Lecture 22

Electroweak Unification

- Chiral Fermions
- EW unification
- Higgs particle

2 Obstacles to Electroweak Unification

- Electromagnetism and the weak force are exactly the same, only different...
  1. While the γ is massless, the weak bosons $W^\pm$ and Z^0 are quite massive.
  2. The QED interaction is purely vector (γ^μ), whereas the weak interaction combines vector and axial terms ($\gamma^\mu(c_V - c_A\gamma^5)$).
- The first difference requires the Higgs mechanism to sort out. For now, we will merely take encouragement from the experimental observation that M_W is so large that g_e and g_w are fairly similar in size.
- The second difference is addressed by the GWS theory.

Hiding the $V - A$

- Looking at the $V - A$ vertex factor for the $W^{\pm}$, we can make this look like a pure vector interaction if we associate part of the interaction with the fermion wavefunction:

$$\frac{-ig_w}{\sqrt{2}}\left[\bar{u}\left(\gamma^{\mu}\,\frac{1-\gamma^5}{2}\right)u\right] = \frac{-ig_w}{\sqrt{2}}\left[\bar{u}\gamma^{\mu}u_L\right]$$

where

$$u_L \equiv \frac{(1-\gamma^5)}{2}\,u$$

At this stage, this is just a definition and notation. Of course, the subscript L suggests that u_L is somehow the left-handed part of u.

γ^5: Chirality vs. Helicity

$$\gamma^5 u = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u_A \\ u_B \end{pmatrix}$$

$$= \begin{pmatrix} u_B \\ u_A \end{pmatrix}$$

$$= \begin{pmatrix} \frac{(\mathbf{p}\cdot\sigma)}{E+m} u_A \\ \frac{(\mathbf{p}\cdot\sigma)}{E-m} u_B \end{pmatrix}$$

$$= \begin{pmatrix} \frac{(\mathbf{p}\cdot\sigma)}{E+m} & 0 \\ 0 & \frac{(\mathbf{p}\cdot\sigma)}{E-m} \end{pmatrix} \begin{pmatrix} u_A \\ u_B \end{pmatrix}$$

- For $m = 0$, we see that γ^5 behaves just like the *helicity operator* $\Sigma \cdot \widehat{p}$.
- γ^5 is defined as the *chirality operator* and it is only in the massless limit that helicity and chirality are the same.

Chiral Fermions

- Since γ^5 acts just like the helicity operator $\Sigma \cdot \widehat{p}$ for massless fermions,

$$u_L \equiv \tfrac{1}{2}(1-\gamma^5)u \quad = \begin{cases} 0 & \text{if } u \text{ has helicity } +1 \\ u & \text{if } u \text{ has helicity } -1 \end{cases}$$

- Similarly, we can project out the right-handed part of a spinor:

$$u_R \equiv \tfrac{1}{2}(1+\gamma^5)u \quad = \begin{cases} u & \text{if } u \text{ has helicity } +1 \\ 0 & \text{if } u \text{ has helicity } -1 \end{cases}$$

Adjoint Spinors

- What about $\bar{u}_L$ and $\bar{u}_R$?

$$\bar{u}_L \;=\; u_L^\dagger \gamma^0 \;=\; u^\dagger \tfrac{1}{2}(1-\gamma^5)\gamma^0 \;=\; u^\dagger \gamma^0 \tfrac{1}{2}(1+\gamma^5) \;=\; \bar{u}\,\tfrac{1}{2}(1+\gamma^5)$$

$$\bar{u}_R \;=\; \bar{u}\,\tfrac{1}{2}(1-\gamma^5)$$

- By using the identity

$$\left[\tfrac{1}{2}(1-\gamma^5)\right]^2 = \tfrac{1}{4}[1-2\gamma^5+(\gamma^5)^2] = \tfrac{1}{2}(1-\gamma^5)$$

we can write

$$\begin{aligned} j^\mu_{weak} \sim \bar{u}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u \quad &= \quad \bar{u}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\tfrac{1}{2}(1-\gamma^5)u \\ &= \quad \bar{u}\,\tfrac{1}{2}(1+\gamma^5)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u \\ &= \quad \bar{u}_L\gamma^\mu u_L \end{aligned}$$

Reinterpreting the Weak Interaction

- Our identity

$$j^{\mu}_{weak} \sim \bar{u}\gamma^{\mu}\tfrac{1}{2}(1-\gamma^{5})u = \bar{u}_L\gamma^{\mu}u_L$$

means that we can think of the charged weak interaction as a pure vector interaction between *left-handed* fermions.

Chiral QED

- The non-chiral QED *current* $\bar{u}\gamma^\mu u$, can be expanded out into four chiral currents ($u = u_L + u_R$):

$$\begin{aligned} j^\mu_{em} \sim \bar{u}\gamma^\mu u &= (\bar{u}_L + \bar{u}_R)\gamma^\mu(u_L + u_R) \\ &= \bar{u}_L\gamma^\mu u_L + \bar{u}_R\gamma^\mu u_R + \bar{u}_L\gamma^\mu u_R + \bar{u}_R\gamma^\mu u_L \end{aligned}$$

- Since

$$\tfrac{1}{2}(1-\gamma^5)\tfrac{1}{2}(1+\gamma^5) = \tfrac{1}{4}[1-(\gamma^5)^2] = 0$$

the LR and RL cross terms in the QED current vanish:

$$\begin{aligned} \bar{u}_L\gamma^\mu u_R &= \bar{u}\,\tfrac{1}{2}(1+\gamma^5)\gamma^\mu\tfrac{1}{2}(1+\gamma^5)u \\ &= \bar{u}\gamma^\mu\tfrac{1}{2}(1-\gamma^5)\tfrac{1}{2}(1+\gamma^5)u \\ &= 0 \end{aligned}$$

This means that only the LL and RR terms survive:

$$j^\mu_{em} \sim \bar{u}\gamma^\mu u = \bar{u}_L\gamma^\mu u_L + \bar{u}_R\gamma^\mu u_R$$

First Intermission

- The charged weak currents, as mediated by the $W^{\pm}$, couple left-handed fermions together:

$$\begin{aligned} j_\mu^- &= \bar{\nu}_L \gamma_\mu e_L \\ j_\mu^+ &= \bar{e}_L \gamma_\mu \nu_L \end{aligned}$$

- The electromagnetic current, as mediated by the γ, couples left-handed fermions together, and it also couples right-handed fermions together:

$$j_\mu^{em} = -\bar{e}_L \gamma_\mu e_L - \bar{e}_R \gamma_\mu e_R$$

Weak Doublets

- Since the W couples left-handed leptons and their neutrinos together, it seems natural to define the weak doublet:

$$\chi_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$$

- In terms of χ_L, the charged weak currents

$$j_\mu^- = \bar{\nu}_L \gamma_\mu e_L \qquad\qquad j_\mu^+ = \bar{e}_L \gamma_\mu \nu_L$$

can be written as

$$j_\mu^\pm = \bar{\chi}_L \gamma_\mu \tau^\pm \chi_L$$

where

$$\tau^+ \equiv \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad\qquad \tau^- \equiv \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Weak Isospin

- The matrices

$$\tau^+ \equiv \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \tau^- \equiv \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

can be constructed from the Pauli spin matrices via

$$\tau^\pm = \tfrac{1}{2}(\tau^1 \pm i\tau^2)$$

- This is looking a lot like isospin (i.e., an internal $SU(2)$ symmetry). Suppose we define a third τ matrix in order to complete the symmetry:

$$\tau^3 \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

A Neutral Current

- From τ^3, we can construct a current (with a factor of $\frac{1}{2}$ for consistency with $j_\mu^\pm$):

$$\begin{aligned} j_\mu^3 &= \bar{\chi}_L \gamma_\mu \tfrac{1}{2}\tau^3 \chi_L \\ &= \tfrac{1}{2}\bar{\nu}_L \gamma_\mu \nu_L - \tfrac{1}{2}\bar{e}_L \gamma_\mu e_L \end{aligned}$$

- Aha! Here is a neutral current!
- Problem: This neutral current is pure $V - A$ and it only involves left-handed particles. The Z^0, conversely, has a more complicated $\gamma_\mu(c_V - c_A\gamma^5)$ structure and, consequently, it also couples to right-handed particles.

Hypercharge

- Although we didn't draw attention to it at the time, there is a relationship called the *Gell-Mann–Nishijima formula* which connects the charge Q, isospin component I^3, baryon number A, and strangeness S, of a quark or hadron:

$$Q = I^3 + \frac{1}{2}(A + S)$$

- The combination $(A + S)$ is defined as the *hypercharge* and is denoted by Y. If we propose some sort of *weak* hypercharge, we can then generalize the Gell-Mann–Nishijima formula to the case of weak isospin:

$$Q = I^3 + \frac{1}{2}Y$$

Hypercharge Current

- From $Q = I^3 + \frac{1}{2}Y$, we can then construct a weak hypercharge current:

$$\begin{aligned} j_\mu^Y &= 2j_\mu^{em} - 2j_\mu^3 \\ &= 2\left(-\bar{e}_L\gamma_\mu e_L - \bar{e}_R\gamma_\mu e_R\right) - 2\left(\tfrac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L - \tfrac{1}{2}\bar{e}_L\gamma_\mu e_L\right) \\ &= -2\bar{e}_R\gamma_\mu e_R - \bar{e}_L\gamma_\mu e_L - \bar{\nu}_L\gamma_\mu\nu_L \end{aligned}$$

- This current is invariant under weak isospin, as the right-handed term is untouched and the left-handed terms

$$\bar{e}_L\gamma_\mu e_L + \bar{\nu}_L\gamma_\mu\nu_L$$

form a weak isospin singlet.

Groups in The Standard Model

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

- The strong force is described by QCD using a color $SU(3)$ symmetry.
- The charged currents of the weak force (i.e, the $W^{\pm}$) make up 2/3 of a weak isospin $SU(2)$ symmetry which acts only on left-handed particles.
- The electromagnetic force is closely connected to a weak hypercharge $U(1)$ symmetry.
- Now we have to explain how the Z^0 and γ arise.

Generalizing to Other Weak Doublets

- Although we have set up this formalism in terms of the electron and its neutrino, we can just as easily use weak doublets for other leptons

$$\chi_L \longrightarrow \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

or for quarks, provided we account for the CKM rotations which distinguish the weak eigenstates from the mass eigenstates:

$$\chi_L \longrightarrow \quad \begin{pmatrix} u \\ d' \end{pmatrix}_L \qquad \begin{pmatrix} c \\ s' \end{pmatrix}_L \qquad \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

Second Intermission

- Let's stop for another review. For a weak doublet χ_L, we construct 3 weak isospin currents

$$\mathbf{j}_\mu = \frac{1}{2}\bar{\chi}_L \gamma_\mu \vec{\tau} \chi_L$$

$j_\mu^\pm$ correspond to the $W^\pm$-mediated currents, and j_μ^3 is some sort of left-handed neutral current.

- The electromagnetic current is

$$j_\mu^{em} = \sum_{i=1,2} Q_i (\bar{u}_{iL} \gamma_\mu u_{iL} + \bar{u}_{iR} \gamma_\mu u_{iR})$$

- We define a weak hypercharge current by

$$j_\mu^Y = 2j_\mu^{em} - 2j_\mu^3$$

Combining Weak Isospin and Hypercharge

- In the GWS model, the weak isospin current $\mathbf{j}_\mu$ couples to a triplet of vector bosons $\mathbf{W}^\mu$ with a coupling strength g_w.
- The weak hypercharge current j_μ^Y couples with strength $\frac{g'}{2}$ to a singlet vector boson B^μ.
- Quantitatively, the interaction terms of $SU(2)_L \otimes U(1)_Y$ are

$$-i\left[g_w \mathbf{j}_\mu \cdot \mathbf{W}^\mu + \frac{g'}{2} j_\mu^Y B^\mu\right]$$

- *None* of the four fields W^1, W^2, W^3, and B correspond directly to the physical particles W^+, W^-, Z^0, and γ.

Origin of the W^+ and W^-

- With a little bit of algebraic manipulation,

$$
\begin{aligned}
\mathbf{j}_\mu \cdot \mathbf{W}^\mu &= j^1_\mu W^{\mu 1} + j^2_\mu W^{\mu 2} + j^3_\mu W^{\mu 3} \\
&= \tfrac{1}{2}\left(j^1_\mu + i j^2_\mu\right)\left(W^{\mu 1} - i W^{\mu 2}\right) \\
&\quad + \tfrac{1}{2}\left(j^1_\mu - i j^2_\mu\right)\left(W^{\mu 1} + i W^{\mu 2}\right) + j^3_\mu W^{\mu 3} \\
&= \tfrac{1}{\sqrt{2}} j^+_\mu W^{\mu +} + \tfrac{1}{\sqrt{2}} j^-_\mu W^{\mu -} + j^3_\mu W^{\mu 3}
\end{aligned}
$$

where we define the W^+ and W^- by

$$
W^\pm_\mu \equiv \tfrac{1}{\sqrt{2}}\left(W^1_\mu \mp i W^2_\mu\right)
$$

W^+ and W^- Vertex Factors

- From the general interaction

$$-i\left[g_w \mathbf{j}_\mu \cdot \mathbf{W}^\mu + \frac{g'}{2} j_\mu^Y B^\mu\right]$$

we see that the coupling involving the W^- is

$$\frac{-ig_w}{\sqrt{2}} j_\mu^- W^{\mu-}$$

- With

$$\begin{aligned} j_\mu^- &= \bar{\nu}_L \gamma_\mu e_L \\ &= \bar{\nu} \gamma_\mu \tfrac{1}{2}(1-\gamma^5) e \end{aligned}$$

we find that the W^- couples to an e and an $\bar{\nu}_e$ with a vertex factor of

$$\frac{-ig_w}{2\sqrt{2}} \gamma_\mu (1-\gamma^5)$$

Spontaneous Electroweak Symmetry Breaking

- The details will have to wait until we look at the Higgs mechanism, but it turns out that the process which endows mass to the $W^{\pm}$ and the Z^0 breaks the $SU(2)_L \otimes U(1)_Y$ symmetry.
- This electroweak SSB allows the neutral states of the two symmetries (W^3 and B) to mix. Here is where the *weak mixing angle*, θ_w, comes in:

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix}$$

Origin of Electromagnetism

- With the electroweak mixing, the interaction terms for the neutral particles are

$$
\begin{aligned}
-i\left[g_w j^3_\mu W^{\mu 3} + \frac{g'}{2} j^Y_\mu B^\mu\right] &= -i\left[g_w \sin\theta_w j^3_\mu + \frac{g'}{2}\cos\theta_w j^Y_\mu\right] A^\mu \\
&\quad -i\left[g_w \cos\theta_w j^3_\mu - \frac{g'}{2}\sin\theta_w j^Y_\mu\right] Z^\mu
\end{aligned}
$$

- We will substitute $j^Y_\mu = 2j^{em}_\mu - 2j^3_\mu$. If A^μ is to represent the electromagnetic field, then

$$
\begin{aligned}
j^{em}_\mu &= \left[g_w \sin\theta_w j^3_\mu + \frac{g'}{2}\cos\theta_w j^Y_\mu\right] \\
&= \left[g_w \sin\theta_w j^3_\mu + g' \cos\theta_w \left(j^{em}_\mu - j^3_\mu\right)\right] \\
&\Rightarrow \boxed{\;g_e = g'\cos\theta_w = g_w \sin\theta_w\;}
\end{aligned}
$$

Origin of the Z^0

- Using $$g_e = g' \cos\theta_w = g_w \sin\theta_w$$

  the Z^0 interaction term is

$$
\begin{aligned}
-i\left[g_w \cos\theta_w j^3_\mu - \frac{g'}{2}\sin\theta_w j^Y_\mu\right] Z^\mu \quad &= \quad -i\left[\left(\frac{g_e \cos\theta_w}{\sin\theta_w}\right) j^3_\mu \right. \\
&\left. - \left(\frac{g_e \sin\theta_w}{2\cos\theta_w}\right) 2(j^{em}_\mu - j^3_\mu)\right] Z^\mu \\
&= \frac{-ig_e}{\sin\theta_w \cos\theta_w}\left[j^3_\mu - \sin^2\theta_w j^{em}_\mu\right] Z^\mu
\end{aligned}
$$

- From this, we define $$g_z = \frac{g_e}{\sin\theta_w \cos\theta_w}$$

Working Out c_V and c_A

- Let's look at the up quark. With

$$
\begin{aligned}
& -ig_z \left[j^3_\mu - \sin^2\theta_w j^{em}_\mu\right] Z^\mu \\
= & \ \frac{-ig_z}{2} Z^\mu \left[\left(\bar{u}_L \gamma_\mu u_L\right) - 2\sin^2\theta_w \frac{2}{3}\left(\bar{u}\gamma_\mu u\right)\right] \\
= & \ \frac{-ig_z}{2} Z^\mu \left[\left(\bar{u}\gamma_\mu \left(\frac{1-\gamma^5}{2}\right) u\right) - 2\sin^2\theta_w \frac{2}{3}\left(\bar{u}\gamma_\mu u\right)\right] \\
= & \ \frac{-ig_z}{2} Z^\mu \left[\bar{u}\gamma_\mu \left\{ \underbrace{\left(\frac{1}{2} - \frac{4}{3}\sin^2\theta_w\right)}_{c_V} - \underbrace{\left(\frac{1}{2}\right)}_{c_A} \gamma^5 \right\} u \right]
\end{aligned}
$$

- In this way, we establish the Z^0 vertex factors to Standard Model fermions of the form $\frac{-ig_z}{2}(c_V - c_A\gamma^5)$.

Couplings to the Z^0

- In a similar fashion, we can work out how the other fermions couple to the Z^0:

f	c_V	c_A
ν_ℓ	$+\frac{1}{2}$	$+\frac{1}{2}$
ℓ^-	$-\frac{1}{2} + 2\sin^2\theta_w$	$-\frac{1}{2}$
q_u	$+\frac{1}{2} - \frac{4}{3}\sin^2\theta_w$	$+\frac{1}{2}$
q_d	$-\frac{1}{2} + \frac{2}{3}\sin^2\theta_w$	$-\frac{1}{2}$

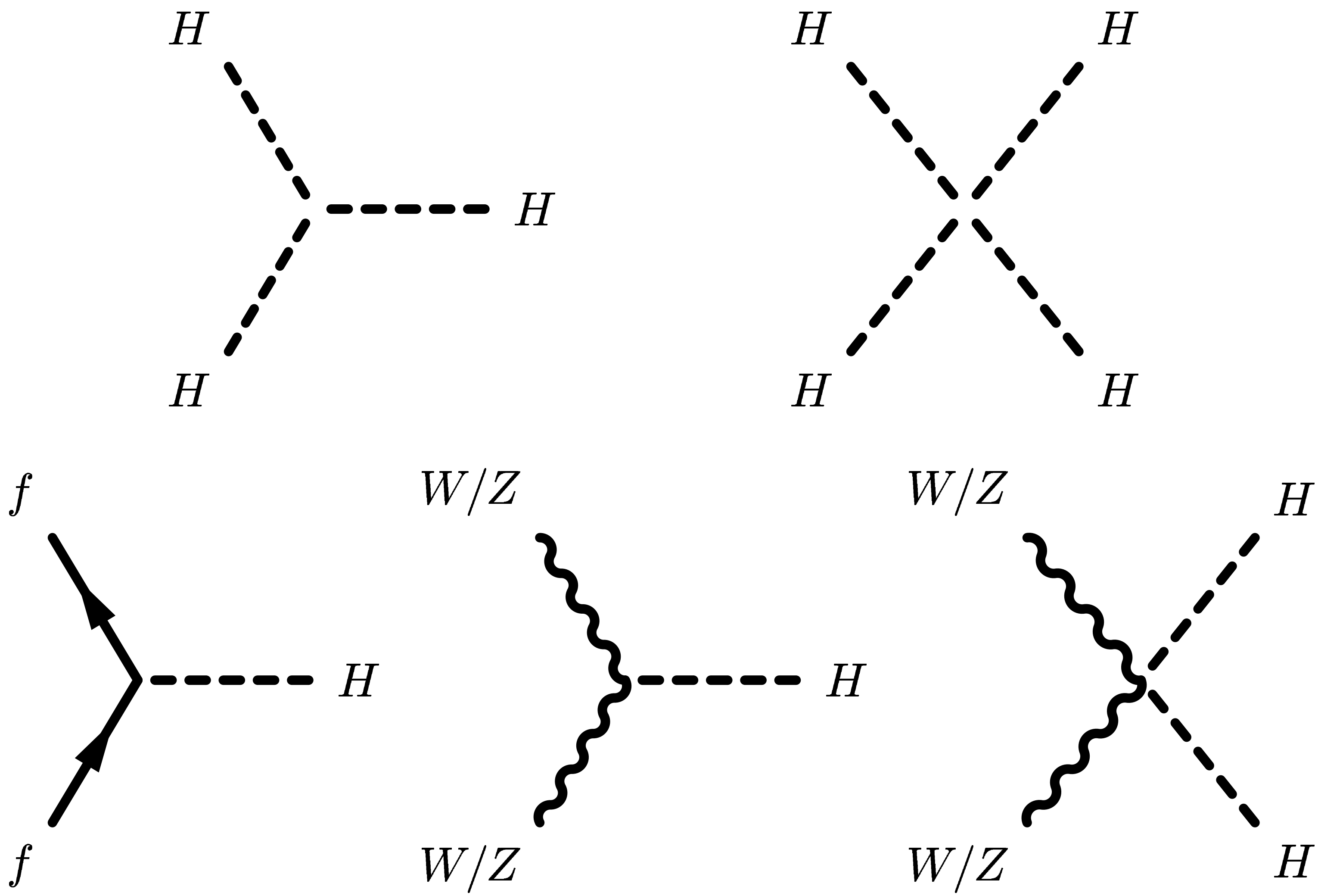
Summary

- We can replace the $V - A$ couplings of the weak interaction with vector couplings between left-handed fermions.
- The W^+ and W^- make up 2/3 of an $SU(2)_L$ weak isospin symmetry. We postulate W^3 as the remaining part.
- Meanwhile, a $U(1)_Y$ weak hypercharge symmetry couples to a field B.
- From electroweak symmetry breaking, the W^3 and B mix to give us the A field (γ) of QED and the Z^0.
- All couplings are related to each other by the weak mixing angle θ_w.

The Standard Model Higgs

- The Higgs couples to *every* massive particle in the Standard Model.

Higgs Boson Vertices

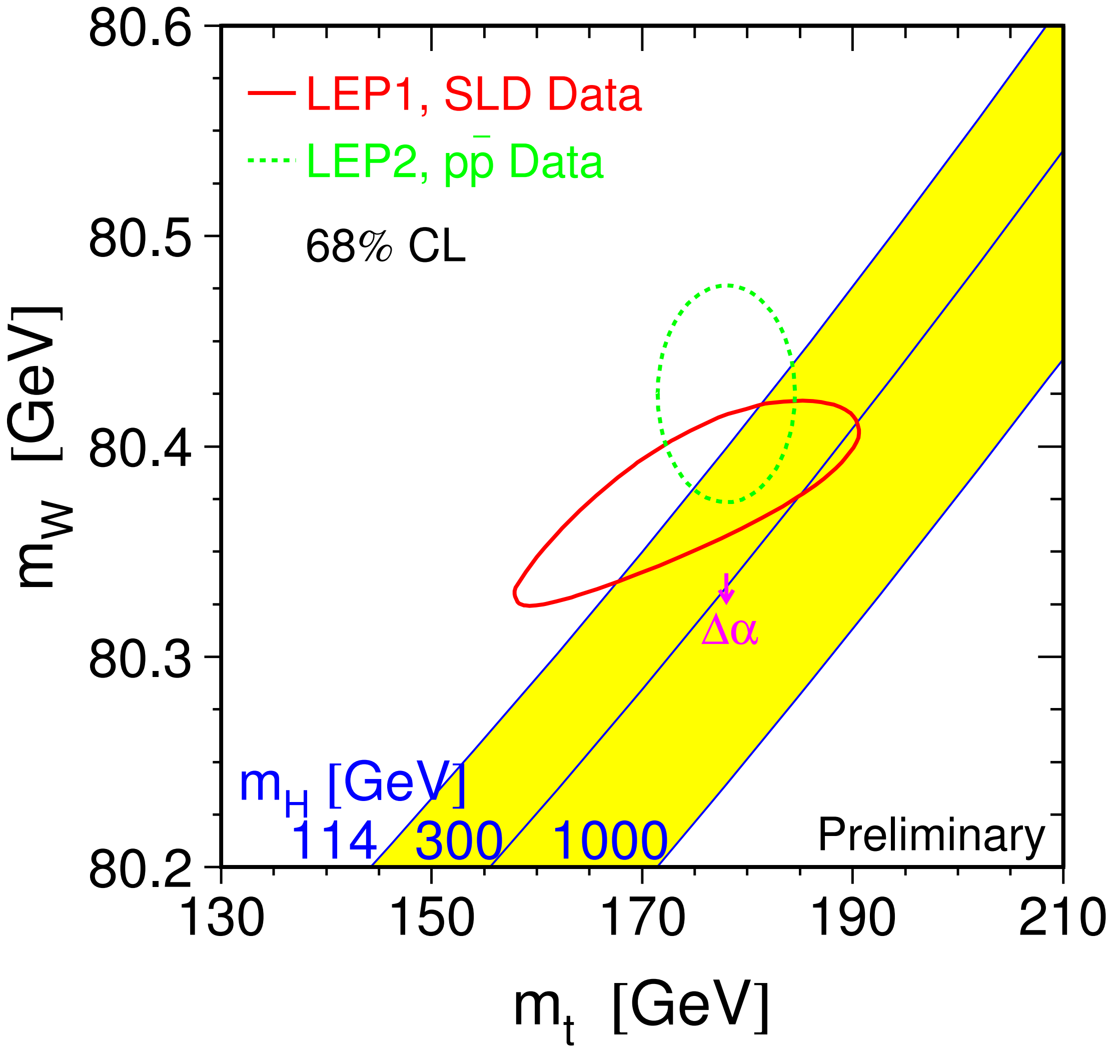


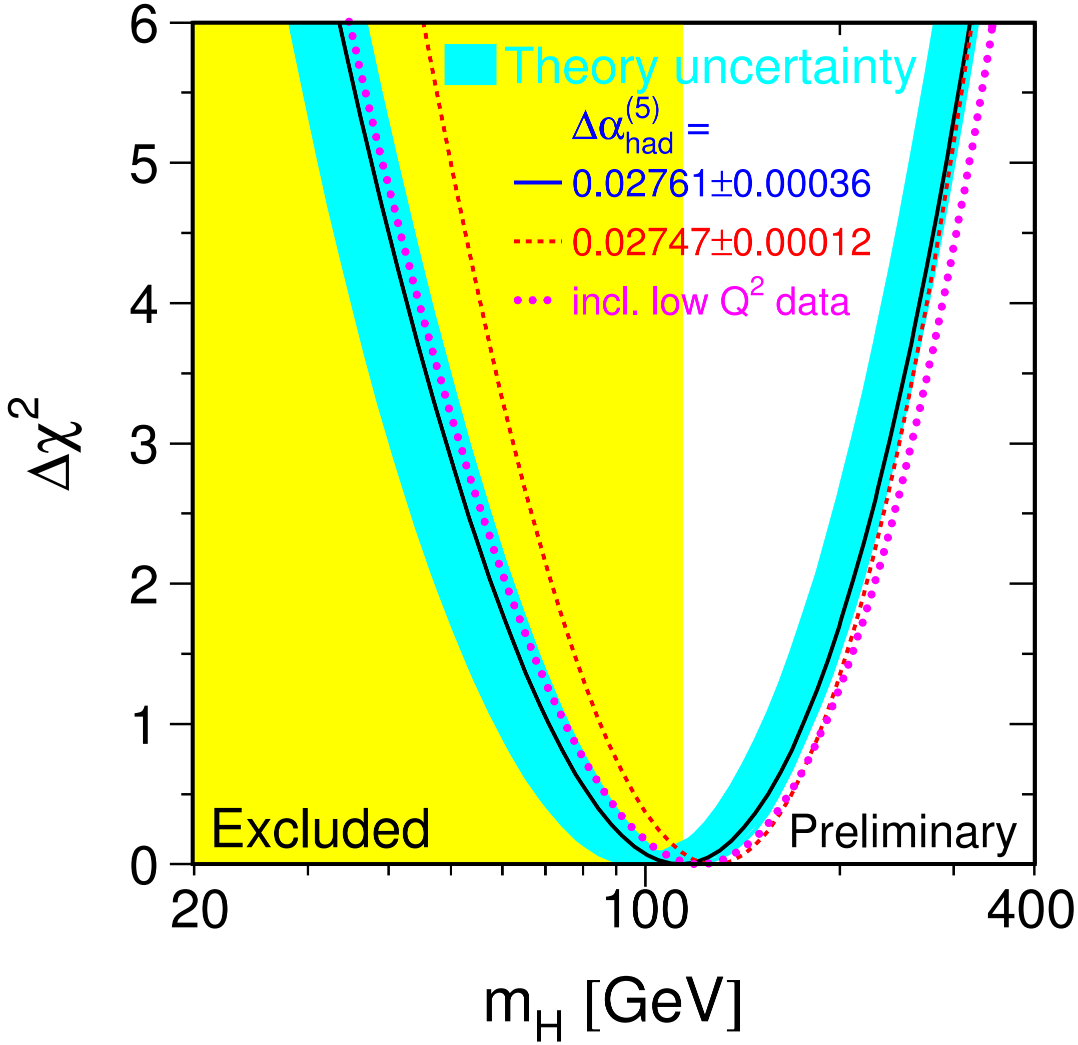
What Makes Us So Sure the Higgs Exists?

- One word: *unitarity*.
- Just as we inferred the existence of the $W^{\pm}$ and the Z^0 based on the pathological high-energy behavior of certain scattering cross sections, we find that high-energy divergences in $W^+ W^- \rightarrow W^+ W^-$ scattering are cured by the Higgs boson.
- Technically, this doesn't mean that the disease *has to* be cured by the Higgs boson, but there had better be *something* new before 1 TeV. The Higgs just happens to be the "simplest something new".

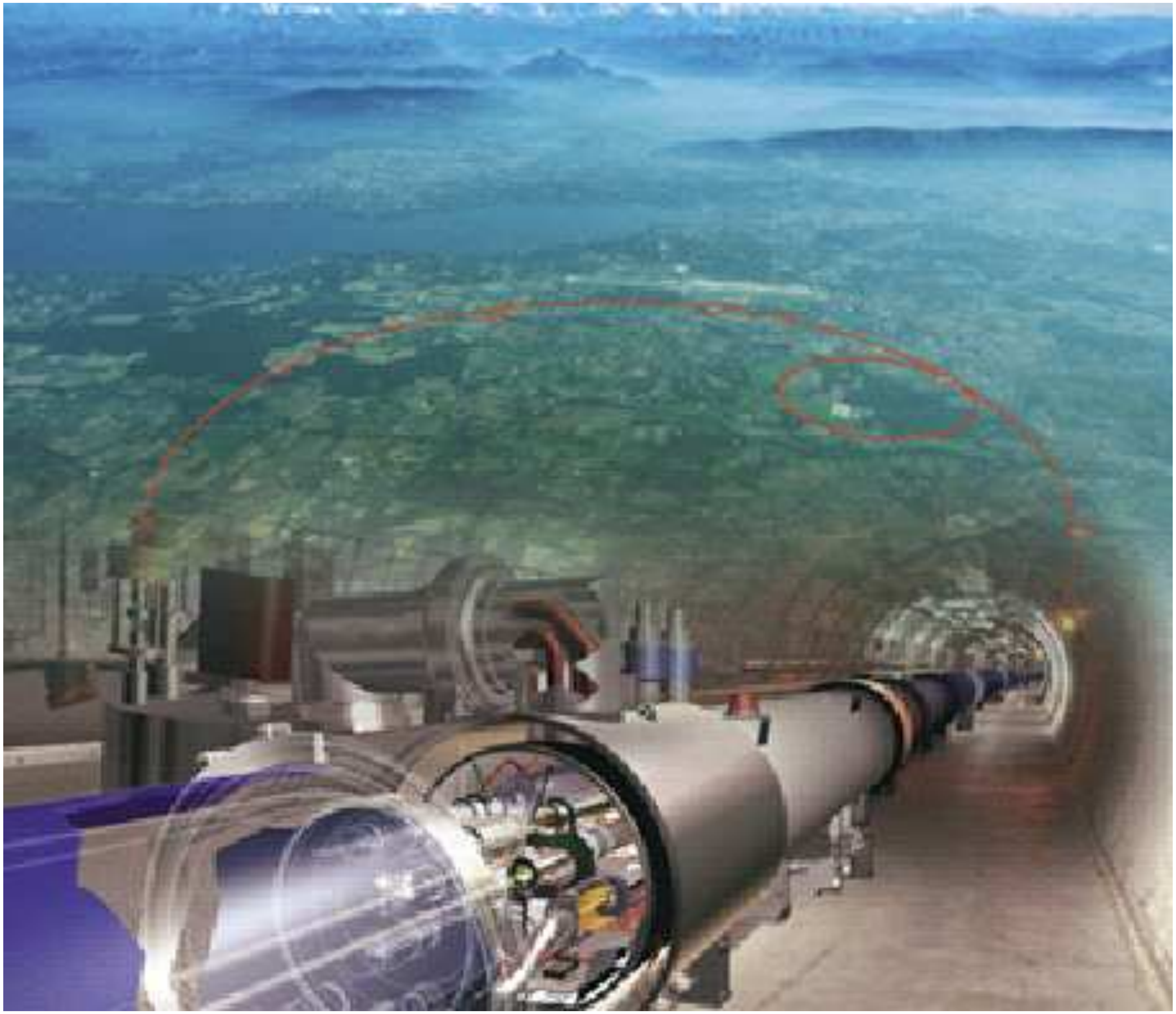
Constraints on m_H

- The mass of the Higgs is $m_H = v\sqrt{\lambda/2}$ where $v = 247$ GeV is fixed by G_F and λ is an unknown dimensionless coupling co nstant.
- m_H can't be too large, lest we violate *unitarity*.
- m_H can't be too small, lest the weak vacuum become unstable (i.e., an even lower-energy state exists elsewhere).
- Even at energies below m_H, the Higgs appears in Standard Model loop diagrams. This allows us to infer the most likely mass of the Higgs.
- The Standard Model Higgs has a mass somewhere between 115 GeV and about 200 GeV. LHC will soon sort this out.


LEP1, SLD Data
LEP2, $p\bar{p}$ Data
68% CL
$\Delta\alpha$
m_H [GeV]
114
300
1000
Preliminary
80.6
80.5
80.4
80.3
80.2
m_W [GeV]
130
150
170
190
210
m_t [GeV]

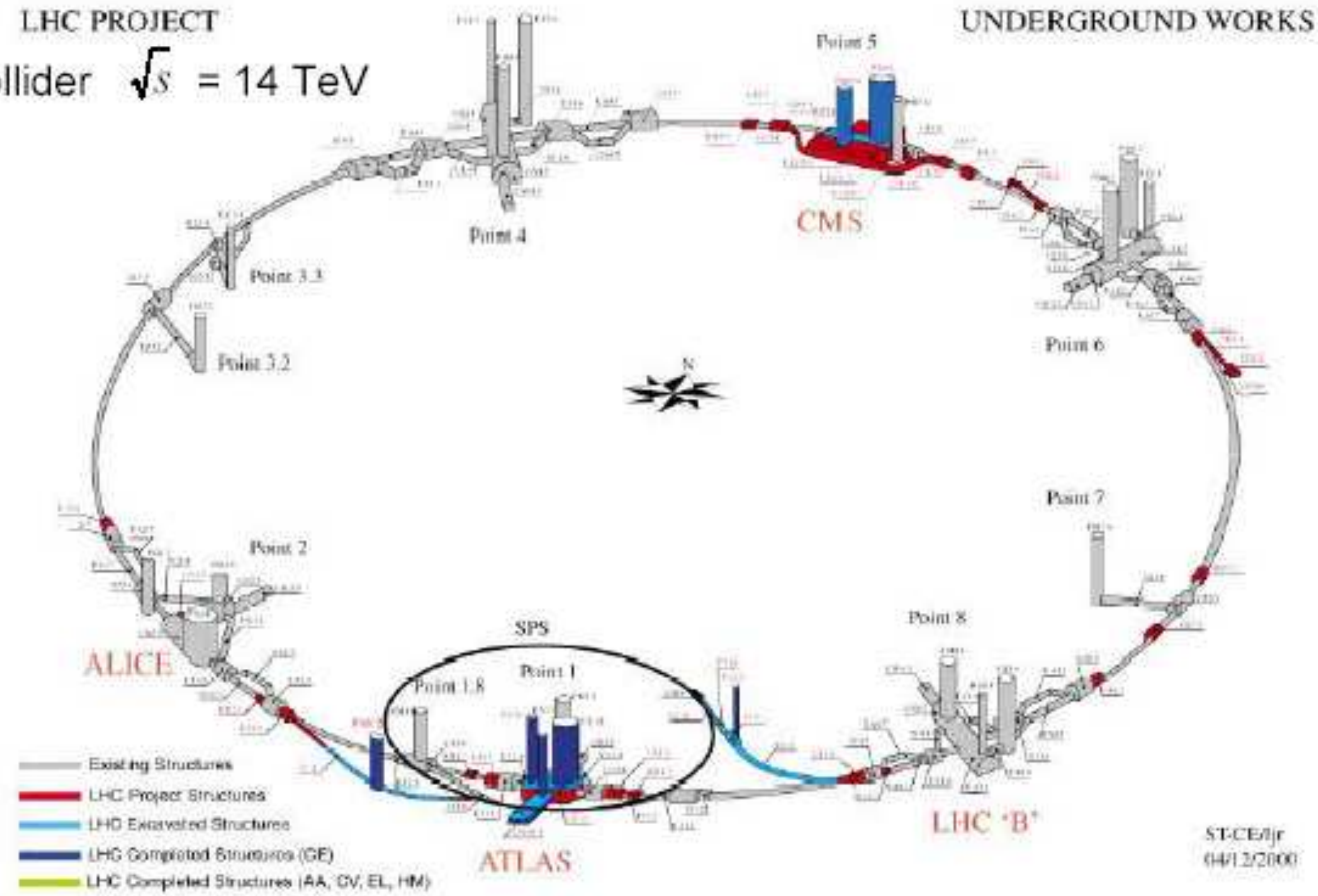


Theory uncertainty
$\Delta\alpha_{had}^{(5)}$ =
0.02761±0.00036
0.02747±0.00012
incl. low Q^2 data
Excluded
Preliminary
$\Delta\chi^2$
m_H [GeV]
0
1
2
3
4
5
6
20
100
400



Large Hadron Collider at CERN

pp collider $\sqrt{s}$ = 14 TeV



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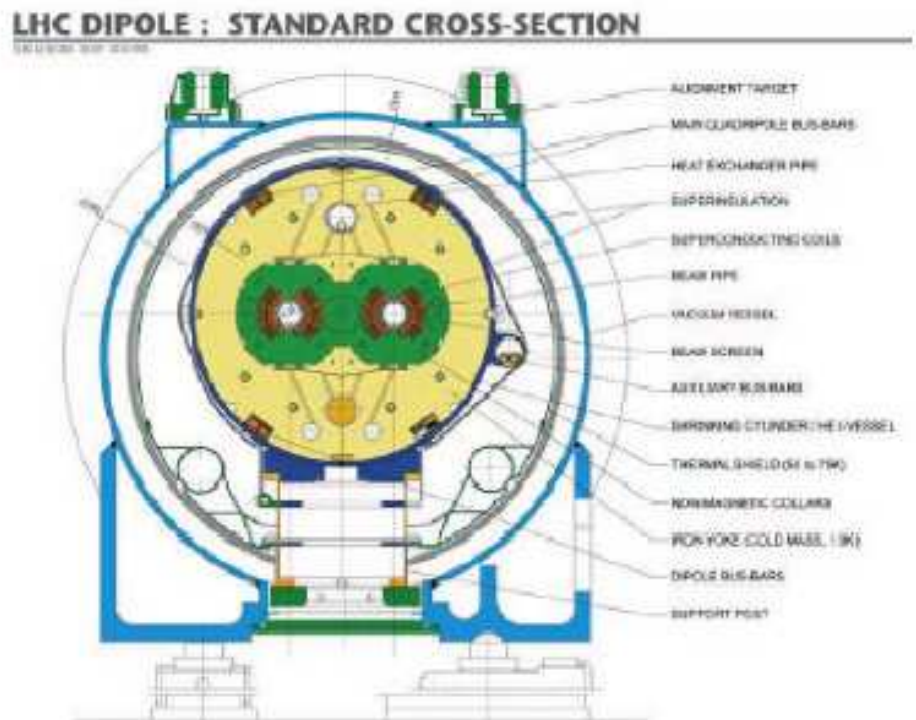
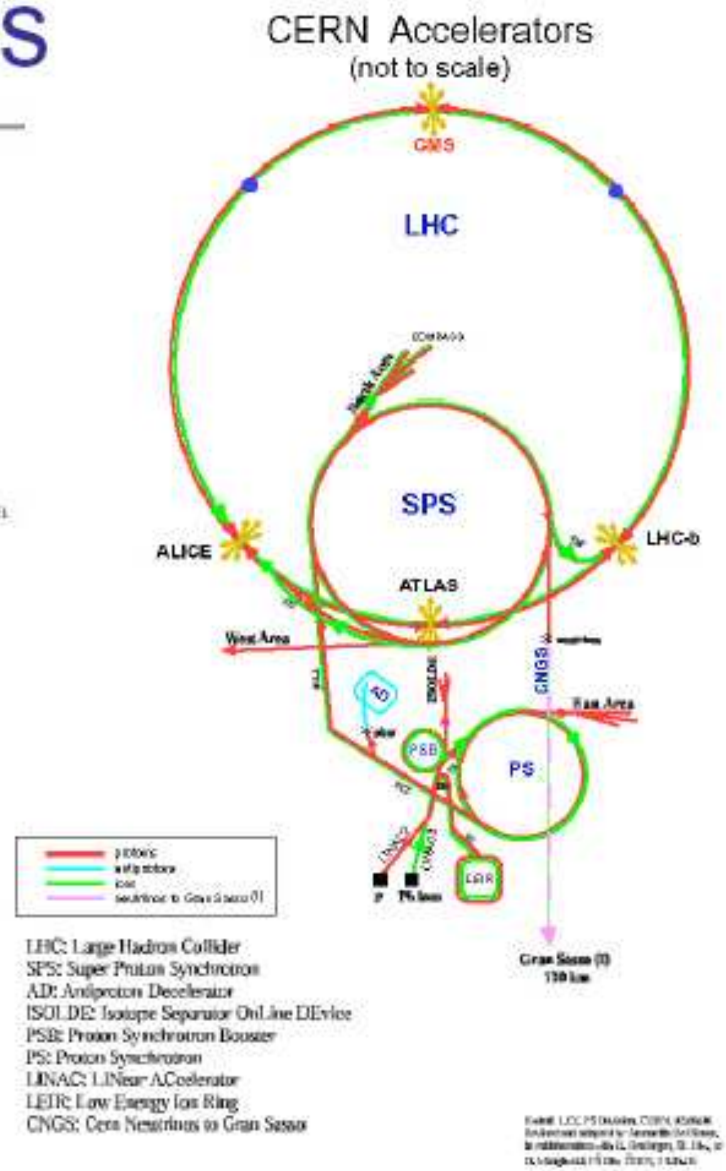
CERN Accelerators





LHC: 7 TeV on 7 TeV proton-proton

Design luminosity of 10^{34} $cm^{-2}s^{-1}$

Higgs Mechanism

A room full of physicists chattering quietly is like space filled with the Higgs field...

... a well-known scientist walks in, creating a disturbance as he moves across the room and attracting a cluster of admirers with each step...

...this increases his resistance to movement, in other words, he acquires mass, just like a particle moving through the Higgs field...

...if a rumor crosses the room...

...it creates the same kind of clustering, but this time among the scientists themselves. In this analogy, these clusters are the Higgs particles

Higgs to be found below about 1 TeV and/or new physics beyond the Standard Model!!!

ATLAS educational web page, adapted from an idea from Dr D. J. Miller

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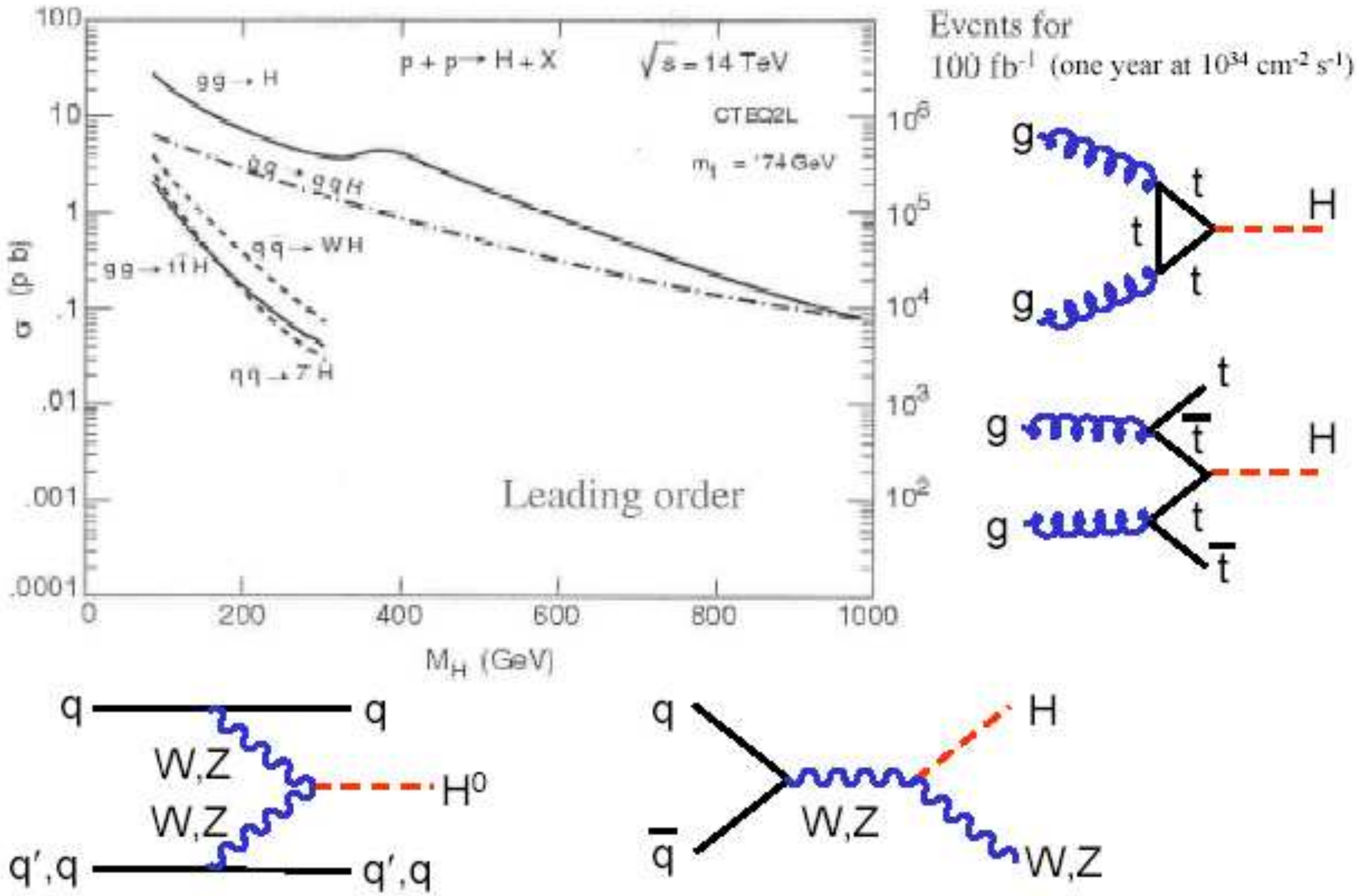
SM Higgs Production at the LHC



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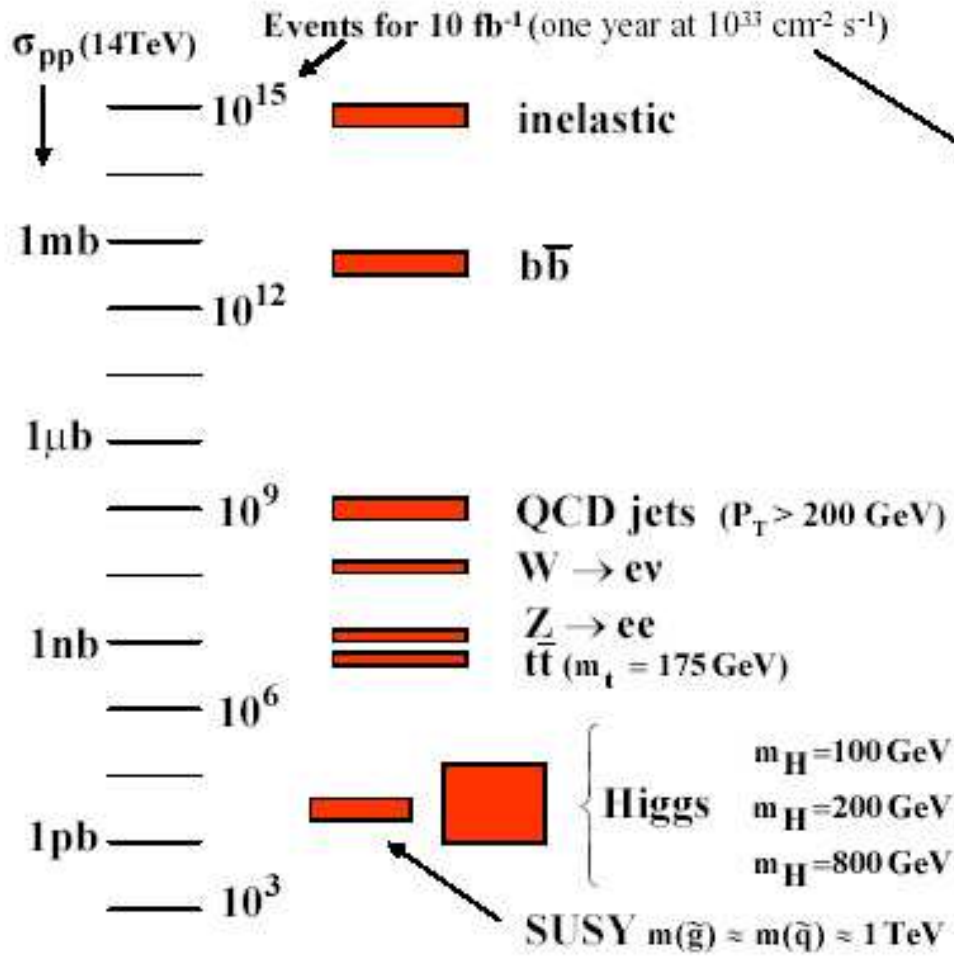
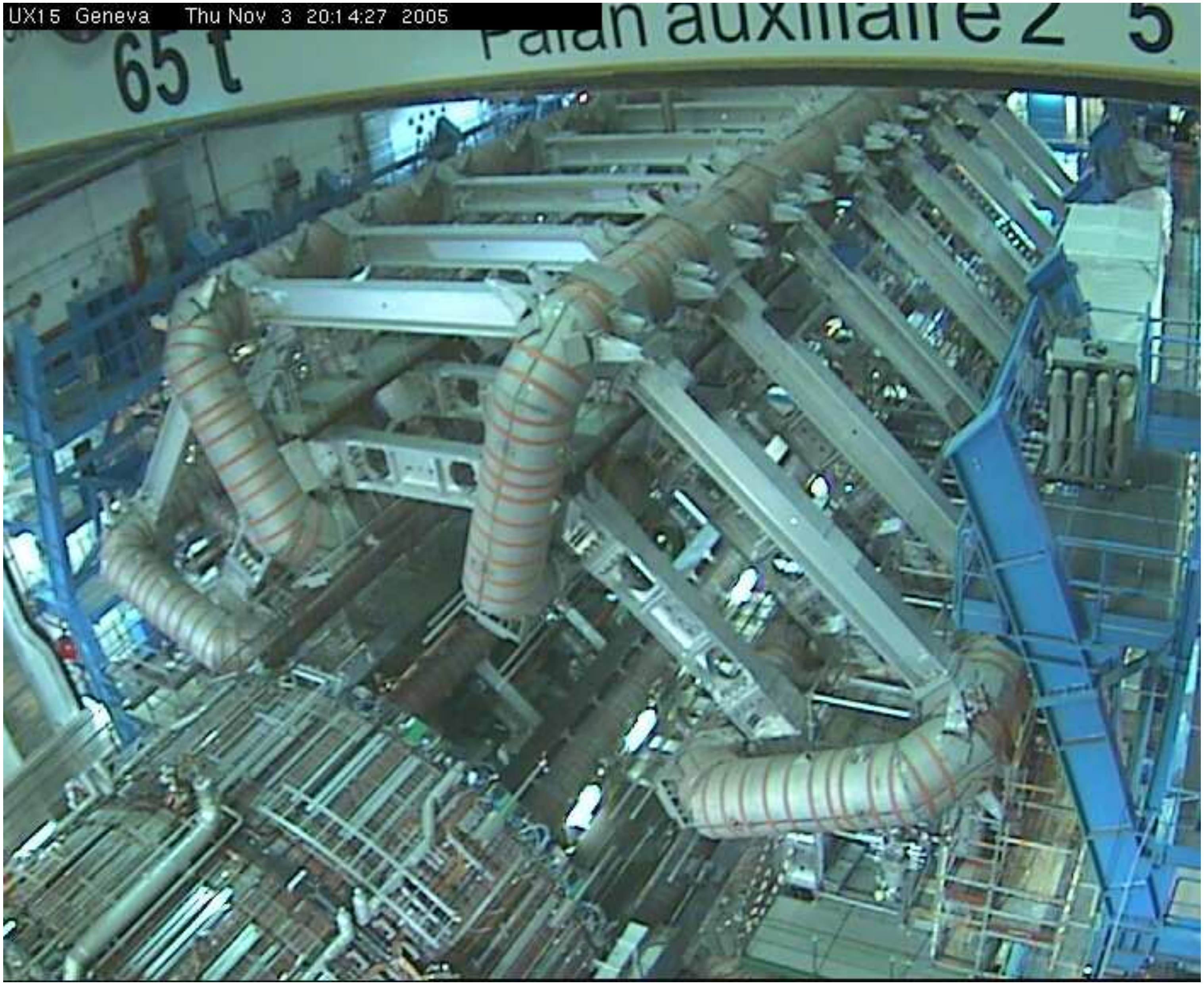
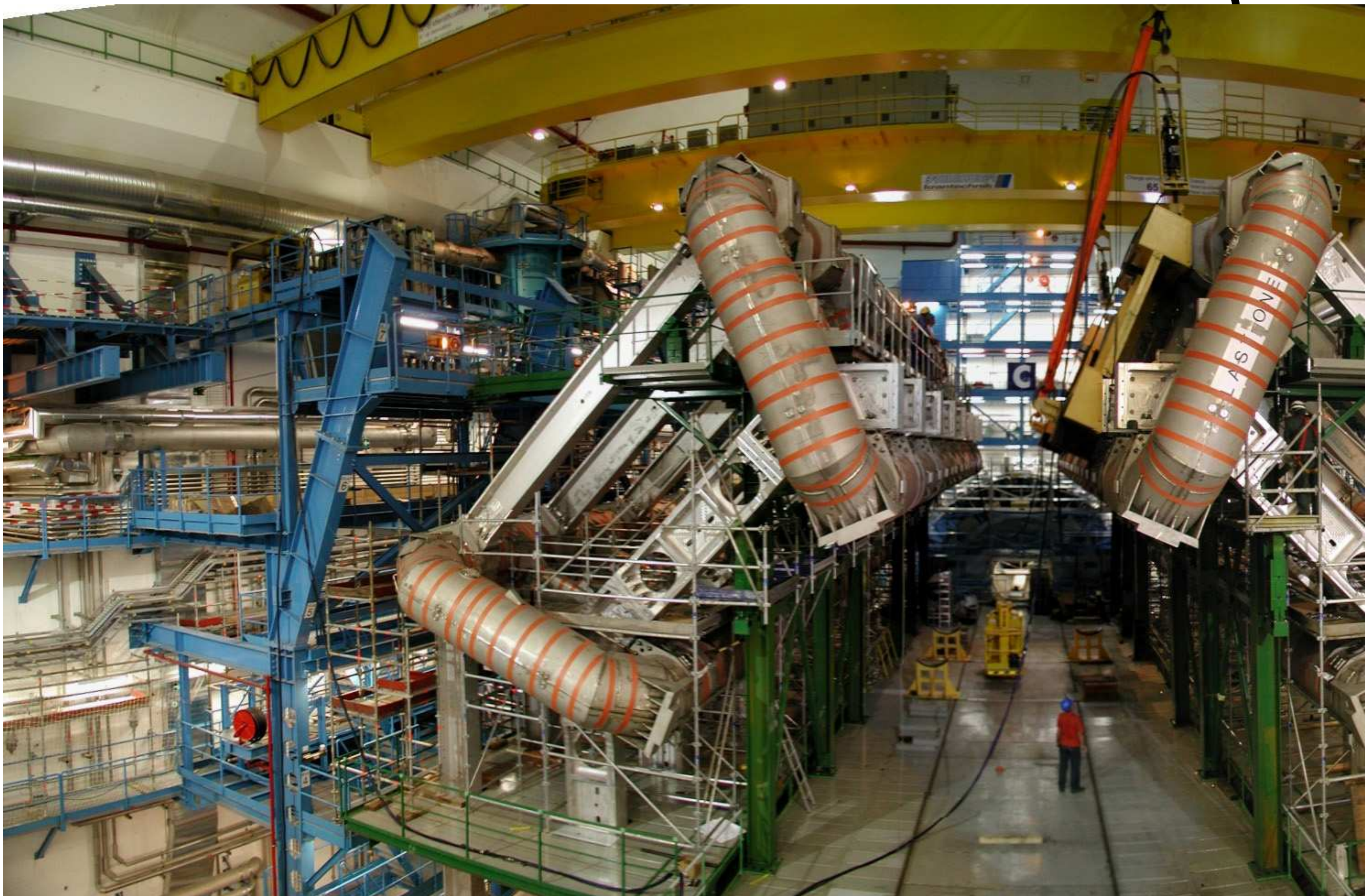
LHC PP Cross Section



- 6.7 times the commissioning luminosity
- 83% of first year luminosity
- 10% of nominal luminosity

It is a challenge to "fish out" events that are more than 10 orders of magnitude rarer than the most common interactions


UX15 Geneva Thu Nov 3 20:14:27 2005
65 t
Palan auxiliaire 2 5


C