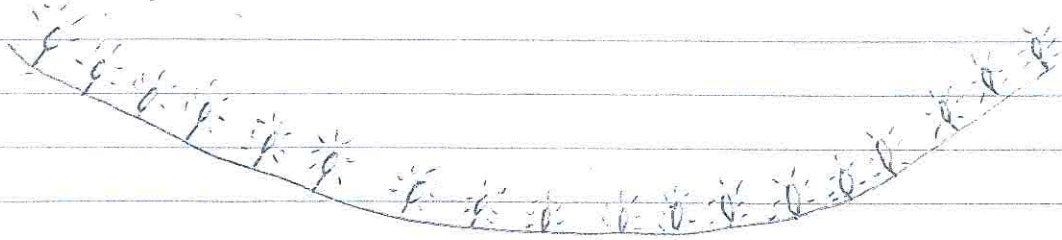
1.10.2 The x-mas lights:



are 20 bulbs connected in series, so their circuit diagram looks like this:

20 lights, each with resistance "R"

One of the bulbs "fuses," which we can interpret as it melting such that its resistance goes away and it effectively becomes a wire. Thus, after this fusing, we are in effect left with 19 bulbs in series (instead of 20).

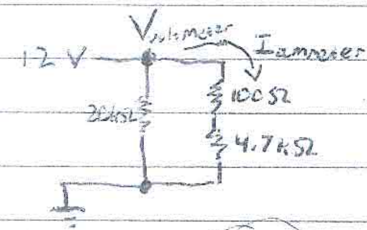
The bulbs are "1 W bulbs," which we can interpret as meaning that <u>before</u> the fusing (when the lights are working as designed), each bulb dissipates 1 watt of <u>power</u>.

From that information, we can figure out the resistance R of each bulb: The <u>voltage</u> across each bulb (before the fusing) must equal $240/20$ volts $= 12$ volts. The power P dissipated in each bulb is given by $P = V^2/R$ (this is from $P = IV$ and $I = V/R$) $= 12^2/R$. Thus, $1 \text{ watt} = \frac{144}{R}$, thus $R = 144\ \Omega$.

<u>Then</u>, we can determine the current flow and power dissipation per bulb <u>after</u> the one bulb fuses. After the bulb fuses, there will be $\frac{240}{19} = 12.63\ V$ across each remaining bulb. The current flow will be $\frac{12.63\ V}{144\ \Omega} = 87.72$ milliamps. And the power dissipated will be $\frac{12.63^2}{144} = 1.11$ watts. //

1.10.5 To find V_{XY}, note that the resistors with 330Ω and 220Ω resistance are in parallel, thus their combined resistance is $\frac{1}{\frac{1}{330}+\frac{1}{220}} = 132\Omega$. This is in series with the 100Ω resistor, so the circuit is acting like a "voltage divider" with $V_{XY} = 6 \times \frac{132}{100+132} = 3.41$ V. Then, we know that $V_{XY} = I_1 \times 330\Omega \Rightarrow I_1 = 10.34$ milliamps, and also that $V_{XY} = I_2 \times 220\Omega \Rightarrow I_2 = 15.51$ milliamps. //

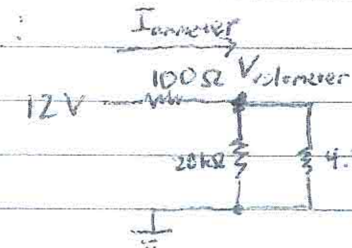
1.10.8 (a) We have:



$V_{voltmeter}$ will clearly be 12 V in this case.

$V_{voltmeter} = I_{ammeter}(100\Omega + 4700\Omega) \Rightarrow I_{ammeter} = \frac{12V}{4800\Omega} =$ 2.5 milliamps.

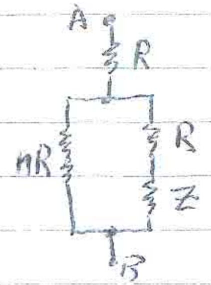
(b) We have:



The $20k\Omega$ and $4.7k\Omega$ in parallel act like a $\frac{1}{\frac{1}{20}+\frac{1}{4.7}} = 3806\Omega$ resistor. Thus, this circuit acts like a "voltage divider" with $V_{voltmeter} = 12 \times \frac{3806}{3806+100} = 11.69V$ and $(12 - V_{voltmeter}) = I_{ammeter} \times 100\Omega \Rightarrow I_{ammeter} = \frac{0.31}{100} =$ 3.1 milliamps. //

Bugg 1.10.13:

Re-drawing the circuit shown, the first part of the question is equivalent to the question:



R_{eq} between points A and B equals Z. Find n in terms of R and Z.

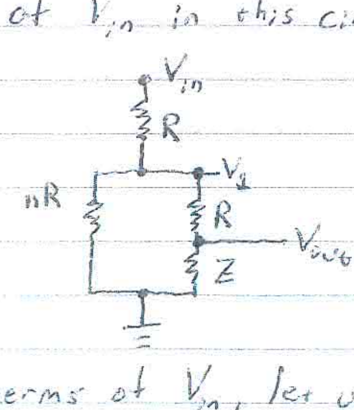
Thus we have that $Z = R + \frac{1}{\frac{1}{nR} + \frac{1}{R+Z}} \Rightarrow \frac{1}{Z-R} = \frac{1}{nR} + \frac{1}{R+Z}$

$$\Rightarrow \frac{1}{nR} = \frac{1}{Z-R} - \frac{1}{Z+R} = \frac{Z+R}{Z^2-R^2} - \frac{Z-R}{Z^2-R^2} = \frac{2R}{Z^2-R^2}$$

$$\Rightarrow nR = \frac{Z^2-R^2}{2R} \Rightarrow n = \frac{Z^2-R^2}{2R^2} = 0.5\left(\frac{Z^2}{R^2} - 1\right) = 0.5(x^2-1)$$

where $x = Z/R$.

To find the constant value A, we would like to find the voltage V_{out} in terms of V_{in} in this circuit:



To find V_{out} in terms of V_{in}, let us first find the voltage V_1 in terms of V_{in}. $V_1 = V_{in}\left(\frac{Z-R}{Z}\right)$. Now let's find V_{out} in terms of V_1.

$$V_{out} = V_1\left(\frac{Z}{Z+R}\right).$$ Now feeding the above equation in as shown,

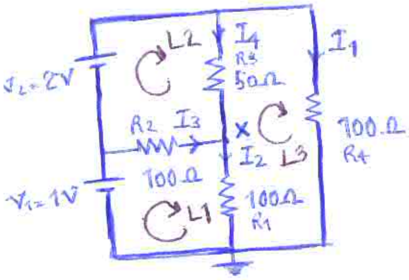
$$V_{out} = V_{in}\left(\frac{Z-R}{Z}\right)\left(\frac{Z}{Z+R}\right) = V_{in}\left(\frac{Z-R}{Z+R}\right) =$$

V_{in}/A where $A = \frac{Z+R}{Z-R} = \frac{Z/R+1}{Z/R-1} = \frac{x+1}{x-1}$ where $x = Z/R$.

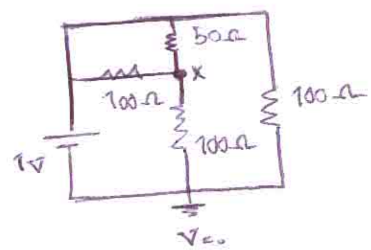
If $Z = 100\Omega$ and A is (a) 2, then $2 = \frac{x+1}{x-1} \Rightarrow 2x-2 = x+1$
$\Rightarrow x = 3 \Rightarrow R = 33.3\Omega$ and $n = 4$; and if A is (b) 10, then
$10 = \frac{x+1}{x-1} \Rightarrow 10x-10 = x+1 \Rightarrow 9x = 11 \Rightarrow x = 11/9 \Rightarrow$
$R = \frac{900}{11}\Omega = 81.8\Omega$ and $n = 0.5\left(\frac{121}{81} - 1\right) = \frac{20}{81}$.

If $A = 10$, then for example a 10% increase in R would drive A from 10 all the way up to 19 (a 90% increase in A), whereas if $A = 2$, then a 10% increase in R would only drive A from 2 to 2.16 (an 8% increase in A). So R needs to be much more stable in the case that $A = 10$ than when $A = 2$.

(I) (II)



$\Rightarrow$



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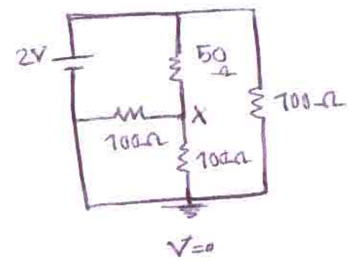

$I_{in} = I_{out}$ $\quad I = \frac{V}{R}$

(I) $\Rightarrow \quad \frac{V_x}{100} + \frac{V_x - 1}{100} + \frac{V_x - 1}{50} = 0 \quad \Rightarrow \quad 4V_x = 3 \quad \Rightarrow \quad \boxed{V_x = \frac{3}{4}}\,(V)$

(II) $\Rightarrow \quad \frac{V_x}{100} + \frac{V_x}{100} + \frac{V_x - 2}{50} = 0 \quad \Rightarrow \quad 4V_x = 4 \quad \Rightarrow \quad \boxed{V_x = 1}\,(V)$

superposition principle $\Rightarrow \quad V_x = 1 + \frac{3}{4} = \boxed{\frac{7}{4}}\,(V)$

$I = \frac{V}{R} \quad \Rightarrow \quad I_1 = \frac{3V}{100\Omega} = 30\ mA \qquad I_3 = \frac{1V - 1.75V}{100\Omega} = -7.5\ mA$

$I_2 = \frac{1.75V}{100\Omega} = 17.5\ mA \qquad I_4 = \frac{3V - 1.75V}{50\Omega} = 25\ mA$

$$\left.\begin{array}{l} L1:\ -V_1 + I_2R_1 + I_3R_2 = 0 \\ L_2:\ -V_2 - I_3R_2 + I_4R_3 = 0 \\ L_3:\ -I_2R_1 + I_1R_4 - I_4R_3 = 0 \end{array}\right\} \checkmark \Leftarrow \text{check}$$

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4)

$Q = CV \quad \Rightarrow \quad Q = 2\mu F \times 12\ V = \boxed{24\ \mu C}$

$E = \frac{1}{2}CV^2 = \frac{1}{2} \times 2\mu F \times (12)^2\ V = \boxed{144\ \mu J} \qquad \#\ E = \frac{Q^2}{2C} = \boxed{144\ \mu J}$

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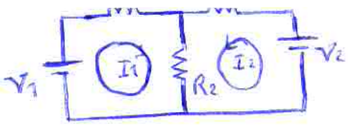
5)

$Q = \int_0^t I\,dt = CV$

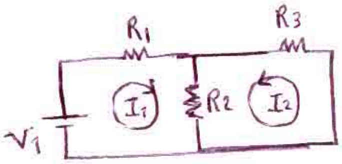
$\Rightarrow \quad V = \frac{1}{C}\int_0^t I\,dt = \frac{1}{C}\int_0^{2.5} (10t - 4t^2)\,dt = \frac{1}{10}\left(-\frac{4}{3}t^3 + 5t^2\right)\ V$

Note that (I ⟶ μA ... V → V



★ $V_1 - I_1R_1 - \left(\frac{R_2R_3}{R_2+R_3}\right)I_1 = 0$

$$I_1 = \frac{V_1(R_2+R_3)}{\underbrace{R_1R_2+R_1R_3+R_2R_3}_{X}} = \boxed{\frac{V_1(R_2+R_3)}{X}}$$



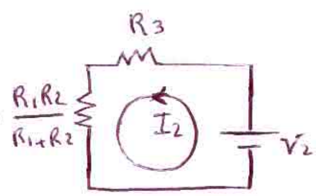
$\Rightarrow \quad V_1 - I_1R_1 - (I_1+I_2)R_2 = 0 \quad \overset{\star}{\Rightarrow}$

$\overset{\star}{\Rightarrow} \quad I_1R_2 + I_2R_2 = \frac{R_2R_3}{R_2+R_3}I_1 \quad \Rightarrow \quad I_1\left(\frac{R_3}{R_2+R_3} - 1\right) = I_2$

$$\frac{V_1(\cancel{R_2+R_3})}{X}\left(\frac{\cancel{R_3}-R_2-\cancel{R_3}}{\cancel{R_2+R_3}}\right) = I_2 \quad \Rightarrow$$

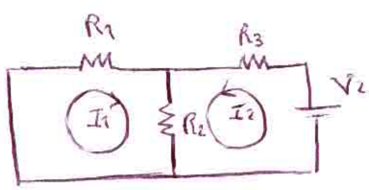
$\Rightarrow \quad \boxed{I_2 = -\frac{V_1R_2}{X}}$

■ V_2 acting alone: $\Rightarrow$



★★ $V_2 - I_2R_3 - I_2\left(\frac{R_1R_2}{R_1+R_2}\right) = 0$

$\Rightarrow \quad \boxed{I_2 = \frac{V_2(R_1+R_2)}{X}}$

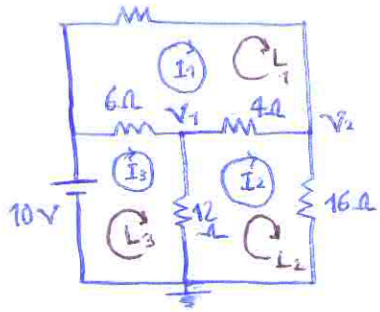


$\Rightarrow \quad V_2 - I_2R_3 - (I_1+I_2)R_2 = 0 \quad \Rightarrow$

$\overset{\star\star}{\Rightarrow} \quad (I_1+I_2)R_2 = I_2\left(\frac{R_1R_2}{R_1+R_2}\right)$

$$I_1R_2 + \frac{V_2(R_1+R_2)R_2}{X} = \frac{I_2R_1R_2}{R_1+R_2}$$

$\Rightarrow \quad \boxed{I_1 = -\frac{V_2R_2}{X}}$



$$I_{in} = I_{out} \quad , \quad I = \frac{V}{R}$$

$$\Rightarrow \left\{ \begin{array}{l} \text{for } V_1 \Rightarrow \quad \dfrac{10-V_1}{6} + \dfrac{V_2-V_1}{4} = \dfrac{V_1}{12} \\[2em] \text{for } V_2 \Rightarrow \quad \dfrac{10-V_2}{48} + \dfrac{V_1-V_2}{4} = \dfrac{V_2}{16} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} V_1 = \dfrac{35}{6} \\[1em] V_2 = 5 \end{array} \right.$$

$$L1 \Rightarrow \quad 0 = 6(I_1 - I_3) + 48 I_1 + 4(I_1 - I_2)$$

$$58 I_1 - 4 I_2 - 6 I_3 = 0 \quad (I)$$

$$L_2 \Rightarrow \quad 0 = 12(I_2 - I_3) + 4(I_2 - I_1) + 16 I_2$$

$$32 I_2 - 12 I_3 - 4 I_1 = 0 \quad (II)$$

$$L_3 \Rightarrow \quad 10 = 6(I_3 - I_1) + 12(I_3 - I_2)$$

$$18 I_3 - 6 I_1 - 12 I_2 = 10 \quad (III)$$

$$I), (II), (III) \Rightarrow \left\{ \begin{array}{l} I_1 = \dfrac{5}{48} \\[1em] I_2 = \dfrac{5}{16} \\[1em] I_3 = \dfrac{115}{144} \end{array} \right.$$