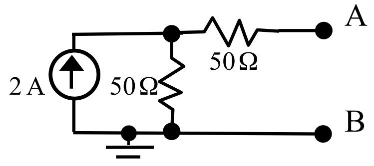
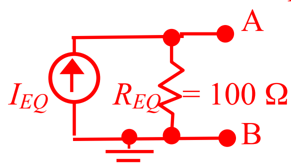
P214 Midterm SOLUTIONS

(50 minutes, plus a 5 – 10 minute break partway for “Great BC Shakeout”; *total 100 points*)

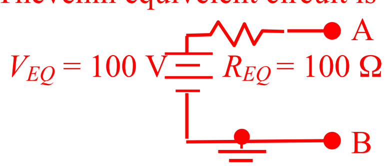
1. Derive the Thevenin and Norton equivalent circuits for the following circuit: (40 points)



We wish to determine the Thevenin V_{EQ} and R_{EQ} and the Norton I_{EQ} for this circuit. To derive the Thevenin V_{EQ}, we want to know the voltage between A and B when nothing is connected between those two points (i.e. as in the circuit above). The current around the loop in the circuit above in 2 amps (due to the current source) and thus the voltage across the 50 ohm resistor in the loop will be $V = IR = 2 \times 50 = 100$ volts. There will be zero voltage difference across the 50 ohm resistor between A and the upper right corner of the loop since there will be no current running through that leg when nothing is connected between A and B, and so the voltage difference between A and B will be 100 volts, and so $\mathbf{V_{EQ} = 100}$ **volts**. R_{EQ} is just the resistance between A and B when the current source is treated as being completely disconnected and the circuit being open where the current source presently is, and so that is just the resistance of two 50 ohm resistors in series, which is just $\mathbf{R_{EQ} = 100}$ **ohms**. To find I_{EQ}, we could either just note that $I_{EQ} = V_{EQ}/R_{EQ} = 1$ amp, or we could also consider what the current I_{EQ} would need to be in the Norton equivalent circuit below:



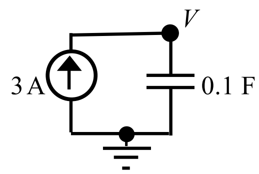
in order to make the voltage between A and B the same as the V_{EQ} calculated above, i.e. 100 volts. The answer is $\mathbf{I_{EQ} = 1}$ **amp**. So the Norton equivalent circuit is as above with $I_{EQ} = 1$ amp, and the Thevenin equivelent circuit is as below:



2. A 50 μF capacitor contains 1.25 mC of charge. What is the energy (in Joules) stored in the capacitor? (15 points)

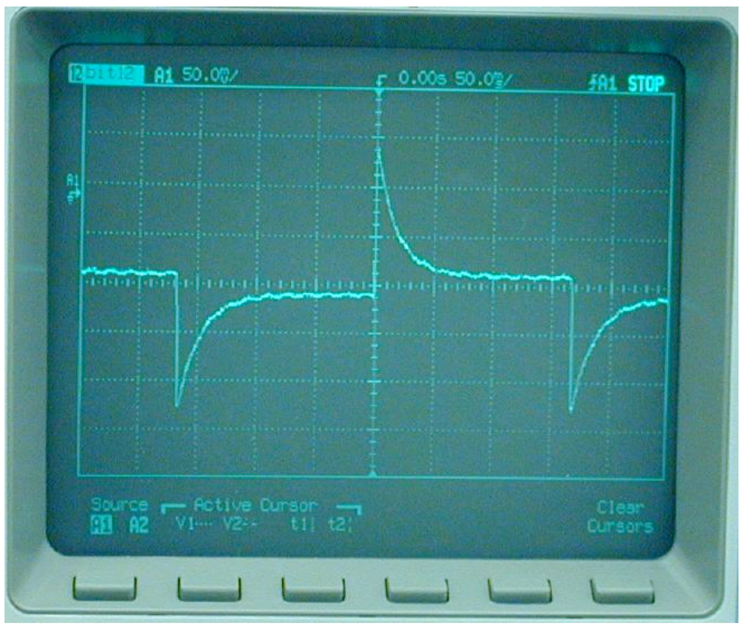
The energy stored in a capacitor is $E = ½CV^2$. The voltage $V = Q/C = (1.25 \times 10^{-3})/(50 \times 10^{-6}) = 25$ volts, thus $E = ½(50 \times 10^{-6})(25)^2 =$ **15.625 millijoules**.

3. At time $t = 0$ the current source is turned on in the following circuit. The capacitor is initially uncharged. Find the voltage V as a function of time. (25 points)



We know that $Q = CV$, and differentiating that, that $I = C(\mathrm{d}V/\mathrm{d}t)$. Thus $3 = 0.1(\mathrm{d}V/\mathrm{d}t)$, and thus $(\mathrm{d}V/\mathrm{d}t) = 30$. Multiplying by $\mathrm{d}t$ and integrating, we get that $V(t) = 30t + V_0$, where V_0 is a constant of integration. We are told that the capacitor is initially uncharged, thus we know that $V_0 = Q_{t=0}/C = 0$, thus $\boldsymbol{V(t) = 30t}$. (Note that this result will, of course, only be the case until either the circuit is disconnected, or the capacitor reaches its voltage limit, at which time the capacitor will, most likely, burn out and no longer function as a capacitor.)

4. An oscilloscope reads out the following voltage as a function of time: (20 points)



The oscilloscope is set at 50 milliseconds per division, and 50 millivolts per division. What are the a) amplitude, b) period, c) frequency, and d) angular frequency of this signal?

The "divisions" on an oscilloscope are the dotted-line divisions on the grid shown on the screen, and thus (a) the amplitude, which is half the peak-to-peak difference, is in the range of approximately **125 – 135 millivolts**, (b) the period is just a bit under 350 milliseconds (perhaps 1/3 second = **333 milliseconds**), (c) the frequency, which is 1/(the period) is about **3 Hz**, and (d) the angular frequency, which is 2π x the frequency, is in the range **17 – 20 radians/second**.