

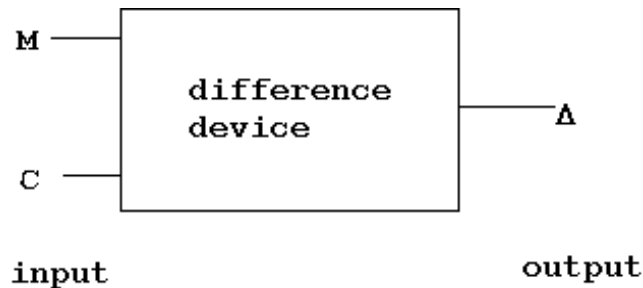
Topics: Wheatstone bridge, practice with Kirchoff's Laws, Thévenin's Theorem

USEFUL SOFTWARE TOOL:

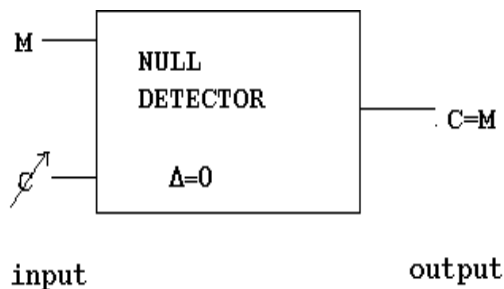
Electronics Workbench. available on PCs @front of the Elliott 139 terminal room

Measurements using DIFFERENCES and NULL measurements

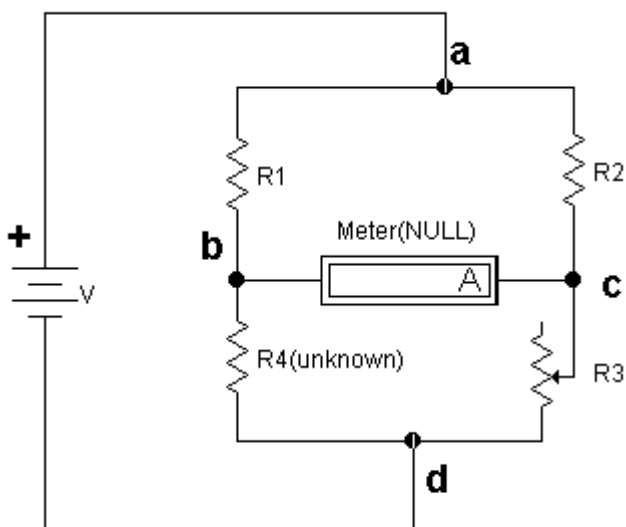
To make precise and accurate measurements often it is necessary to measure the difference between a unknown value of a quantity (eg voltage, current, resistance, etc) and a known or calibrated value. If Δ is the measured difference, and C is the calibration value, then the unknown value to be measured is just calculated as: $M = \Delta + C$



Another approach is to measure Δ and adjust C until $\Delta=0$. This is achieved with a NULL DETECTOR.



Wheatstone Bridge is an example of a NULL detector used for measuring resistance.



The Meter is a 'NULL detector' (voltmeter or ammeter)
R1 and R2 are fixed and known resistances
R4 is unknown and to be measured
R3 is a variable resistor whose value is known.

R3 is varied until the Meter reads 0
'At Balance': $R1/R4 = R2/R3$
or $R4 = R1 \times R3/R2$

Example of Use:

R4 could represent a 'thermistor', this is a device whose resistance varies with temperature as $R_T = Ae^{B/T}$. Measuring the resistance gives the temperature if the thermistor calibration constants, A and B, are known: $T = \frac{B}{\ln(R_T / A)}$.

The thermistor is an example of a *transducer*. Converts temperature into resistance. We measure the resistance (using eg a Wheatstone Bridge) then calculate a temperature using the measured resistance and calibration constants. In the lab you'll do this. In fact, you'll calibrate the thermistor by measuring the resistance at fixed temperatures determined from a calibrating thermometer. The calibration curve is obtained by measuring the slope and intercept of $\ln R_T = \ln A + B\left(\frac{1}{T}\right)$.

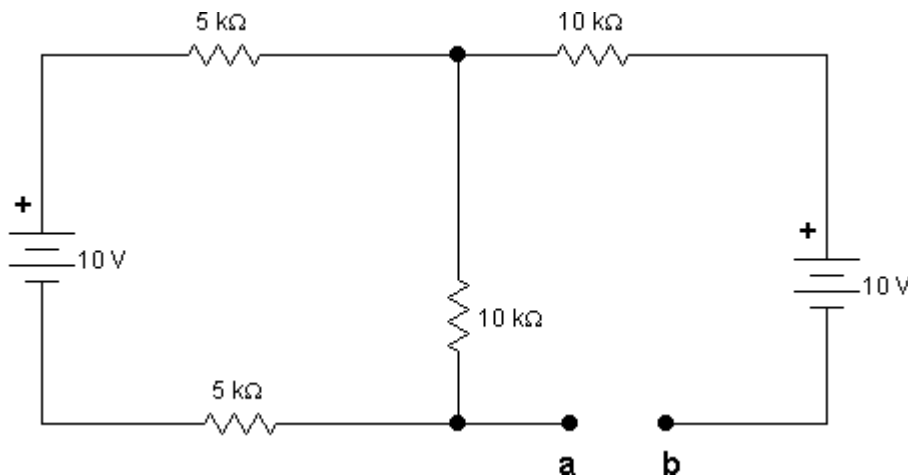
ANALYSIS of Wheatstone's Bridge using Thévenin's Theorem

Thévenin's Theorem is a very powerful tool for simplifying complicated circuits into a very simple one:

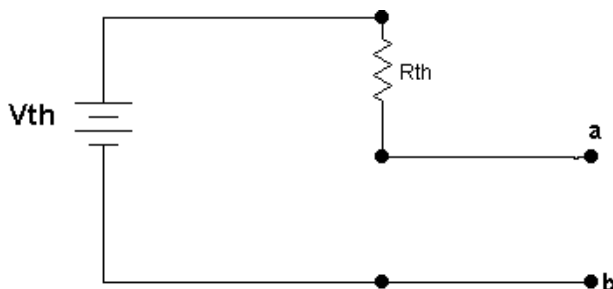
Thévenin's Theorem

A network of resistances, current sources and voltage sources can be replaced by a single equivalent voltage source, V_{TH} , in series with a single equivalent resistance, R_{TH} .

For Example: we want to know the voltage drop across a and b in the following. Thévenin's Theorem says we can replace this:



by this:

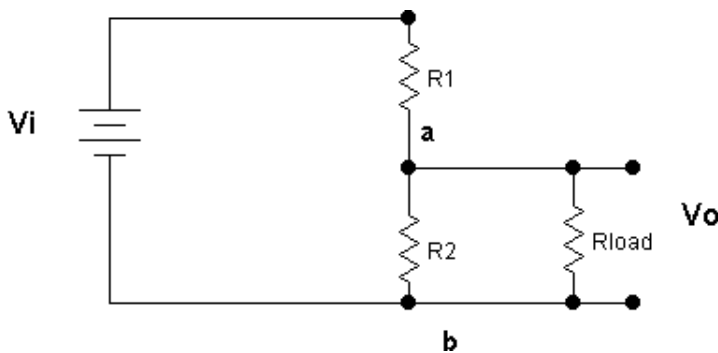


MUCH SIMPLER, EH?

1. The voltage drop across the element to be studied is labelled V_{th} and the element is removed from the circuit. Then calculate V_{th} (eg, using Kirchhoff's rules) with the element out.
2. Simply replace all voltage sources with "plain old wires", then calculate R_{th} .
3. Place the element which you removed in 1. back into the circuit in series with V_{th} and R_{th} .
4. you're done.

Example 1:

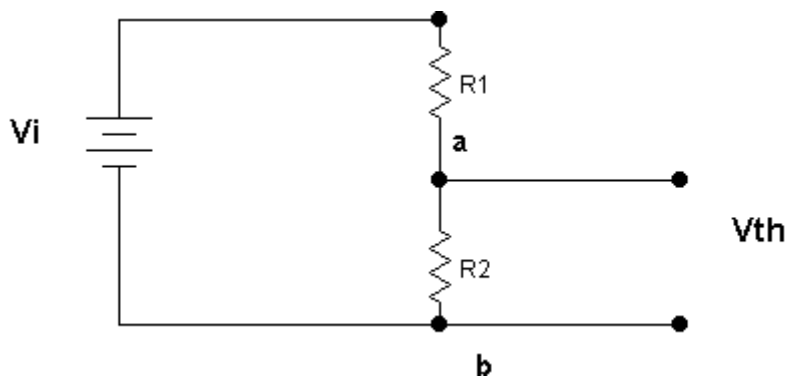
The voltage divider is used to setup a voltage $R_2/(R_1+R_2) V_i$. Then a load is put on the output



The 'load' on a two-terminal output of a circuit is the follow-on stage of the circuit that draws current. There is an equivalent resistance associated with the load, denoted as R_{LOAD} , through which the load current flows. For example, when powering a radio with a battery, the circuitry of the radio is considered the 'load' on the battery.

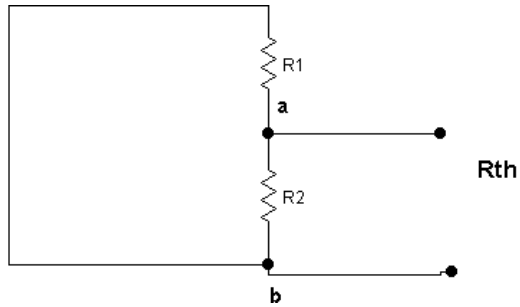
of the divider, R_{load} . We want to know the voltage across R_{load} .

1. Remove R_{load} , since that is what we want to study. The voltage V_{ab} is now V_{th} .



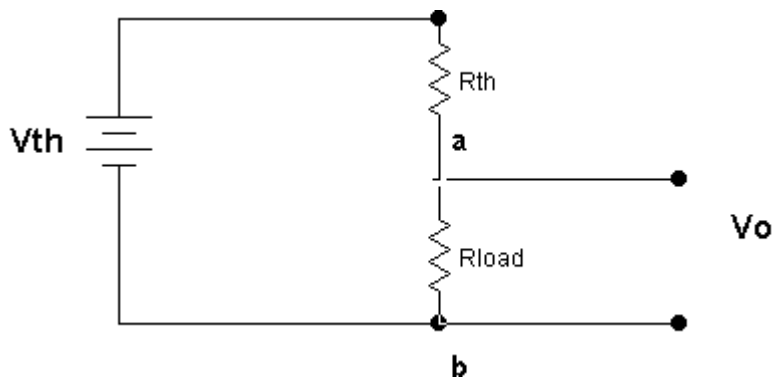
Now calculate $V_{th} = V_i R_2 / (R_1 + R_2)$

2. Now short out V_i and calculate R_{th} :



$$R_{th} = R_1 R_2 / (R_1 + R_2)$$

3. Put the R_{load} into the Thevenin circuit.

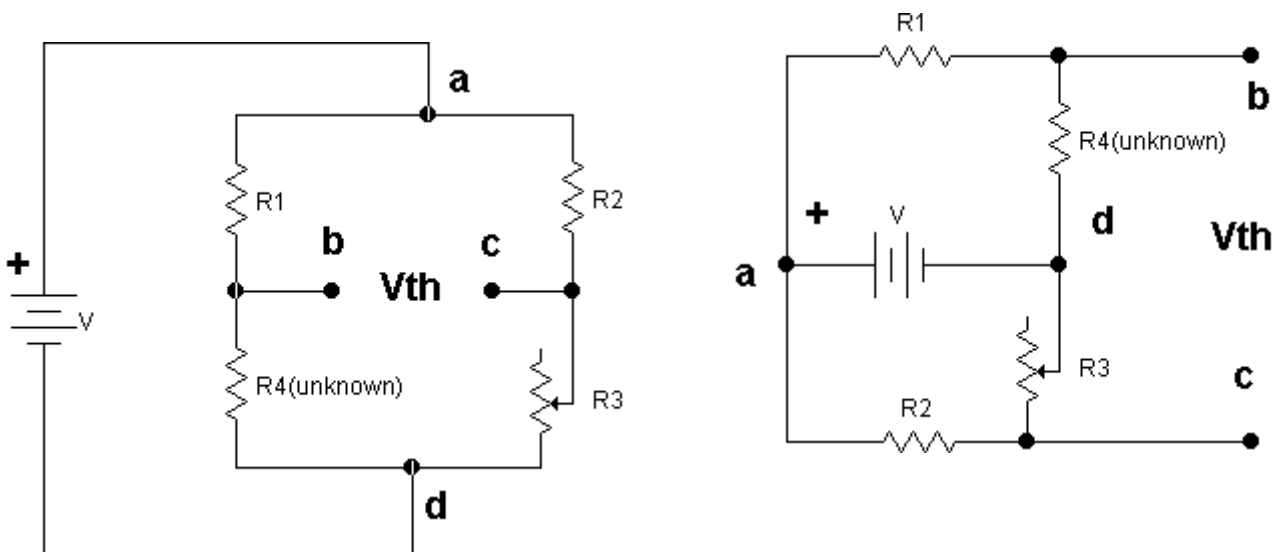


So now we can determine that $V_o = V_{th} R_{load} / (R_{load} + R_{th})$

EXAMPLE 2:

Wheatstone's Bridge analysis

The meter is the element of interest and has some resistance R_m . At balance, the voltage across the two terminals is zero. We'll start the analysis by letting that be V_{th} :



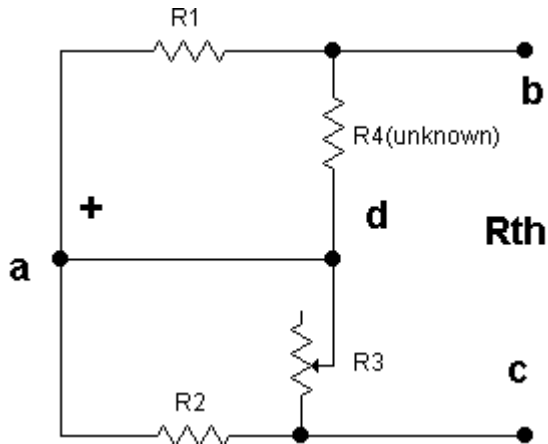
$$V_{th} = V_{bc} = V \times \left(\frac{R_4}{R_1 + R_4} - \frac{R_3}{R_2 + R_3} \right)$$

AT BALANCE:

$$V_{th} = V_{bc} = 0$$

$$\frac{R_1}{R_4} = \frac{R_2}{R_3}$$

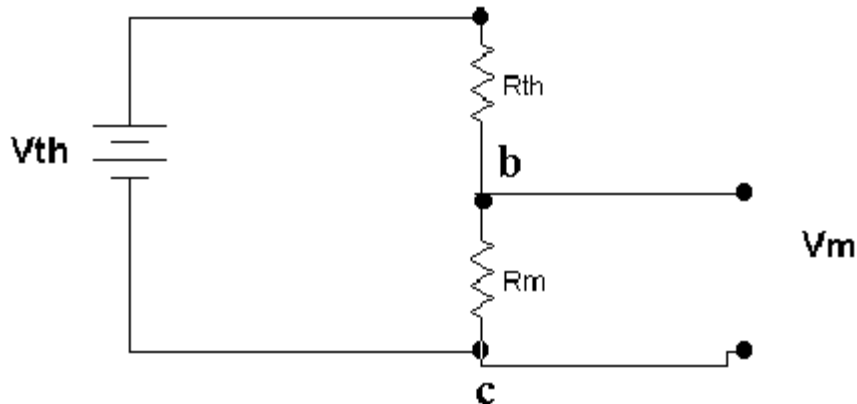
Now find R_{th} :



$$R_{th} = \frac{R_1 \times R_4}{R_1 + R_4} + \frac{R_2 \times R_3}{R_2 + R_3}$$

OFF BALANCE:

$$V_m = V_{th} \frac{R_m}{R_m + R_{th}}$$

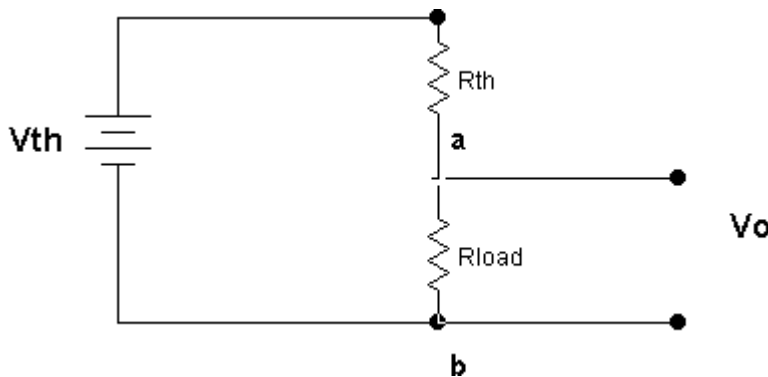


Maximum Power Transfer to a Load

Power delivered to a load is, of course the current squared times the load resistance.

The question arises: what is the value of a load resistance when the power delivered to it is a maximum? This is important when a source cannot deliver much power, for example when receiving a weak radio transmission. In that case, one wants as much power delivered to the load as possible.

Considered the circuit:



The power delivered to R_{LOAD} is $I^2 R_{LOAD}$ and $I = V_{th} / (R_{th} + R_{LOAD})$ so the power transferred to R_{LOAD} is:

$$P_{LOAD} = V_{th}^2 R_{LOAD} / (R_{th} + R_{LOAD})^2.$$

This is a maximum when $dP_{LOAD} / d R_{LOAD} = 0$.

$$dP_{LOAD} / d R_{LOAD} = V_{th}^2 ((R_{th} + R_{LOAD})^2 - 2 R_{LOAD} (R_{th} + R_{LOAD})) / (R_{th} + R_{LOAD})^4 = 0.$$

$$\text{so } (R_{th} + R_{LOAD})^2 - 2 R_{LOAD} (R_{th} + R_{LOAD}) = 0$$

$$\rightarrow (R_{th} + R_{LOAD}) - 2 R_{LOAD} = 0$$

$$\rightarrow R_{LOAD} = R_{th}$$

This means the maximum power you can transfer to a load occurs when $R_{LOAD} = R_{th}$ and the maximum power is $V_{th}^2 R_{LOAD} / (2R_{LOAD})^2 = V_{th}^2 / (4R_{LOAD})$.