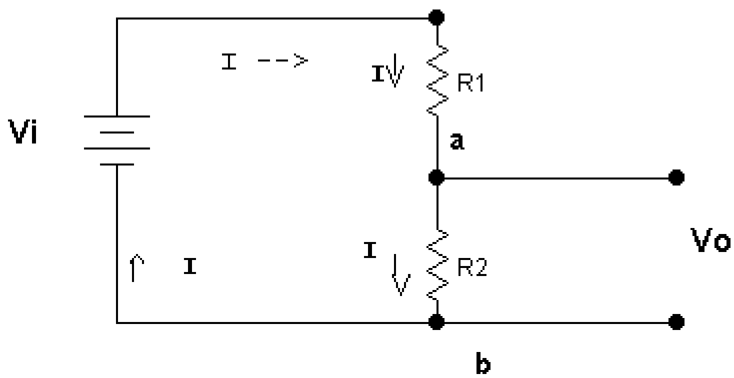
Topics: the voltage divider, loading errors, Kirchoff's laws

The Voltage Divider:

application of series resistance equation:

$$\mathbf{R_{eq} = R_1 + R_2 + R_3 + \ldots = \Sigma R_I}$$



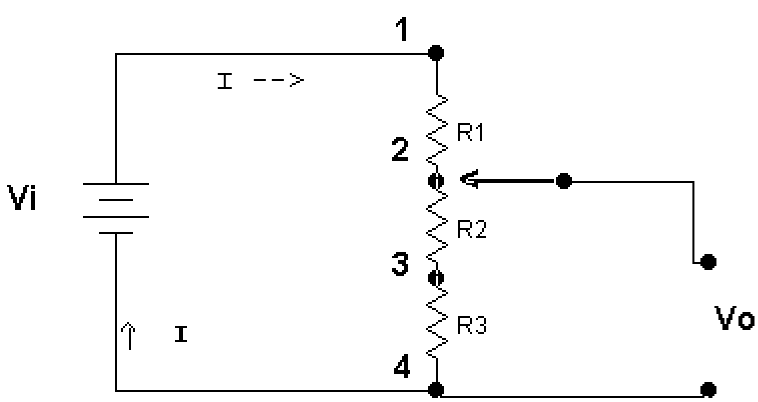
$$V_o = V_{ab} = I \times R_2$$

$$I = \frac{V_i}{R_1 + R_2} \quad \text{since } R_{eq} = R_1 + R_2$$

$$\therefore V_o = \frac{R_2}{R_1 + R_2} V_i$$

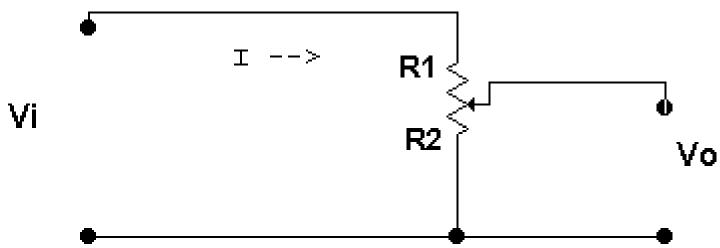
$$\text{Attenuation Factor} = \frac{V_o}{V_i} = \frac{R_2}{R_1 + R_2}$$

Switched resistor ladder produces a **switched voltage divider:**



Switch Position	V_o
1	V_i
2	$\frac{R_2 + R_3}{R_1 + R_2 + R_3} V_i$
3	$\frac{R_3}{R_1 + R_2 + R_3} V_i$
4	0

Extension of this to VARIABLE ATTENUATORS (Variable resistors)
using a **POTENTIOMETER** ('Pot')



$R = R_1 + R_2$ is fixed position of wiper sets R_1 and R_2:

$$V_o = \frac{R_2}{R} V_i$$

This is a three terminal device which provides output voltages between 0 and V_i

EFFECTS OF METER LOADING: LOADING ERROR

Because a voltmeter has an internal resistance, $R_{vm} = R_m + R_v$ in the diagram below, the voltage reading is affected by the current used by the meter, to make the measurement. This is the LOAD of the meter.
In the example shown, one measures the current, I_{vm}, flowing through R_{vm} and multiplies by R_{vm} to determine the voltage. This assumes that I_{vm} is negligible and does not modify the circuit. Consider what happens if we want to measure a voltage V_0 across a resistor, R, in a voltage divider. When we insert the meter, notice that R_{vm} and R are in parallel and the voltage across R changes because the meter is present.

Voltage drop without meter is: $Vo = I \times R$

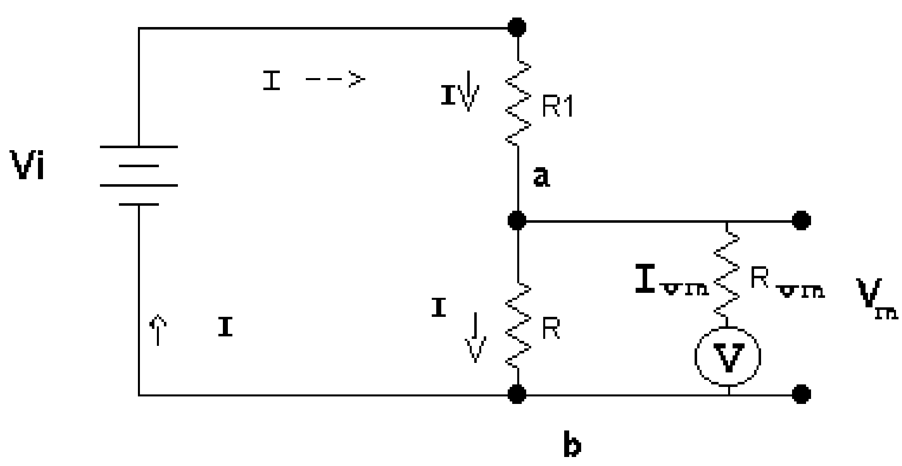
Voltage drop with the meter is:

$$Vm = I \times (1/R + 1/R_{vm})^{-1}$$

or $Vm = I \times R \times R_{vm} / (R \times R_{vm}) = V \times R_{vm} / (R + R_{vm})$

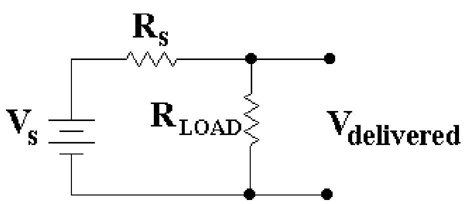
So there is a loading error of $1 - R_{vm} / (R + R_{vm})$

if $R << R_{vm}$ the error is small.



Ideal and Real Voltage Sources

An ideal voltage source provides the nominal voltage at its two terminals regardless of the load on the source. Real sources, such as batteries, can often be usefully modelled as an ideal source in series with an internal resistance. When the output is loaded with an external R_{LOAD}, one has the circuit presented immediately below: in this case R_s is the internal resistance of the source, V_s, is the internal voltage of the source (sometimes called the electromotive force, or emf, of the source).



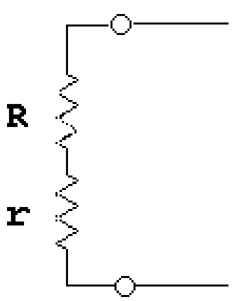
$$V_{delivered} = V_s R_{LOAD} /(R_s + R_{LOAD})$$

If the internal resistance is very low relative to the load resistance, then the delivered voltage is nearly equal to the internal voltage. One can measure the internal resistance of a battery by precisely measuring the delivered voltage of the battery under varying loads.

APPROXIMATIONS:

If two resistors, R and r, are in series and they have greatly different values (eg $R > 10 \times r$) then the equivalent resistance is approximately R.

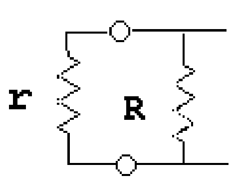
In series, BIG resistances are important: since they limit the current.



For R and r in series
If R>> r then $R_{eq} \approx R$ (+ a little)

If two resistors, R and r, are now in parallel and $R > 10 \times r$ then the equivalent resistance is approximately r.

In parallel, SMALL resistances are important: since they determine the current.



For R and r in parallel
If R>> r then $R_{eq} \approx r$(-a little)

KIRCHOFF's LAWS
Used for determining voltage drops and currents in a circuit.

NODE LAW: (or current law)
total current flowing into a node equals the total current flowing out of the node

$$\sum_{i \text{ at node}} I_i = 0$$

this is a statement of conservation of charge.

LOOP LAW: (or voltage law)
the sum of the voltage differences around a any CLOSED loop is zero.

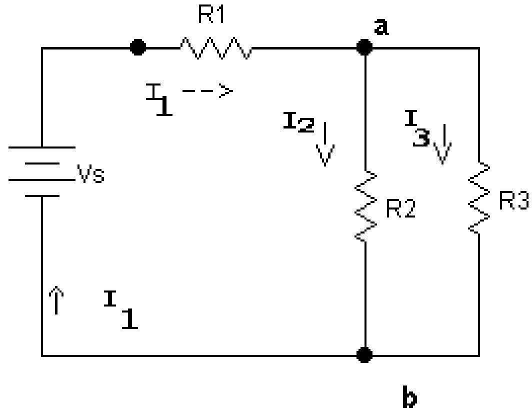
$$\sum_{i \text{ around a closed loop}} V_i = 0$$

this is a statement of conservation of energy

APPLYING KIRCHOFF'S LAWS TO SOLVE CIRCUIT PROBLEM

1. **Assume a positive current direction in each resistor.(this direction is arbitrary)**

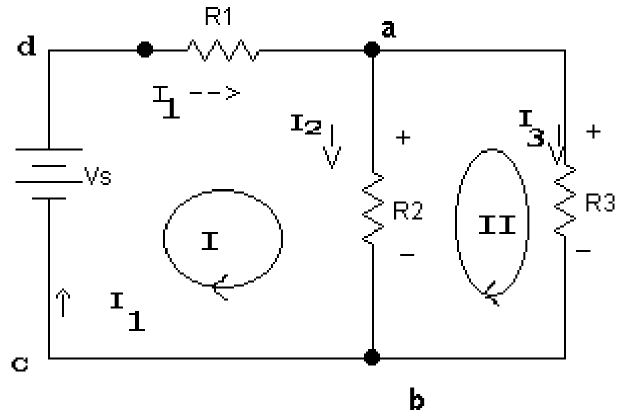
NODE LAW



$I_1 = I_2 + I_3$

$I_1 - I_2 - I_3 = 0$

LOOP LAW



Loop I:

$V_{da} + V_{ab} + V_{cd} = 0$

So: $V_{da} + V_{ab} = - V_{cd} = V_s$

Loop II:

$V_{ab} + V_{ba} = 0$

So: $V_{ab} = - V_{ba}$

2. **Mark the polarity (+ or -) of the voltage across each resistor using Ohm's Law and the current directions defined in 1.**
3. **Apply Kirchoff's Node and Loop Laws to get the same number of independent simultaneous linear equations as the number of unknowns.**

EXAMPLE:

Find currents flowing through each of the resistors in the following circuit:

Node Laws

$I_1 = I_2 + I_3$

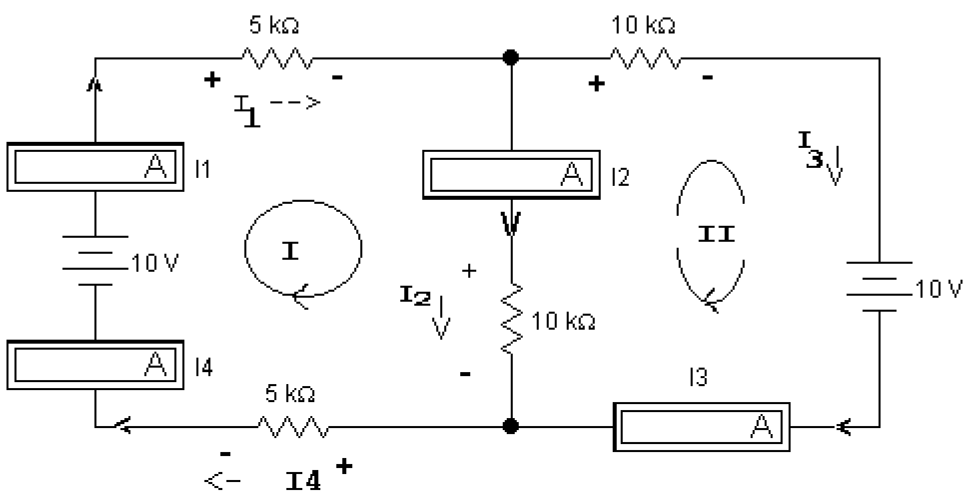
$I_4 = I_2 + I_3$

Loop I:

$-I_1 R_1 - I_2 R_2 - I_4 R_4 + 10V = 0$

Loop II:

$-I_3 R_3 - 10V + I_2 R_2 = 0$



Four equations and four unknowns…
solve this set of linear equations

Solution:

I1 = I4 = +1/3 mA
I2 = +2/3mA
I3=-1/3 mA