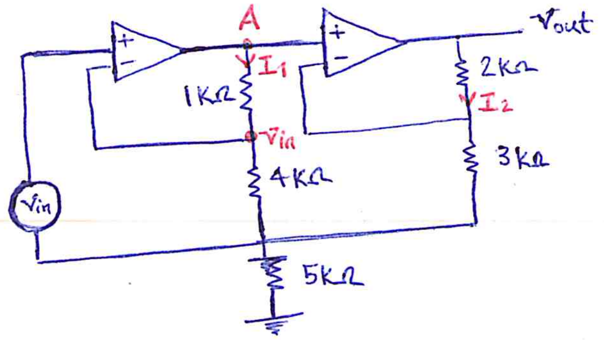
1)

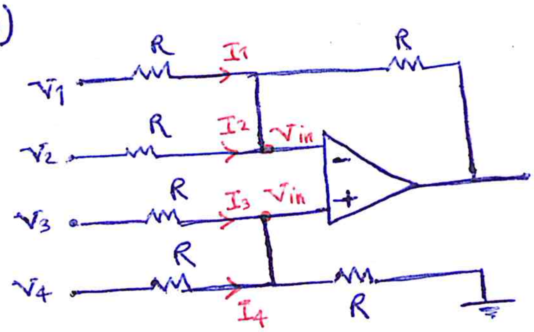


$$\begin{cases} V_{out} = 2I_2 + 3I_2 + 5(I_1 + I_2) = 10I_2 + 5I_1 \\ V_{in} = 4I_1 + 5(I_1 + I_2) = 5I_2 + 9I_1 \end{cases} \Rightarrow \frac{V_{out}}{V_{in}} = \frac{9I_1 + 5I_2}{5I_1 + 10I_2}$$

we need to calculate the relation between I_1 and I_2:

$$\begin{cases} V_A = I_1 + 4I_1 + 5I_1 + 5I_2 = 10I_1 + 5I_2 \\ V_A = V_{out} - 2I_2 \end{cases} \Rightarrow \boxed{I_2 = \frac{5}{3} I_1} \Rightarrow \boxed{\frac{V_{out}}{V_{in}} = \frac{65}{12}} \checkmark$$

2) 7.29)



$V_o = V_4 + V_3 - V_2 - V_1$

$$V_o = V_1 - I_1R - (I_1 + I_2)R \quad (*)$$

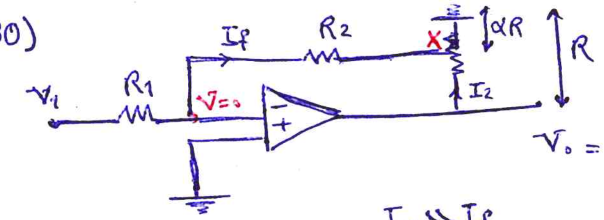
$$V_1 = V_{in} + RI_1$$

$$\begin{cases} V_4 = RI_4 + I_3R + RI_4 \\ V_4 = V_{in} + RI_4 \end{cases} \Rightarrow V_{in} = RI_4 + RI_3$$

$$\Rightarrow V_1 = RI_4 + RI_3 + RI_1$$

$$(*) \Rightarrow V_o = RI_4 + RI_3 - RI_1 - RI_2 \Rightarrow \boxed{V_o = V_4 + V_3 - V_1 - V_2} \checkmark$$

2) 7.30)

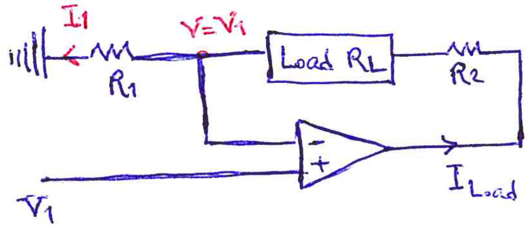


$$V_o = -\frac{R_2 V_1}{\alpha R_1}$$

$I_2 \gg I_f$

$I_2 \gg I_f$: $\begin{cases} V_o = I_2 R \\ V_x = I_2 \alpha R \\ \dfrac{V_1 - V}{R_1} = \dfrac{V - V_x}{R_2} \quad , V = 0 \end{cases} \Rightarrow V_o = \dfrac{V_x}{\alpha} = \boxed{\dfrac{-R_2 V_1}{R_1 \alpha}}$ ✓

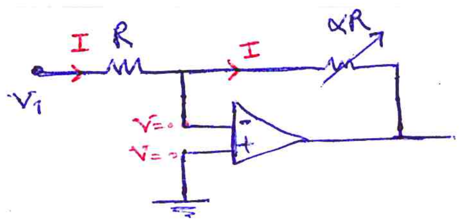
7.31)



$$I_{Load} = \frac{V_1}{R_1}$$

$V = V_1 = R_1 I_1 \Rightarrow I_1 = I_{Load} \Rightarrow \boxed{V_1 = R_1 I_{Load}}$ ✓

7.32)



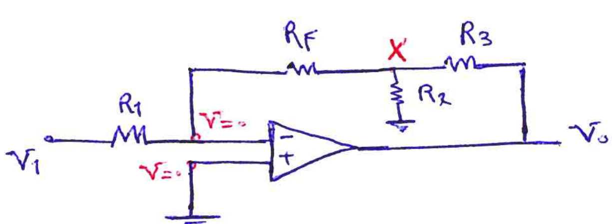
$V_o = -\alpha V_1$

$V_1 = IR$

$V = 0 = I(\alpha R) + V_o \Rightarrow V_o = -I\alpha R$

$\boxed{V_o = -V_1 \alpha}$ ✓

7.33)



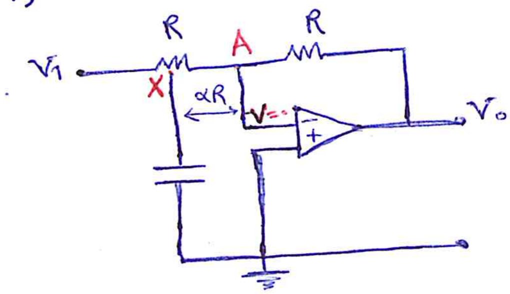
$$\frac{V_o}{V_1} = -\frac{R_F}{R_1}\left\{1 + \frac{R_3}{R_2}\left(1 + \frac{R_2}{R_F}\right)\right\}$$

$\dfrac{V_1 - V}{R_1} = \dfrac{V_x - V}{R_F} \quad , V = 0 \Rightarrow \boxed{\dfrac{V_1}{R_1} = \dfrac{V_x}{R_F}}$ (I)

$\dfrac{V_x - V}{R_F} + \dfrac{V_x}{R_2} + \dfrac{V_x - V_o}{R_3} = 0 \quad ; V = 0$ and (I) $\Rightarrow \boxed{\dfrac{V_o}{V_1} = -\dfrac{R_F}{R_1}\left\{1 + \dfrac{R_3}{R_2}\left(1 + \dfrac{R_2}{R_F}\right)\right\}}$ ✓

2) 7.34)

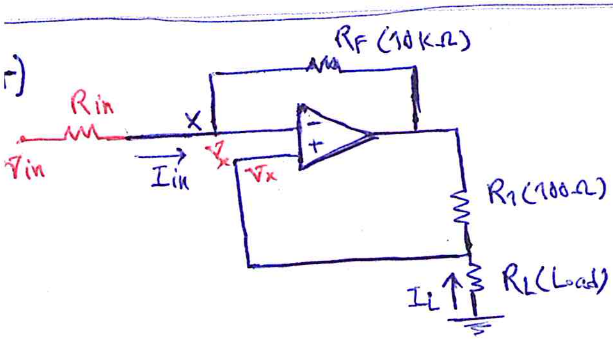
$V_o = -V_1/(1+j\omega t)$, $T = (\alpha-\alpha^2)CR$



$$A: \quad \frac{V - V_x}{\alpha R} = \frac{V_o - V}{R} \overset{V=0}{\Rightarrow} V_x = -\alpha V_o \quad (I)$$

$$X: \quad \frac{V_x - V_1}{R - \alpha R} + \frac{V_x}{Z} + \frac{V_x - \cancel{V}}{\alpha R} = 0 \quad (II)$$

$$\left.\begin{matrix}(I)\\(II)\end{matrix}\right\} \Rightarrow \boxed{V_o = \frac{-V_1}{1 + CR(\alpha-\alpha^2)\omega j}} \checkmark$$



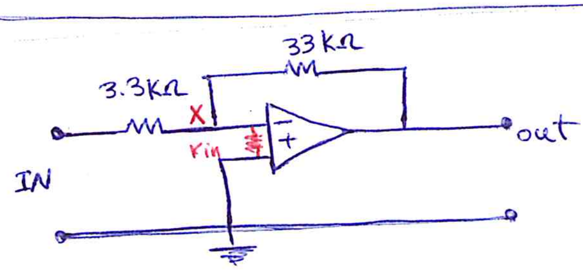
$V_x = I_L R_L$

$V_x = V_{in} - I_{in} R_{in}$

$$I_L R_L = V_{in} - I_{in} R_{in} \Rightarrow \frac{V_{in}}{I_{in}} = \frac{I_L R_L}{I_{in}} + R_{in} \quad (*)$$

$$I_{in} R_F + I_L R_1 = 0 \Rightarrow \frac{I_{in}}{I_L} = -\frac{R_1}{R_F} \Rightarrow \frac{I_L}{I_{in}} = -\frac{R_F}{R_1} \overset{(*)}{\Rightarrow} \boxed{Z_{in} = \frac{V_{in}}{I_{in}} = -\frac{R_F R_L}{R_1} + R_{in}} \checkmark$$

)



$$\frac{V_x}{1\,k\Omega} + \frac{V_x - V_{out}}{33\,k\Omega} + \frac{V_x - V_{in}}{3.3\,k\Omega} = 0 \quad (*)$$

$$V_{out} = G(\cancel{V_+} - V_-) \Rightarrow V_{out} = -GV_- \; ; \; V_x = V_- = -\frac{V_{out}}{G} \quad (**)$$

5) Cont'd ...

$$(*) \text{ and } (**) \Rightarrow A = \frac{V_{out}}{V_{in}} = \boxed{\frac{\frac{1}{33\,k\Omega}}{\left(-\frac{1}{G}\left(\frac{1}{750\,\Omega}\right) - \frac{1}{33\,k\Omega}\right)}}$$ ✓

$$I_{in} = \frac{V_{in} - V_x}{3.3\,k\Omega} \Rightarrow \left.\begin{array}{l} I_{in} = \dfrac{V_{in} + \frac{V_{out}}{G}}{3.3\,k\Omega} \\ V_{out} = A V_{in} \end{array}\right\} \Rightarrow I_{in} = V_{in}\left(\frac{1 + A/G}{3.3\,k\Omega}\right)$$

$$\boxed{Z_{IN} = \frac{V_{in}}{I_{in}} = \frac{3.3\,k\Omega}{1 + \frac{A}{G}}}$$ ✓

$G = 10$ → $A = -1.85$, $R_{IN} = 4.05\,k\Omega$

$G = 100$ → $A = -6.94$, $R_{IN} = 3.55\,k\Omega$

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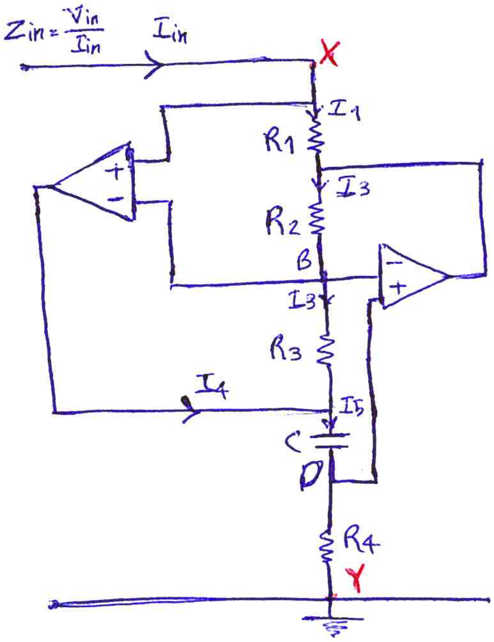
9) $Z_{in} = \frac{V_{in}}{I_{in}}$



$I_{in} = I_1$

$V_X = V_{in} = V_B = V_D$

$X \longrightarrow B \Rightarrow V_{in} - I_1 R_1 - I_3 R_2 = V_{in}$

$$\Rightarrow \boxed{I_{in} = -\frac{I_3 R_2}{R_1}}$$

$B \longrightarrow D \Rightarrow V_{in} - I_3 R_3 - I_5 Z_c = V_{in}$

$$\boxed{I_3 = -\frac{I_5 Z_c}{R_3}}$$

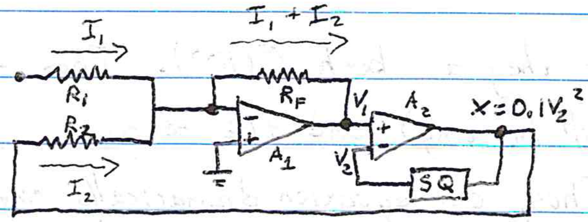
where $Z_c = \frac{1}{j\omega C}$

$D \longrightarrow Y \Rightarrow V_{in} - I_5 R_4 = 0 \Rightarrow \boxed{V_{in} = I_5 R_4}$

$$\left.\begin{array}{l} I_3 = -\dfrac{I_{in} R_1}{R_2} \\ I_5 = -\dfrac{I_3 R_3}{Z_c} \end{array}\right\} \Rightarrow I_5 = \frac{I_{in} R_1 R_3}{R_2 Z_c} = \frac{j\omega C R_1 R_3 I_{in}}{R_2}$$

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{I_5 R_4}{I_{in}} = \frac{j\omega C \cancel{I_{in}} R_1 R_3 R_4}{\cancel{I_{in}} R_2} = \boxed{\frac{j\omega C R_1 R_3 R_4}{R_2}}$$ ✓

Bugg 7.15.3



From the fact that amplifier A_1 has a voltage gain of $A_1 \approx -10^4$:

1) $V_1 = A_1(0 - V_-) = -A_1 V_-$.

From the fact that amplifier A_2 has a voltage gain of $A_2 \approx +10^4$, and SQ's output is $x = SV_2^2 \approx 0.1 V_2^2$:

$A_2(V_1 - V_2) = x$, and $V_2 = \sqrt{\frac{x}{S}} \Rightarrow A_2\left(V_1 - \sqrt{\frac{x}{S}}\right) = x \Rightarrow$

$\sqrt{\frac{x}{S}} = V_1 - \frac{x}{A_2} \Rightarrow \frac{x}{S} = V_1^2 - \frac{2V_1 x}{A_2} + \frac{x^2}{A_2^2} \Rightarrow$

$\frac{x^2}{A_2^2} - x\left(\frac{2V_1}{A_2} + \frac{1}{S}\right) + V_1^2 = 0$. Using 1) $V_1 = -A_1 V_-$, we get

2) $\frac{x^2}{A_2^2} + x\left(\frac{2A_1V_-}{A_2} - \frac{1}{S}\right) + A_1^2 V_-^2 = 0$.

And from the three instances of Ohm's Law, we have:

3) $e - V_- = I_1 R_1$, $\Rightarrow I_1 = \frac{e - V_-}{R_1}$,

4) $x - V_- = I_2 R_2$, $\Rightarrow I_2 = \frac{x - V_-}{R_2}$,

5) $V_- - V_1 = (I_1 + I_2) R_F$.

From 5) and 1), we have: $(1 + A_1)V_- = (I_1 + I_2)R_F$, and then incorporating 3) and 4), we have: $(1 + A_1)V_- = \left(\frac{e - V_-}{R_1} + \frac{x - V_-}{R_2}\right)R_F$

$\Rightarrow \left(1 + A_1 + \frac{R_F}{R_1} + \frac{R_F}{R_2}\right)V_- = \left(\frac{e}{R_1} + \frac{x}{R_2}\right)R_F$

$$\Rightarrow 5')\quad V_- = \frac{\left(\frac{e}{R_1} + \frac{x}{R_2}\right)R_F}{1 + A_1 + \frac{R_F}{R_1} + \frac{R_F}{R_2}}.$$

Feeding this back into 2), we have:

$$\frac{x^2}{A_2^2} + x\left(\frac{2A_1R_F\left(\frac{e}{R_1} + \frac{x}{R_2}\right)}{A_2\left(1 + A_1 + \frac{R_F}{R_1} + \frac{R_F}{R_2}\right)} - \frac{1}{S}\right) + \frac{A_1^2 R_F^2\left(\frac{e}{R_1} + \frac{x}{R_2}\right)^2}{\left(1 + A_1 + \frac{R_F}{R_1} + \frac{R_F}{R_2}\right)^2} = 0.$$

This is a quadratic equation in x, which could "in principle" be solved, however its coefficients are much too messy to just use the quadratic equation to solve it. We need to make some assumptions. One assumption we can make is that $|A_1|$ and $|A_2|$

are large, which is fair since they are both $O(10^4)$. Thus, the term $\frac{x^2}{A_2^2}$ will go away (it will be tiny) and the terms $(1 + A_1 + \frac{R_F}{R_1} + \frac{R_F}{R_2}) \approx A_1$. Thus, our equation dramatically simplifies:

$$-\frac{x}{S} + R_F^2\left(\frac{e}{R_1} + \frac{x}{R_2}\right)^2 = 0$$

$$\Rightarrow \frac{R_F^2}{R_2^2}x^2 + x\left(\frac{2eR_F^2}{R_1R_2} - \frac{1}{S}\right) + \frac{R_F^2e^2}{R_1^2} = 0,$$ and thus, from the quadratic equation, $x = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha}$ if $\alpha \equiv \frac{R_F^2}{R_2^2}$, $\beta \equiv \left(\frac{2eR_F^2}{R_1R_2} - \frac{1}{S}\right)$, and $\gamma \equiv \frac{R_F^2e^2}{R_1^2}$.

If $S = a$, $\frac{R_F}{R_2} = b$, and $e = \frac{cR_1}{R_F} \Rightarrow c = \frac{eR_F}{R_1}$, the quadratic equation for x looks like: $b^2x^2 + x(2bc - \frac{1}{a}) + c^2 = 0$, which is somewhat similar to, although certainly different than, $ax^2 + bx + c = 0$ — both are of course quadratic equations, although their coefficients are different. $b^2x^2 + x(2bc - \frac{1}{a}) + c^2 = 0$ is the correct one. Its solutions are of course $x = \frac{\frac{1}{a} - 2bc \pm \sqrt{\frac{-4bc}{a} + \frac{1}{a^2}}}{2b^2}$. The limitations mentioned in the problem answers of course all apply -- we have directly used the assumptions $|A_2| \gg \frac{R_2}{R_F}$ and $|A_1| \gg \left(\frac{R_F}{R_1} + \frac{R_F}{R_2}\right)$, and the other limitations are necessary as well.