Optimal Jet Finder

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Disclaimer

We use the OJF name as given by the authors. We do not necessarily agree with the name. If you do not agree with the name, contact the OJF author.

Optimal Jet Finder (OJF)

- Documentation
  - proposed by Fyodor Tkachov
  - short introduction: Phys. Rev. Lett. 91, 061801 (2003)
  - Int. Journal of Mod. Phys. A, Vol 17, No 21 (2002) 2783-2884.
  - authors webpage (with links to source code, etc.)
    - http://www.inr.ac.ru/~ftkachov/projects/jets/welcome.html
  - first presentation in the context of ATLAS: Damir Lelas 2007/01/10
    - http://indico.cern.ch/conferenceDisplay.py?confId=7765

- Implementation in Athena in progress
  - Rolf Seuster, Damir Lelas
    - significant recent progress: many bugs have been corrected
  - Discussion with other experts has started

Optimal Jet Finder (OJF)

HEP event: list of **particles** $p_a, \quad a = 1, 2, \ldots, n_{\text{parts}}$
(partons • hadrons • calorimeter cells • towers • preclusters)

recombination matrix $\{z_{aj}\}_{n_{\text{parts}} \times n_{\text{jets}}}$

$$q_j = \sum_{a=1}^{n_{\text{parts}}} z_{aj} p_a$$

the 4-momentum q_j of the j-th jet expressed by 4-momenta p_a of the particles

result: list of **jets** $q_j, \quad j = 1, 2, \ldots, n_{\text{jets}}$

E. Jankowski

Recombination matrix

- Fixes the jet configuration

$$q_j = \sum_{a=1}^{n_{\text{parts}}} z_{aj} \, p_a$$

the 4-momentum q_j of the j-th jet expressed by 4-momenta p_a of the particles (a=1,2,...,n_{parts})

$$z_{aj} \geq 0$$

the fraction of the energy of the a-th particle can be positive only

$$\bar{z}_a \equiv 1 - \sum_{j=1}^{n_{\text{jets}}} z_{aj}$$

the fraction of the energy of the a-th particle that does not go into any jet

$$\bar{z}_a \geq 0$$

i.e. no more than 100% of each particle is assigned to jets

Soft (transverse) energy

- Cylindrical kinematics
  - OJF can also be formulated in terms of spherical kinematics
  - Here we use cylindrical kinematics, relevant to hadron colliders

$$E_{\text{soft}}^{\perp} \equiv \sum_{a=1}^{n_{\text{parts}}} \bar{z}_a E_a^{\perp} \qquad E_{\text{tot}}^{\perp} \equiv \sum_{a=1}^{n_{\text{parts}}} E_a^{\perp}$$

transverse energy left outside the jets

total transverse energy

$$p_a = E_a^{\perp}\left(\cosh \eta_a, \cos \varphi_a, \sin \varphi_a, \sinh \eta_a\right) \qquad p_a^2 = 0$$

$$E_a^{\perp} \equiv \sqrt{\left(p_a^{\mathrm{x}}\right)^2 + \left(p_a^{\mathrm{y}}\right)^2}$$

Fuzzyness

- Define the fuzziness of the event

$$Y \equiv 2\sum_{j=1}^{n_{\text{jets}}} q_j \cdot \tilde{q}_j$$

fuzziness

$$q_j \equiv \left(E_j, \mathbf{q}_j\right) \equiv \sum_{a=1}^{n_{\text{parts}}} z_{aj} p_a \qquad \eta_j \equiv \frac{\sum_{a=1}^{n_{\text{parts}}} z_{aj} E_a^{\perp} \eta_a}{\sum_{a=1}^{n_{\text{parts}}} z_{aj} E_a^{\perp}} \qquad \frac{\mathbf{q}_j^{\perp}}{\left|\mathbf{q}_j^{\perp}\right|} \equiv \left(\cos\varphi_j, \sin\varphi_j\right)$$

$$\tilde{q}_j \equiv \left(\cosh\eta_j, \cos\varphi_j, \sin\varphi_j, \sinh\eta_j\right) \qquad \tilde{q}_j^2 = 0$$

Final jet configuration

- The recombination matrix is found by minimizing Ω given by

$$\Omega\left[\left\{z_{aj}\right\},\left\{p_a\right\}\right]E_{\text{tot}}^{\perp} \equiv \frac{1}{R^2}Y + E_{\text{soft}}^{\perp}$$

R weights the relative contributions

- Ω is dimensionless, and linear in the jet energies
- For cylindrical kinematics:

$$\Omega E_{\text{tot}}^{\perp} = \frac{4}{R^2}\sum_{a=1}^{n_{\text{parts}}} E_a^{\perp} \sum_{j=1}^{n_{\text{jets}}} z_{aj}\left(\sinh^2\frac{\eta_a-\eta_j}{2}+\sin^2\frac{\varphi_a-\varphi_j}{2}\right)+\sum_{a=1}^{n_{\text{parts}}}\bar{z}_a E_a^{\perp}$$

Algorithm: fixed n_{jets} case

1. Fix the parameter R

2. Start with some (random) value of the recombination matrix z_{aj} and minimize Ω with respect to z_{aj} for some given p_a

3. Repeat this a few times, each time starting with a different (random) z_{aj}

4. The value of z_{aj} that corresponds to the smallest of the minima of Ω is the final jet configuration for the required (fixed) number of jets

Algorithm: general case

- If the number of jets is to be determined

1. Start with $n_{\text{jets}} = 1$
2. Find the corresponding n_{jets} configuration
3. Check if $\Omega < \omega_{\text{cut}}$ is fulfilled; if yes stop here
4. If not, increase n_{jets} by one and repeat at step 2

- The parameter ω_{cut} is some small positive number, analogous to the jet resolution parameter of conventional recombination algorithms
  - related to the jet resolution y_{cut} of conventional recombination algorithms
  - ω_{cut} is effectively the upper limit on $E^{\perp}_{\text{soft}} / E^{\perp}_{\text{tot}}$.

OJF features

- The authors claim it is based on an optimal jet definition that solves the problem of jet definition in general
  - OJF is infrared and collinear safe
  - no issues intrinsic with seeds
  - no issues with overlapping cones: all jets are “ready to use”
- Particle energy can be shared among jets (continuous z_{aj})
  - Hadronization is always an effect of the interaction of at least two hard partons evolving into two jets, so some hadrons that emerge in this process can belong partially to both jets
  - conventional jet algorithms have z_{aj} equal to 0 or 1, i.e. a particle either entirely belongs to some jet or does not belong to that jet at all;
- the association between particles and jets is obtained through the minimization of a global function
  - global variables are a bi-product of this procedure
- Particles (or part of them) are allowed to be outside all jets
  - but with a penalty in the global function to minimize

Ω function

$$\Omega E_{\text{tot}}^{\perp} = \frac{4}{R^2} \sum_{a=1}^{n_{\text{parts}}} E_a^{\perp} \sum_{j=1}^{n_{\text{jets}}} z_{aj} \left(\sinh^2 \frac{\eta_a - \eta_j}{2} + \sin^2 \frac{\varphi_a - \varphi_j}{2} \right) + \sum_{a=1}^{n_{\text{parts}}} \overline{z}_a E_a^{\perp}$$

- Some insight in the meaning of Ω and *R* can be gained by considering the narrow jet limit:

if $\Delta\eta_{aj} \equiv \eta_a - \eta_j \ll 1$ and $\Delta\varphi_{aj} \equiv \varphi_a - \varphi_j \ll 1$ then

$$\Omega \simeq \frac{\sum_{a=1}^{n_{\text{parts}}} E_a^{\perp} \sum_{j=1}^{n_{\text{jets}}} z_{aj} \left(\frac{\Delta R_{aj}}{R} \right)^2}{\sum_{a=1}^{n_{\text{parts}}} E_a^{\perp}} + \frac{\sum_{a=1}^{n_{\text{parts}}} \overline{z}_a E_a^{\perp}}{\sum_{a=1}^{n_{\text{parts}}} E_a^{\perp}} = \left\langle \sum_{j=1}^{n_{\text{jets}}} z_{aj} \left(\frac{\Delta R_{aj}}{R} \right)^2 \right\rangle_{E_a^{\perp}} + \left\langle \overline{z}_a \right\rangle_{E_a^{\perp}}$$

where $(\Delta R_{aj})^2 \equiv (\Delta\eta_{aj})^2 + (\Delta\varphi_{aj})^2$

average (transverse energy weighted) over all particles

Connection with cone algorithms

- The exact relation between the threshold angle and *R* depends on how transverse energy is distributed between the particles
- The OJF forms jets following the structure of energy flow within the correlation angle *R*
- In the narrow jet limit:
  - If an infinitesimally soft (transverse energy) particle distance to the nearest jet is less than or equal to *R*, then it is included in that jet;
  - for non-infinitesimally soft particle, the threshold angle is less than *R*;
    - if an infinitesimal hard parton is split into to equal transverse energy fragments separated by 2θ, then the OJF would include them both into one jet if $\theta \leq \frac{1}{\sqrt{2}} R$.
  - the parameter *R* relates to the conventional cone algorithm R_{cone} through $R_{\text{cone}} \approx \frac{1}{\sqrt{2}} R$