the weak force is inherently stronger than the electromagnetic force.

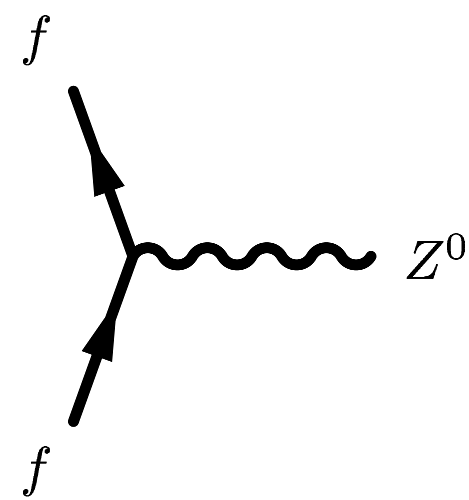
Feynman Rules for the Z^0

- The Z^0 propagator looks just like that of the W:

$$\frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_Z^2}\right)}{q^2 - M_Z^2}$$

- The Z^0 bosons mediate neutral current (NC) weak interactions. They couple to fermions via

$$\frac{-ig_z}{2}\gamma^\mu(c_V - c_A\gamma^5)$$



Fermion Couplings to the Z^0

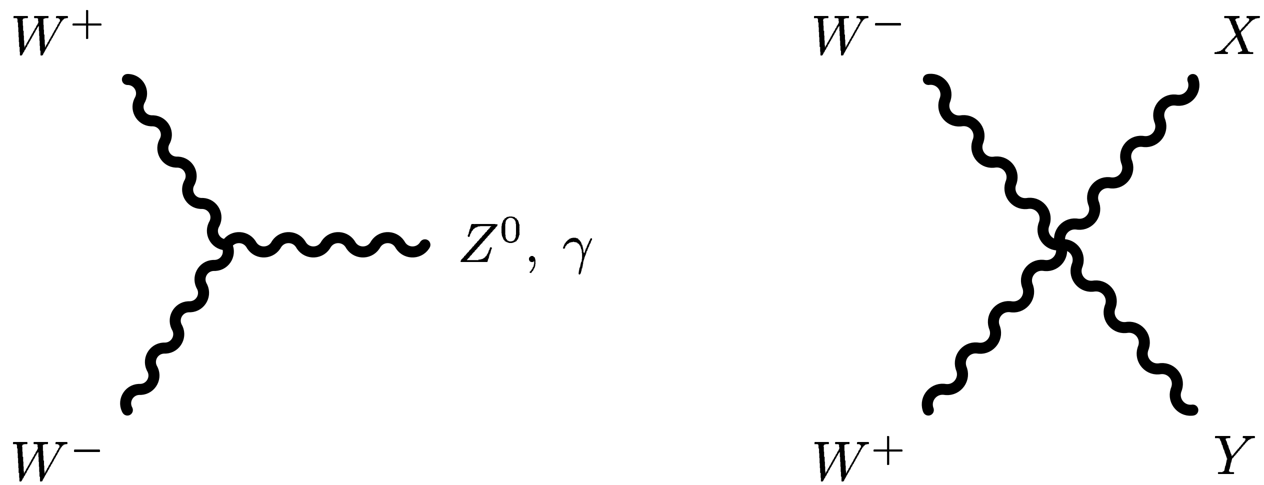
- The vector and axial couplings c_V and c_A are specified by the Glashow-Weinberg-Salam model:

f	c_V	c_A
ν_ℓ	$+\frac{1}{2}$	$+\frac{1}{2}$
ℓ^-	$-\frac{1}{2} + 2\sin^2\theta_w$	$-\frac{1}{2}$
q_u	$+\frac{1}{2} - \frac{4}{3}\sin^2\theta_w$	$+\frac{1}{2}$
q_d	$-\frac{1}{2} + \frac{2}{3}\sin^2\theta_w$	$-\frac{1}{2}$

- The Z^0 does not change the lepton or quark flavor. The SM has no *flavor-changing neutral currents* (FCNC) at tree level.

Gauge Boson Self-Couplings

- Just like QCD, the electroweak bosons carry (weak interaction) charge and can interact with each other:



where (X, Y) can be (γ, γ), (γ, Z^0), (Z^0, Z^0), or (W^+, W^-). Consult Appendix D of Griffiths for vertex factors.

γ vs. Z^0

- The Z^0 couples to every charged fermion, just like the photon does.

$$Z \, / \, \gamma \rightarrow f \, \overline{f}$$

This made it difficult to detect the Z^0 because at low energies, the QED effects dominate. Nevertheless, there are always small weak effects in otherwise electromagnetic systems (e.g. atomic parity violation).

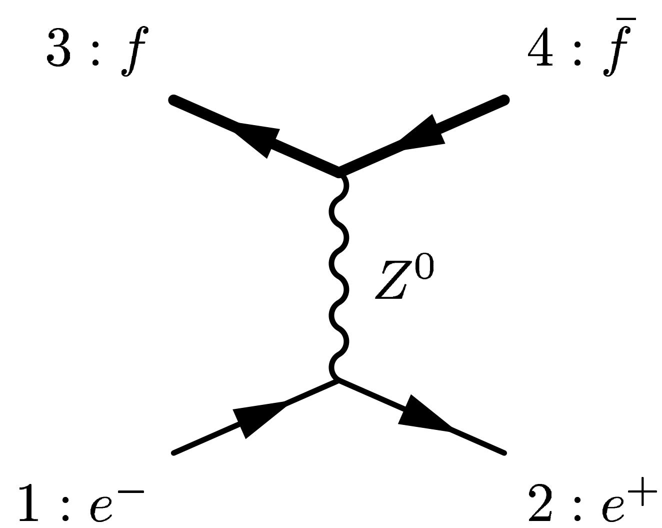
- Unlike the photon, the Z^0 also couples to neutrinos.

$$Z \rightarrow \nu \, \overline{\nu}$$

Neutrino experiments are never easy, but at least they allow us to isolate the weak interaction.

Example: $e^+ \, e^- \rightarrow f \, \bar{f}$

- We first considered this interaction in the context of extending QED in order to predict hadron production rates. Now we would like to see how the Z^0-mediated s-channel diagram compares to the corresponding γ-mediated diagram:



The Scattering Amplitude

- The amplitude is

$$\begin{aligned}\mathcal{M} &= i\left[\bar{u}_4\left(\frac{-ig_z}{2}\gamma^\mu(c_V^f - c_A^f\gamma^5)\right)v_3\right]\left[\frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_Z^2}\right)}{q^2 - M_Z^2}\right] \\ &\quad\times \left[\bar{v}_2\left(\frac{-ig_z}{2}\gamma^\nu(c_V^e - c_A^e\gamma^5)\right)u_1\right]\end{aligned}$$

- At low energies, $q^2 \ll M_Z^2$, and we would eventually find that, up to some factors of c_V, c_A, and $\sin^2\theta_w$, the Z^0-mediated diagram would be like the QED diagram only with α replaced by $G_F E^2$.

At Higher Energies

- If q^2 is not small, we can no longer simplify the Z^0-propagator. Keeping the full propagator,

$$\mathcal{M} = -\frac{g_z^2}{4(q^2 - M_Z^2)}\left[\bar{u}_4\gamma^\mu(c_V^f - c_A^f\gamma^5)v_3\right]\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_Z^2}\right) \times \left[\bar{v}_2\gamma^\nu(c_V^e - c_A^e\gamma^5)u_1\right]$$

- Assuming that we can neglect all fermion masses, the $\frac{q_\mu q_\nu}{M_Z^2}$ part of the propagator will contribute nothing, since we can write q as either $p_1 + p_2$ or $p_3 + p_4$. Then the $\not{q}$ factors lead to combinations like $\bar{u}_4\not{p}_4$ and $\not{p}_3 v_3$, which, by the Dirac equation, are $\bar{u}_4 m_4$ and $-m_3 v_3$.

Moving Along...

$$\mathcal{M} = -\frac{g_z^2}{4(q^2 - M_Z^2)} \left[\bar{u}_4 \gamma^\mu (c_V^f - c_A^f \gamma^5) v_3\right] \times \left[\bar{v}_2 \gamma_\mu (c_V^e - c_A^e \gamma^5) u_1\right]$$

$$\left\langle |\mathcal{M}|^2 \right\rangle = \left[\frac{g_z^2}{8(q^2 - M_Z^2)}\right]^2 \mathrm{Tr}\left[\gamma^\mu (c_V^f - c_A^f \gamma^5) \not{p}_3 \gamma^\nu (c_V^f - c_A^f \gamma^5) \not{p}_4\right] \times \mathrm{Tr}\left[\gamma_\mu (c_V^e - c_A^e \gamma^5) \not{p}_1 \gamma_\nu (c_V^e - c_A^e \gamma^5) \not{p}_2\right]$$

- The traces are best evaluated by first bringing the c_V and c_A terms together:

$$(c_V - c_A \gamma^5) \not{p}_3 \gamma^\nu (c_V - c_A \gamma^5) = (c_V - c_A \gamma^5)^2 \not{p}_3 \gamma^\nu = (c_V^2 + c_A^2) \not{p}_3 \gamma^\nu - 2 c_V c_A \gamma^5 \not{p}_3 \gamma^\nu$$

One can show...

- ... that after taking the traces, writing the momenta in terms of E and $\sin\theta$, and then using Fermi's Golden Rule, that the cross section for Z^0-mediated $e^+ \, e^- \to f \; \bar{f}$ is

$$\sigma \;=\; \frac{1}{3\pi}\left(\frac{g_z^2 E}{4[(2E)^2 - M_Z^2]}\right)^2 [(c_V^f)^2 + (c_A^f)^2][(c_V^e)^2 + (c_A^e)^2]$$

- As it stands, it looks like this cross section blows up when $E = M_Z/2$. This is much more serious than the infinite cross section for Rutherford scattering because this (Z^0) divergence can be traced all the way back to the amplitude.

Unstable Particles

- The source of the problem is that the kinematics are such that $e^+ \, e^- \rightarrow Z^0$ is a physically allowable process *even without* a subsequent decay to $f \; \bar{f}$.
- As a result, we need to modify the Z^0-propagator in order to account for the instability of the Z^0. Here's what we do:
  1. We recall the familiar configuration-space wavefunction of a stable particle: $\Psi(\mathbf{r}, t) = \psi(\mathbf{r}) e^{-iEt}$
  2. Since the particle is stable, the probability of finding the particle somewhere is always equal to 1 since the wavefunction is normalized:

$$P(t) = \int |\Psi|^2 \, d^3\mathbf{r} = 1$$

3. If the particle is unstable, we expect the probability of finding the particle to fall off with time according to the decay rate Γ

$$P(t) = \int |\Psi|^2 \, d^3\mathbf{r} = e^{-\Gamma t}$$

4. In the particle rest frame, this means that

$$\Psi(\mathbf{r}, t) = \psi(\mathbf{r}) e^{-iMt - \frac{\Gamma t}{2}}$$

5. We then apply the substitution $M \to M - \frac{i\Gamma}{2}$ to the propagator of an unstable particle and assume that Γ is sufficiently small that we can neglect the Γ^2 term:

$$\begin{aligned} \frac{1}{q^2 - M^2} &\to \frac{1}{q^2 - (M - i\Gamma/2)^2} \\ &\simeq \frac{1}{q^2 - M^2 + iM\Gamma} \end{aligned}$$

Back to the Z^0 Peak...

- With the modification to the Z^0 propagator,

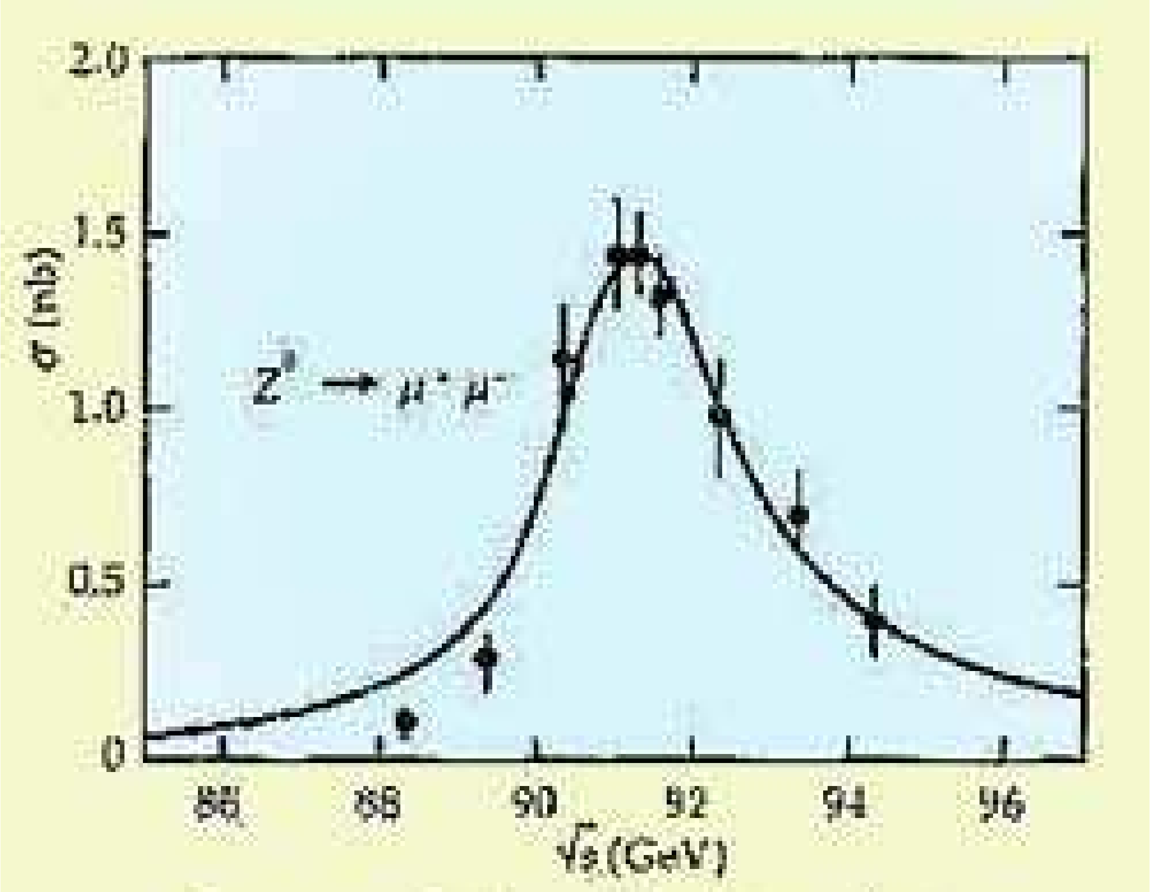
$$\frac{1}{q^2 - M_Z^2} \quad \rightarrow \quad \frac{1}{q^2 - M_Z^2 + iM_Z\Gamma_Z}$$

the cross section takes the form

$$\sigma \quad \sim \quad \frac{1}{[(2E)^2 - M_Z^2]^2 + (M_Z\Gamma_Z)^2}$$

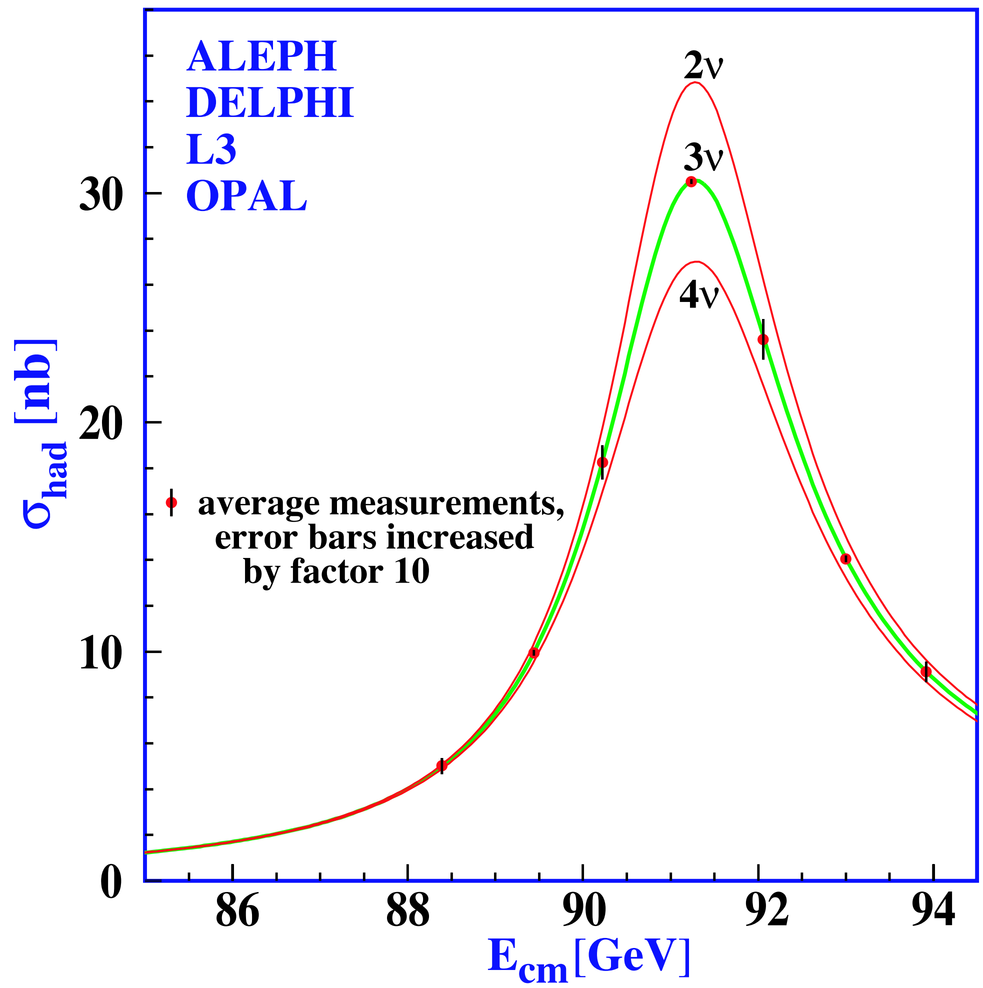
This is known as a *Breit-Wigner resonance*. Both the height and width of the resonance peak are determined by the decay width Γ_Z.

Measurement of the Z^0 Peak in dimuons



Z_0-resonance in the reaction $e^+e^- \rightarrow \mu^+\mu^-$

Final lineshape of the Z^0 Peak



More on the Z^0 Peak

- While QED dominates $e^+ e^- \rightarrow f \bar{f}$ at low energies

$$\frac{\sigma_Z}{\sigma_\gamma} \simeq 2 \left(\frac{E}{M_Z} \right)^4$$

it is the Z^0-mediated process which dominates near the resonance. At the peak,

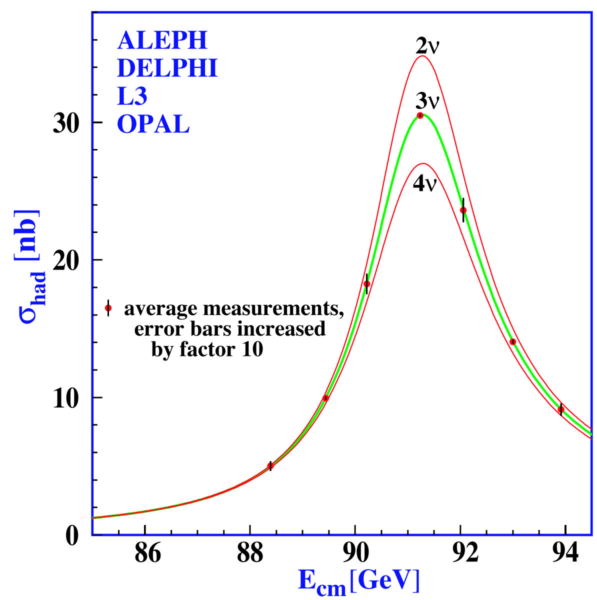
$$\frac{\sigma_Z}{\sigma_\gamma} \simeq \frac{1}{8} \left(\frac{M_Z}{\Gamma_Z} \right)^2 \simeq 200$$

- Γ_Z can be calculated in the Standard Model by putting a Z^0 in the initial state. When this is done it is found that there cannot be a 4th lepton generation with a light neutrino.

Number of light neutrino generations

The Z can decay into neutrinos $Z^0 \to \nu\bar{\nu}$ which each neutrino species contributing to the total width.

The cross section is proportional to the decay width.



The Z^0 Peak at CERN

- Precise measurements of electroweak parameters (M_W, M_Z, and $\sin^2\theta_w$) also shed light on other Standard Model parameters such as m_t and m_H.
- In the early days at LEP (started in 1989), a number of unusual systematic effects needed to be accounted for in order to measure these parameters accurately:
  1. Tidal distortions of the ring
  2. Water levels in nearby Lake Geneva
  3. Correlations with the TGV

Water levels in nearby Lake Geneva

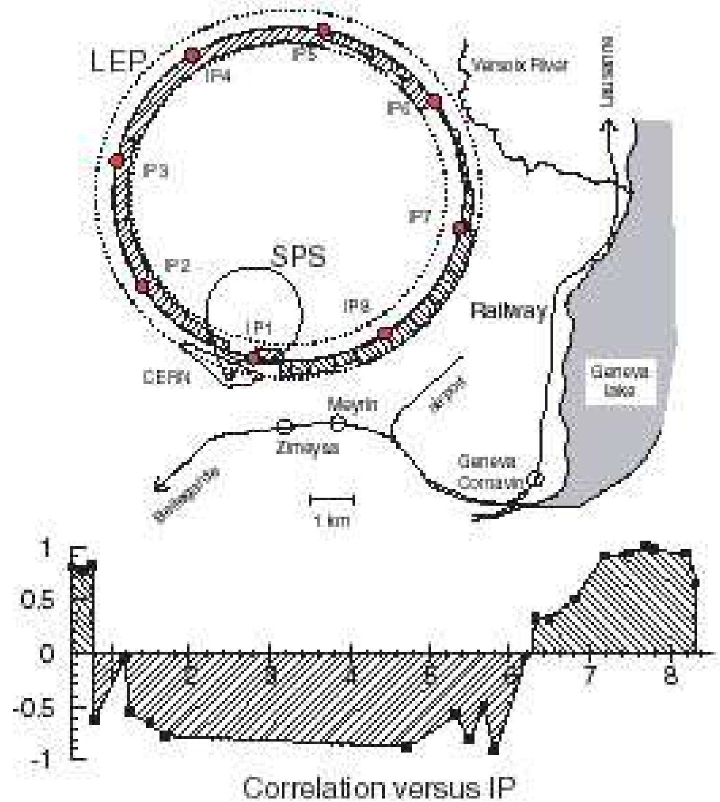


Figure 10: The TVG train line in relation to the LEP ring. The ground return for the train includes the LEP ring and has an effect of order 1-3 MeV on the effective beam energy.

Tidal distortions of the ring

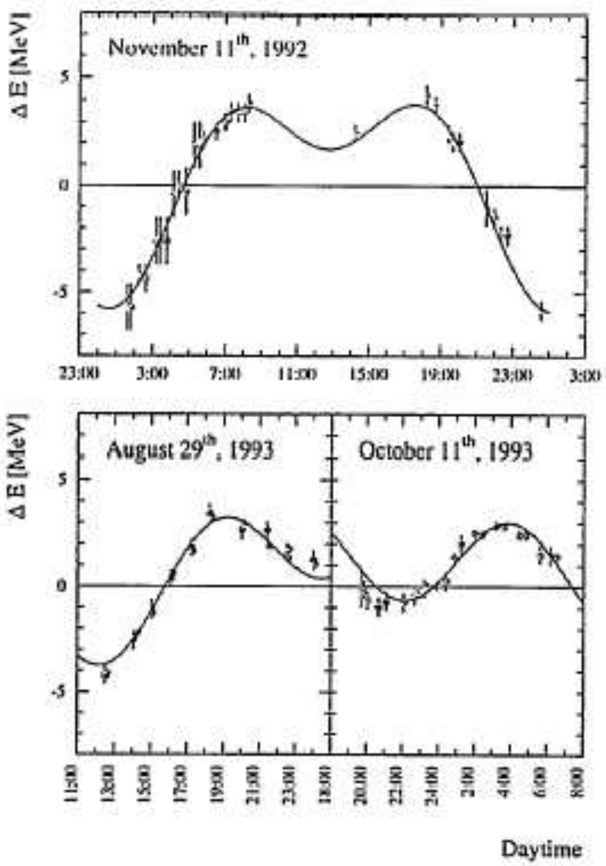


Figure 12. The LEP beam energy correlation to the moon, revealing the effect of tides.

Correlations with the TGV

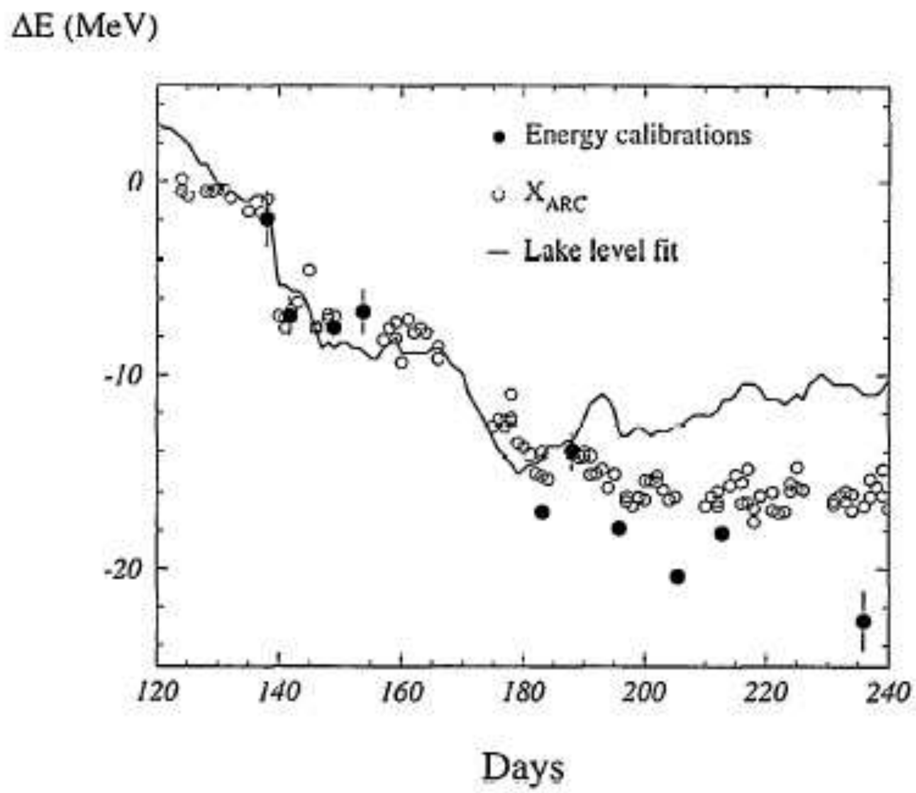


Figure 13. The LEP beam energy correlation to the water level of Lake Geneva.

Summary

- Unitarity bounds suggest the existence of the $W^\pm$ and, subsequently, the Z^0.
- The electroweak parameters (masses and couplings) are connected by the Weinberg angle θ_w.
- Z^0-mediated processes are usually dominated by QED processes except for
  1. Processes involving neutrinos
  2. Processes at high energies
- Much can be learned from measurements of the Z^0 resonance peak.