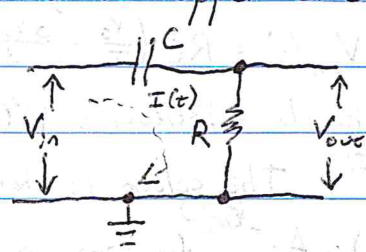
Physics 214 Problem Set #3 SOLUTIONS

Bugg problem # 3.12.11) Let's consider the upper circuit first:



I have drawn in the current, $I(t)$ and a ground, for reference. (There will be no current flow into V_{out} -- consider it to be like an oscilloscope.) From Ohm's Law, we have that

$$V_{out}(t) = I(t)R, \text{ and thus } I(t) = \frac{V_{out}(t)}{R}$$

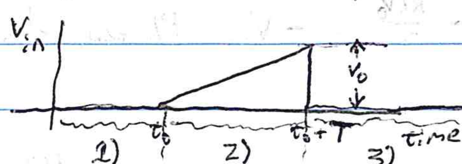
From the capacitor relation $Q = CV$, we have that

$$Q(t) = C\left(V_{in}(t) - V_{out}(t)\right), \text{ and thus}$$

(differentiating this): $I(t) = C\left(\frac{dV_{in}(t)}{dt} - \frac{dV_{out}(t)}{dt}\right)$. We want to find $V_{out}(t)$, so let's substitute $\frac{V_{out}(t)}{R}$ in for $I(t)$:

$$V_{out}(t) = RC\left(\frac{dV_{in}(t)}{dt} - \frac{dV_{out}(t)}{dt}\right). \quad ①$$

This differential equation, ①, is the equation governing the behaviour of the above circuit. To go any further, we need to input the $V_{in}(t)$ that we are provided with, ie.:



There are 3 parts to this waveform, denoted by 1), 2), and 3) above. Let's first consider part 1), where $\frac{dV_{in}(t)}{dt} = 0$, and the capacitor is initially uncharged. We have $V_{out}(t) = RC\left(\frac{dV_{in}(t)}{dt} - \frac{dV_{out}(t)}{dt}\right) = -RC\frac{dV_{out}(t)}{dt}$, thus

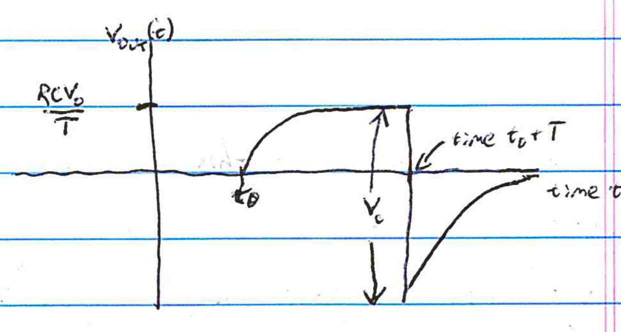
$$\int dt = -RC\int \frac{dV_{out}(t)}{V_{out}(t)} \Rightarrow \ln\left[V_{out}(t)\right] = -\frac{t}{RC} + (\text{constant of integration}) \Rightarrow$$

$$V_{out}(t) = Ae^{-t/RC},$$ where A is a constant of integration.

To find the constant A, we note that the initial condition is that the capacitor is uncharged, and thus $V_{out}(t)$ is initially zero, as is $V_{in}(t)$. Thus $A=0$, and thus, for part 1), $V_{out}(t)$ is just 0.

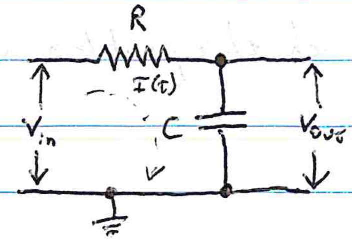
For part 2), $V_{in}(t)$ has a slope of $\frac{V_0}{T}$ (its rise over run), thus $\frac{dV_{in}(t)}{dt} = \frac{V_0}{T}$. Thus (inputting this into equation ①): $V_{out}(t) = RC\left(\frac{V_0}{T} - \frac{dV_{out}(t)}{dt}\right)$. The way to solve that equation is to make the substitution $V'(t) \equiv V_{out}(t) - \frac{RCV_0}{T}$. Then we have that $V'(t) = -RC\frac{dV'(t)}{dt}$. The solution to that differential equation (as for $V_{out}(t)$ in part 1) is $V'(t) = Ae^{-(t-t_0)/RC}$ for some constant A, and thus $V_{out}(t) = Ae^{-(t-t_0)/RC} + \frac{RCV_0}{T}$. We know that at time $t=t_0$, $V_{out}(t)=0$, thus A must equal $-\frac{RCV_0}{T}$, and thus $V_{out}(t) = \frac{RCV_0}{T}\left(1-e^{\frac{-(t-t_0)}{RC}}\right)$.

For part 3), we are back to the situation where $V_{in}(t)=0$ and thus $V_{out}(t) = Ae^{\frac{-(t-t_0-T)}{RC}}$ for some constant A. To find A, we note that the charge state of the capacitor cannot suffer a discontinuity (as having one would imply an infinite current), and thus $Q(t_0+T) = C(V_{in}(t_0+T) - V_{out}(t_0+T))$ must be the same right before and right after the end of the ramp. Right before the end of the ramp, we have that $Q(t_0+T) = C\left(V_0 - \frac{RCV_0}{T}\right) = CV_0\left(1-\frac{RC}{T}\right)$. Right after the end of the ramp, we have that $Q(t_0+T) = C(-V_{out}(t_0+T)) = -CAe^{\frac{-(t-t_0-T)}{RC}} = -CA$. Thus $-CA = CV_0\left(1-\frac{RC}{T}\right) \Rightarrow A = -V_0\left(1-\frac{RC}{T}\right)$
$\Rightarrow V_{out}(t) = -V_0\left(1-\frac{RC}{T}\right)e^{\frac{-(t-t_0-T)}{RC}}$. The jump in $V_{out}(t)$ at $t=t_0+T$ is $-V_0\left(1-\frac{RC}{T}\right) - \frac{RCV_0}{T} = -V_0$. Plotted, this is



For the lower circuit, we have:

3.2.11 (cont'd)



From Ohm's Law we have $V_{in}(t) - V_{out}(t) = I(t)R$, and from the capacitor relation we have $Q(t) = CV_{out}(t)$, and thus

$$I(t) = C\frac{dV_{out}(t)}{dt}$$

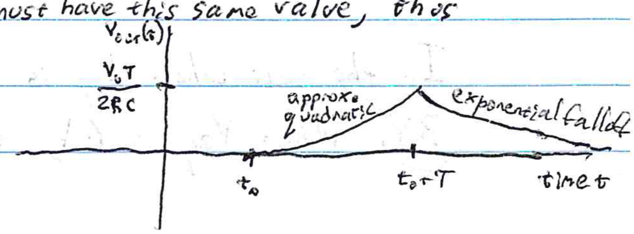
Thus we have that $\frac{V_{in}(t) - V_{out}(t)}{R} = C\frac{dV_{out}(t)}{dt}$, and thus

$$V_{out}(t) = V_{in}(t) - RC\frac{dV_{out}(t)}{dt}.$$

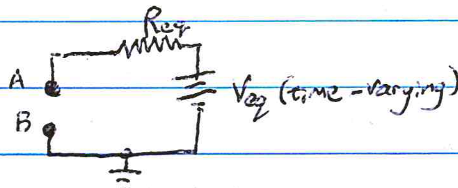
Let's consider the 3 parts of the input waveform again. In part 1), $V_{in}(t) = 0$, and thus $V_{out}(t) = -RC\frac{dV_{out}(t)}{dt}$, thus $V_{out}(t) = Ae^{-t/RC}$ where A is a constant of integration, and since both V_{in} and V_{out} are initially zero, $A=0$. For part 2), $V_{in}(t) = \frac{V_0}{T}(t-t_0)$, thus $V_{out}(t) = \frac{V_0}{T}(t-t_0) - RC\frac{dV_{out}(t)}{dt}$.

Make the substitution $V'(t) = V_{out}(t) - \frac{V_0}{T}(t-t_0) + \frac{RCV_0}{T}$. Then $V'(t) = -RC\frac{dV'(t)}{dt}$, which implies $V'(t) = Ae^{\frac{-(t-t_0)}{RC}}$, and thus $V_{out}(t) = Ae^{\frac{-(t-t_0)}{RC}} + \frac{V_0}{T}(t-t_0) - \frac{RCV_0}{T}$. We know that at $t=t_0$, $V_{out}(t) = 0$, thus $A = \frac{RCV_0}{T}$, thus $V_{out}(t) = \frac{RCV_0}{T}(e^{\frac{-(t-t_0)}{RC}} - 1) + \frac{V_0}{T}(t-t_0)$.

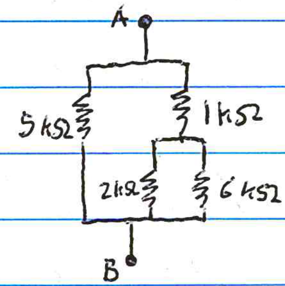
We know that $e^{\frac{-(t-t_0)}{RC}} \approx 1 - \frac{(t-t_0)}{RC} + \frac{(t-t_0)^2}{2R^2C^2} - \ldots$ (these are the first few terms of the Taylor series expansion for e^x), thus $V_{out}(t) \approx \frac{RCV_0}{T}\left(-\frac{(t-t_0)}{RC} + \frac{(t-t_0)^2}{2R^2C^2}\right) + \frac{V_0}{T}(t-t_0) = \frac{V_0(t-t_0)^2}{2TRC}$. For part 3), we are back to the situation where $V_{in}(t) = 0$, and thus $V_{out}(t) = Ae^{\frac{-(t-t_0-T)}{RC}}$ for some constant A. Again we note that the charge state of the capacitor cannot suffer a discontinuity, and thus $Q(t) = CV_{out}(t)$ must be the same before and right after the end of the ramp. Right before the end of the ramp, $V_{out}(t) \approx \frac{V_0T}{2RC}$, and thus right after the ramp, it must have this same value, thus $A = \frac{V_0T}{2RC}$, and thus $V_{out}(t) = \frac{V_0T}{2RC}e^{\frac{-(t-t_0-T)}{RC}}$.



3.12.12) I think the simplest way to approach this problem is to make a Thevenin equivalent circuit:

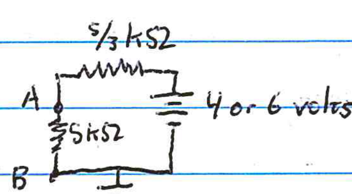


To determine R_{eq}, we can redraw the circuit (with voltage sources changed to wires):



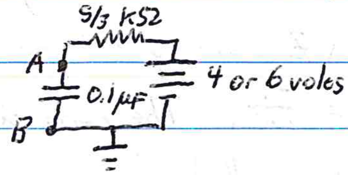
Thus $R_{eq} = \dfrac{1}{\frac{1}{5} + \frac{1}{1 + \frac{1}{\frac{1}{2}+\frac{1}{6}}}}\ k\Omega = \frac{5}{3}\ k\Omega$. The value of V_{eq} will depend on whether we are inside a pulse, or outside of one. In any case, we can find V_{eq} via solving for current in current loops (when nothing is between A and B) -- we get the two equations $24 - 8I_2 - 2I_1 = 0$ and $V_1 - 8I_1 - 2I_2 = 0$, where $V_1 = 6$ or 0 depending on whether we are in a pulse or not. We have $V_{eq} = 3I_1 + 2I_2 = 3(-0.8 + \frac{2V_1}{15}) + 2(3.2 - \frac{V_1}{30}) = 4 + \frac{V_1}{3}$, ie. either 4 or 6 volts.

(a) When a 5 kΩ resistor is connected between A and B, we have the equivalent circuit:



In the case where V_{eq} is 4 V, the voltage at A will be $4 \times \frac{5}{\frac{20}{3}} =$ 3 V. When V_{eq} is 6 V, the voltage at A will be $6 \times \frac{5}{\frac{20}{3}} =$ 4.5 V.

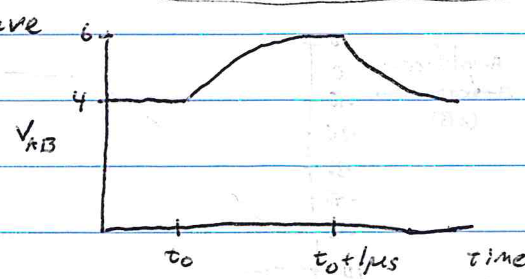
3.2.12 (b) When a $0.1\,\mu F$ capacitor is connected between A and B, we have the equivalent circuit:



We consider 3 parts of the input voltage "waveform": 1) V_{eq} is 4 volts and hasn't yet ever risen to 6 V, 2) V_{eq} enters a pulse and is thus 6V, and 3) V_{eq} falls back to 4V after a pulse. In all cases, we have Ohm's Law: $(V_{eq} - V_{AB}) = I \times \frac{5000}{3}$, and the capacitor relation $Q = V_{AB} \times 10^{-7} \Rightarrow I = 10^{-7}\frac{dV_{AB}}{dt}$, and thus together they imply $V_{eq} - V_{AB} = \frac{5\times10^{-4}}{3}\frac{dV_{AB}}{dt}$. In parts 1) and 3), $V_{eq} = 4 \Rightarrow$ $4 - V_{AB} = \frac{5\times10^{-4}}{3}\frac{dV_{AB}}{dt}$. Let $V' = V_{AB} - 4 \Rightarrow V' = -\frac{5\times10^{-4}}{3}\frac{dV'}{dt} \Rightarrow$ $V'(t) = Ae^{-\frac{\cancel{5\times10^{-4}}\,3t}{\cancel{3}\,5\times10^{-4}}} \Rightarrow V_{AB}(t) = 4 + Ae^{-\frac{\cancel{5\times10^{-4}}\,3t}{\cancel{3}\,5\times10^{-4}}}$. For part 1), ~~the~~ V_{AB} is initially "charged up" to exactly 4 volts, so $A = 0$. We will go on to part 2) before solving for A in part 3). In part 2), $V_{eq} = 6 \Rightarrow$ $6 - V_{AB} = \frac{5\times10^{-4}}{3}\frac{dV_{AB}}{dt} \Rightarrow V_{AB}(t) = 6 + Be^{-\frac{\cancel{5\times10^{-4}}\,3(t-t_0)}{\cancel{3}\,5\times10^{-4}}}$ We know that at the start of the pulse, $V_{AB}(t_0) = 4 \Rightarrow B = -2$ and $V_{AB}(t) = 6 - 2e^{-\frac{3\cancel{5\times10^{-4}}(t-t_0)}{5\times10^{-4}\cancel{3}}}$.

Now for part 3), $V_{AB}(t) = 4 + Ae^{-\frac{3\cancel{5\times10^{-4}}(t-t_0-1\mu s)}{5\times10^{-4}\cancel{3}}}$, and we know that $V_{AB}(t_0 + 1\mu s) = 6$, thus $A = 2$ and $V_{AB}(t) = 4 + 2e^{-\frac{3\cancel{5\times10^{-4}}(t-t_0-1\mu s)}{5\times10^{-4}\cancel{3}}}$.

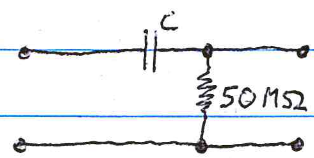
Plotting these out, we have



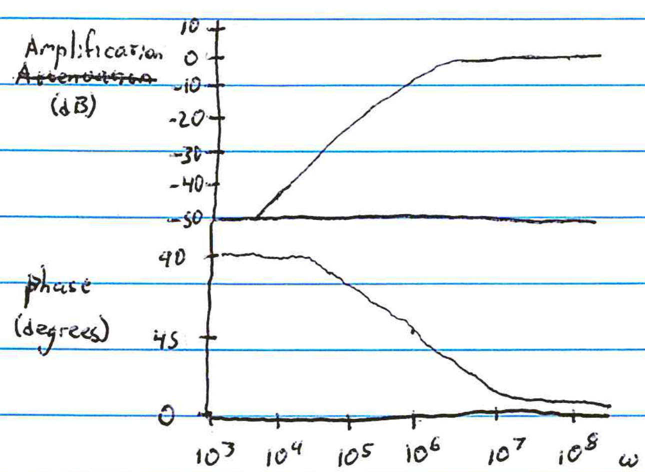
where the time constant $\tau = \frac{5\times10^{-4}}{3}$ seconds $= 0.167$ ms.

4.11.2) (a) The impedance of a capacitor is given by the formula $Z_{capacitor} = \frac{1}{i\omega C}$. (Z is the symbol for impedance,* and ω is the angular frequency) $= \frac{-i}{\omega C}$. The norm of this imaginary number $\left\|\frac{-i}{\omega C}\right\|$ is $\frac{1}{\omega C}$, ~~th~~ which is called the reactance. The impedance of a $1\,\mu F$ capacitance at (a) 50 Hz frequency, which corresponds to $\omega = 100\pi \frac{rad}{s}$ angular frequency, is thus $\frac{-i}{100\pi 10^{-6}} = -\frac{i}{\pi}\times 10^4$ ohms, and its reactance is $\frac{1}{\pi}\times 10^4$ ohms. At (b) 10^6 Hz frequency, which corresponds to $\omega = 2\pi\times 10^6 \frac{rad}{s}$ angular frequency, the impedance is thus $\frac{-i}{2\pi\times10^6\times10^{-6}} = \frac{-i}{2\pi}$ ohms and its reactance is $\frac{1}{2\pi}$ ohms.

A high-pass RC filter has the form:



The capacitance C for a cutoff angular frequency $\omega = \frac{1}{RC} = 10^6 \frac{rad}{s}$
$\Rightarrow C = \frac{1}{50\times10^6\times10^6} = \frac{1}{50\times10^{12}} = 0.02\,pF$. The Bode plot(s) plot the attenuation (in dB) and the phase shift, both as a function of the logarithm of the angular frequency ω. Thus the Bode plots will have the form

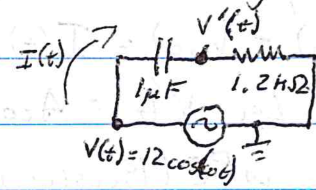


per figure 4.8 in the textbook.

4.11.5) (a) This is simply a voltage divider. To find $I(t)$, note that the total resistance is $80\,k\Omega$, and thus $I(t) = \frac{V(t)}{R} = \frac{3}{20}\cos(10^3 t)$ milliamps, the voltage across the $33k\Omega$ resistor is $V_{33k\Omega}(t) = \frac{99}{20}\cos(10^3 t)$ volts, and $V_{47k\Omega}(t) = \frac{141}{20}\cos(10^3 t)$ volts.

(b) This acts like a voltage divider as well. $Q(t) = CV(t) \Rightarrow I(t) = C\frac{dV(t)}{dt}$ ~~with~~ with the total capacitance $C = \frac{1}{\frac{1}{1}+\frac{1}{3}} = 0.75\,\mu F$. Thus $I(t) = -0.75\times10^{-6} \times 12\omega\sin(\omega t) = -(9\times10^{-3})\sin(10^3 t)$ amps $= -9\sin(10^3 t)$ milliamps. The voltage across the $1\mu F$ capacitor will be $V_{1\mu F}(t) = \frac{1}{1\times10^{-6}} \times \int I(t)dt = 9\cos(10^3 t)$ volts, and the voltage across the $3\,\mu F$ capacitor will be $V_{3\mu F}(t) = \frac{1}{3\times10^{-6}} \times \int I(t)dt = 3\cos(10^3 t)$ volts.

(c) Call the voltage between the capacitor and the resistor $V'(t)$, and call the right-hand side of the voltage source ground as shown:



From Ohm's Law, we know $V'(t) = I(t) \times 1200$, and from the capacitor relation $Q = CV$, we know that $Q(t) = 10^{-6}\times(12\cos(10^3 t) - V'(t))$ and thus $I(t) = 10^{-6}\times(-12000\sin(10^3 t) - \frac{dV'(t)}{dt})$. Sub in ~~$\frac{V'(t)}{1200}$ for $I(t)$ and we get~~ $I(t)\times1200$ for $V'(t)$ and we get

Eq. 1: $I(t) = -1.2\times10^{-3}\times(10\sin(10^3 t) + \frac{dI(t)}{dt})$. To find the solution to that, assume that $I(t)$ is of the form $A\cos(10^3 t) + B\sin(10^3 t)$. Substituting that into eq. 1, we have:

$$A\cos(10^3 t) + B\sin(10^3 t) = -1.2\times10^{-3}\times(10\sin(10^3 t) - 10^3 A\sin(10^3 t) + 10^3 B\cos(10^3 t))$$

Collecting the coefficients in front of the cos terms and the sin terms separately:

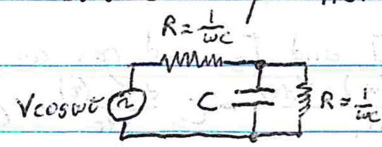
$$\Rightarrow \begin{cases} A = -1.2B \\ B = 1.2A - 0.012 \end{cases}$$

The solution to those simultaneous equations is $A = 5.90\times10^{-3}$ and $B = -4.92\times10^{-3}$.

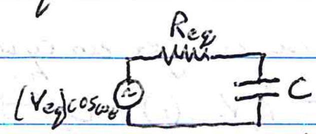
4.11.5 (c) (cont'd) Thus we have that $I(t) = (5.90\cos(10^3 t) - 4.92\sin(10^3 t))$ milliamps. This equals $C\cos(\omega t+\phi)$ milliamps, with $C = \sqrt{5.90^2+4.92^2} = 7.68$ and $\phi = \arctan\left(\frac{4.92}{5.90}\right) = 39.8^\circ$. The voltage $V'(t)$ across the resistor will equal $I(t)R = (1200\times 7.68\times 10^{-3})\cos(\omega t+\phi)$ volts $= 9.22\cos(\omega t+39.8^\circ)$ volts. The voltage across the capacitor will equal $12\cos(\omega t) - 9.22\cos(\omega t+39.8^\circ) = 12\cos(\omega t) - \frac{9.22\times 5.90}{7.68}\cos(\omega t) + \frac{9.22\times 4.92}{7.68}\sin(\omega t) = 4.92\cos(\omega t) + 5.90\sin(\omega t) = D\cos(\omega t+\eta)$ volts, with $D = \sqrt{4.92^2+5.90^2} = 7.68$ and $\eta = -\arctan\left(\frac{5.90}{4.92}\right) = -50.2^\circ$.

4.11.8) This problem is somewhat similar to 4.11.5(c) above. Call the voltage at the bottom of the circuit ground, and let $V'(t)$ be the voltage between the resistor and the capacitor. Then $Q(t) = CV'(t) \Rightarrow I(t) = C\frac{dV'(t)}{dt}$ and $V\cos(\omega t) - V'(t) = I(t)R$, thus $V\cos(\omega t) - V'(t) = RC\frac{dV'(t)}{dt}$. Let $V'(t) = A\cos(\omega t) + B\sin(\omega t) \Rightarrow$ $V\cos(\omega t) - A\cos(\omega t) - B\sin(\omega t) = \omega RC(B\cos(\omega t) - A\sin(\omega t)) \Rightarrow$ $V - A = \omega RCB$ and $-B = -\omega RCA \Rightarrow V - A = \omega^2R^2C^2A \Rightarrow$ $A = \frac{V}{1+\omega^2R^2C^2}$ and $B = \frac{\omega RCV}{1+\omega^2R^2C^2}$. The power dissipation in the circuit (which is all within the resistor) is $I^2(t)R$, and $I(t)$ will equal $\omega C(B\cos(\omega t) + A\sin(\omega t)) = \frac{\omega CV}{1+\omega^2R^2C^2}(\omega RC\cos(\omega t) + \sin(\omega t))$ ①. Thus $P(t) = \frac{\omega^2C^2V^2R}{(1+\omega^2R^2C^2)^2}(\omega^2R^2C^2\cos^2(\omega t) + \sin^2(\omega t) + 2\omega RC\sin(\omega t)\cos(\omega t))$. To find the average power dissipation $\bar{P}$, we note that $\cos^2(x)$ and $\sin^2(x)$ both always average out to $1/2$, and $\sin(x)\cos(x)$ averages to zero. Thus $\bar{P} = \frac{\omega^2C^2V^2R}{2(1+\omega^2R^2C^2)^2}(1+\omega^2R^2C^2) = \frac{\omega^2RC^2V^2}{2(1+\omega^2R^2C^2)} = \frac{RV^2}{2(R^2+(\frac{1}{\omega C})^2)}$. This reaches a maximum when $\frac{d\bar{P}}{dR} = 0 \Rightarrow \frac{V^2}{2(R^2+(\frac{1}{\omega C})^2)} - \frac{R^2V^2}{(R^2+(\frac{1}{\omega C})^2)^2} = 0$ $\Rightarrow \frac{V^2}{2(R^2+(\frac{1}{\omega C})^2)}\left(1 - \frac{2R^2}{(R^2+(\frac{1}{\omega C})^2)}\right) = 0 \Rightarrow 2R^2 = R^2 + \left(\frac{1}{\omega C}\right)^2 \Rightarrow$ $R = \frac{1}{\omega C}$. $\bar{P}$ will have the same value if $R = \frac{1}{\omega C}+\alpha$ and $R = \frac{1}{\omega C}-\alpha$ for very small values of α. Setting $R = \frac{1}{\omega C}$ and putting

4.11.8 (cont'd) an equivalent resistance in parallel with C, we have the circuit



The Thevenin equivalent circuit across C:



has $R_{eq} = \frac{1}{2\omega C}$ and $V_{eq} = \frac{V}{2}$. Thus the current $I(t)$ will equal (from ①) $\frac{\omega C V}{2(1+\omega^2 R_{eq}^2 C^2)}\left(\omega R_{eq} C\cos(\omega t) + \sin(\omega t)\right) = \frac{\cancel{4}\omega C V}{5}\left(\cos(\omega t) + 2\sin(\omega t)\right)$ $= \alpha\omega C V\cos(\omega t + \phi)$ for $\alpha = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = \sqrt{\frac{5}{25}} = \frac{1}{\sqrt{5}}$ and $\phi = \arctan(2) = 63.4°$, thus $I(t) = \left(\frac{\omega C V}{\sqrt{5}}\right)\cos(\omega t + 63.4°)$.

4.11.9) The decibel scale is a logarithmic scale used to describe the ratio of two values of a quantity, which is often (but certainly not always!) power or intensity. Quantity Q, in decibels, with reference to a reference value Q_0, is given by $dB_Q \equiv 10\log_{10}\left(\frac{Q}{Q_0}\right)$ dB. This is 10 times its value in the (little-used) bel scale, $B_Q = \log_{10}\left(\frac{Q}{Q_0}\right)$ B, hence <u>deci</u>bel. Power in decibels can be expressed in terms of voltage ratios because the power P dissipated in a resistor equals V^2/R, thus $dB_P \equiv 10\log_{10}\left(\frac{P}{P_0}\right) = 10\log_{10}\left(\frac{V^2/R}{V_0^2/R}\right) = 20\log_{10}\left(\frac{V}{V_0}\right)$. If we have a reduction in voltage by a factor of 20, we have $dB_P = 20\log_{10}\left(\frac{1}{20}\right)$ amplification $= -26\, dB_P$ amplification $=$ $+26$ dB attenuation.

4.11.10) Consider two loop currents $I_1(t)$ and $I_2(t)$ in the left and right (sort of) (middle) loops respectively, both going clockwise. Then from Kirchhoff's loop law, we have that:

1) $V_S\cos(\omega t) - R_1 I_1(t) = V_x(t)$, 2) $V_x(t) - V_{out}(t) = I_2(t) R_2$,

4.11.10 (cont'd) 3) $I_1(t) - I_2(t) = C_1 \frac{dV_x(t)}{dt}$, and 4) $I_2(t) = C_2 \frac{dV_{out}(t)}{dt}$. Combining 1), 3), and 4), we can eliminate I_1 and I_2 and get an equation in terms of V_{out} and V_x: $\frac{V_s\cos(\omega t) - V_x(t)}{R_1} - C_2 \frac{dV_{out}(t)}{dt} = C_1 \frac{dV_x(t)}{dt}$, and combining 2) and 4) we canget V_x in terms of V_{out}:
$V_x(t) = R_2 C_2 \frac{dV_{out}(t)}{dt} + V_{out}(t)$. Thus we can get an equation purely for V_{out}: $\frac{V_s}{R_1}\cos(\omega t) - \frac{R_2C_2}{R_1}\frac{dV_{out}(t)}{dt} - \frac{1}{R_1}V_{out}(t) - C_2\frac{dV_{out}(t)}{dt} =$
$C_1R_2C_2 \frac{d^2V_{out}(t)}{dt^2} + C_1\frac{dV_{out}(t)}{dt} \Rightarrow$

$$① \quad C_1R_2C_2 \frac{d^2V_{out}(t)}{dt^2} + \left(C_1 + C_2\left(1+\frac{R_2}{R_1}\right)\right)\frac{dV_{out}(t)}{dt} + \frac{1}{R_1}V_{out}(t) = \frac{V_s}{R_1}\cos(\omega t).$$

As usual, assume $V_{out}(t)$ is of the form $A\cos(\omega t) + B\sin(\omega t)$ and solve for A and B. $\frac{dV_{out}(t)}{dt} = \omega B\cos(\omega t) - \omega A\sin(\omega t)$ and $\frac{d^2V_{out}(t)}{dt^2} = -\omega^2 A\cos(\omega t) - \omega^2 B\sin(\omega t)$, thus feeding those into ~~the~~ equation ①, ~~we get~~ and equilibrating the cos terms and the sin terms, we get the two equations

$$② \quad -C_1R_2C_2\omega^2 A + \left(C_1 + C_2\left(1+\frac{R_2}{R_1}\right)\right)\omega B + \frac{1}{R_1}A = \frac{V_s}{R_1} \quad \text{and}$$

$$③ \quad -C_1R_2C_2\omega^2 B - \left(C_1 + C_2\left(1+\frac{R_2}{R_1}\right)\right)\omega A + \frac{1}{R_1}B = 0.$$

~~From equation ③, we get that $A = \frac{\frac{1}{R_1} - C_1R_2C_2\omega^2}{(C_1+C_2(1+\frac{R_2}{R_1}))\omega} B$. Inserting that into equation ②, we get~~

$$\cancel{\left[\frac{(\frac{1}{R_1} - C_1R_2C_2\omega^2)^2}{(C_1+C_2(1+\frac{R_2}{R_1}))\omega} + \left(C_1+C_2\left(1+\frac{R_2}{R_1}\right)\right)\omega\right]B = \frac{V_s}{R_1}}.$$

~~For very large angular frequencies ω, this becomes~~

$$\cancel{\frac{C_1^2R_2^2C_2^2\omega^3}{C_1+C_2(1+\frac{R_2}{R_1})}B = \frac{V_s}{R}}$$

For very large angular frequencies ω, equations ② and ③ become

$$②' \quad -C_1R_2C_2\omega^2 A \approx \frac{V_s}{R_1} \Rightarrow A \approx -\frac{V_s}{\omega^2 R_1R_2C_1C_2}$$

$$③' \quad -C_1R_2C_2\omega^2 B \approx 0 \Rightarrow B \approx 0.$$

Thus for large frequencies, $V_{out}(t) \approx -\frac{V_s}{\omega^2 R_1R_2C_1C_2}\cos(\omega t)$, and thus $V_{out}(t)/V_{in}(t) \approx -1/\omega^2R_1R_2C_1C_2$.

5.9.2) We know that $V = L\frac{dI}{dt}$ = (in this case) $0.1 \times \frac{d}{dt}(25\cos(100t)) =$
$0.1 \times (-2500\sin(100t)) =$ $-250\sin(100t)$ volts.

5.9.3) Since all current must flow through <u>both</u> the resistance <u>and</u> the inductance, the inductance will appear in series with R. The effective cutoff frequency of the filter this will form is $\omega = \frac{R}{L} = 4 \times 10^7 \frac{rad}{s}$, and since current must flow through the inductor, the inductor will ~~i~~ inhibit the passing of high-frequency signals, and will thus form a low-pass filter which will ~~the~~ tend to reject ~~freq~~ angular frequencies greater than $4 \times 10^7 \frac{rad}{s}$.