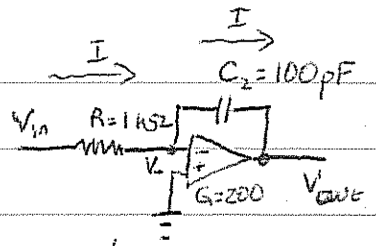
Bugg 8.6.1(a) We have



and thus we have the 3 equations:

1) $V_{out} = -200V_-$ (gain of the amplifier)

2) $V_- - V_{in} = IR$ (Ohm's law)

3) $I = C_2\left(\frac{dV_{out}}{dt} - \frac{dV_-}{dt}\right)$ $(Q = CV)$

Inserting 2) in 3), we get $V_- - V_{in} = RC_2\left(\frac{dV_{out}}{dt} - \frac{dV_-}{dt}\right)$, and then replacing all V_-'s by $\frac{-V_{out}}{200}$ per 1), we have $V_{out} + 200V_{in} = -201RC_2\frac{dV_{out}}{dt}$. Defining $V' \equiv V_{out} + 200V_{in}$, we have $V' = -201RC_2\frac{dV'}{dt} \Rightarrow V' = Ae^{\frac{-t}{201RC_2}}$ and thus $V_{out} = V' - 200V_{in} = Ae^{-t/\tau} - 200V_{in}$, where the time constant $\tau = 201RC_2 = 201 \times 1000 \times 10^{-10} = 2.01 \times 10^{-5}$ seconds.

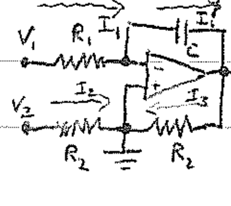
(b) The easiest way to do this part is to use equation 8.11: the bandwidth $\omega_0 = \dfrac{\frac{1}{R_2} + \frac{1}{R_1'(G+1)}}{C_2 + \frac{C_1}{G+1}}$. We have (initially) $R_2 = \infty$, $R_1' = R_1 = 1000$, $G = 200$, $C_2 = 10^{-10}F$, and $C_1 =$ (initially) 0.

Thus $\omega_0 = \dfrac{\frac{1}{1000 \times 200}}{10^{-10}} = 5 \times 10^4 \frac{rad}{s}$.

If we add capacitor $C_1 = 10pF = 10^{-11}F$ and resistor $R_2 = 10^4\Omega$, then the easiest way to find the amplification of the circuit at low frequency is to use the (un-numbered) equation at the bottom of p. 157, i.e. the amplification $\approx -\frac{R_2}{R_1} = -\frac{10000}{1000} = -10$.

To find the new bandwidth, we use equation 8.11 again: $\omega_0 = \dfrac{\frac{1}{10^4} + \frac{1}{2.01 \times 10^5}}{10^{-10} + \frac{10^{-11}}{201}} \approx \frac{10^{10}}{10^4} = 10^6 \frac{rad}{s}$.

8.6.2 First, let's demonstrate the relation given for the "Compute" setting: $V_{out} = -\int_0^t \frac{V_1 dt}{CR_1} - V_2$. Let's redraw the circuit when both switches are in their "Compute" A=0, B=0 setting:



We then have the following

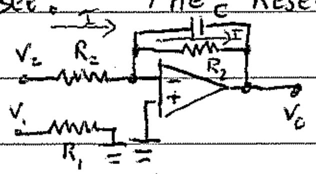
equations: 1) $V_1 = I_1 R_1$ (Ohm's law)

2) $I_1 = -C\frac{dV_0}{dt}$ $(Q = CV)$

3) $V_0 = I_3 R_2$ (Ohm's law)

4) $V_2 = I_2 R_2$ (Ohm's law)

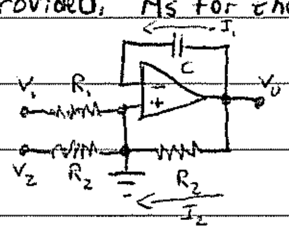
Combining 1) and 2), we have that $\frac{V_1}{R_1} = -C\frac{dV_0}{dt} \Rightarrow \frac{dV_0}{dt} = \frac{-V_1}{CR_1}$
$\Rightarrow V_0 = -\int_0^t \frac{V_1 dt}{CR_1} +$ constant, where the constant of integration is equal to the value of V_0 at time $t = 0$. To determine what that value is, we need to know the initial condition. Let's take that initial condition to be the value V_0 takes after the circuit has been "Reset." The "Reset" $(A = 1, B = 1)$ circuit is:



After sufficient time has passed, there will be no current flowing in the capacitor, and thus we have the two equations: 1') $V_2 = IR_2$

2') $-V_0 = IR_2$,

and thus combing those equations we have $-V_0 = \frac{V_2 R_2}{R_2} \Rightarrow V_0 = -V_2$.
We take this to be the initial condition to the "Compute" output, and thus, during "Compute," $V_0 = -\int_0^t \frac{V_1 dt}{CR_1} - V_2$, like the formula provided. As for the "Hold" setting, we then have:



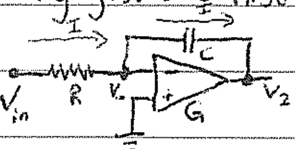
giving the equations: 1'') $I_1 = C\frac{dV_0}{dt} = 0$

2'') $\cancel{I_2}\, V_0 = I_2 R_2$

etc.

Equation 1'') is the only one that matters for this setting. We have $\frac{dV_0}{dt} = 0$, and thus V_0 remains constant while the ~~hold~~ "Hold" setting remains in effect. Thus, each of the three settings do as they claim.

8.6.3 i) Considering just the first half of the network, we have:



Since the gain of the amplifier is G, we have 3 equations:

1) $V_2 = -GV_-$ (gain of amp)

2) $V_{in} - V_- = IR$ (Ohm's law)

3) $I = C\left(\frac{dV_-}{dt} - \frac{dV_2}{dt}\right)$ $(Q = CV)$

Combining 2) and 3), we have $V_{in} - V_- = RC\left(\frac{dV_-}{dt} - \frac{dV_2}{dt}\right)$. Then replacing all V_-'s with $\frac{-V_2}{G}$, per 1), we have $\frac{1}{CR}\left(\frac{V_2}{G} + V_{in}\right) = -\left(1 + \frac{1}{G}\right)\frac{dV_2}{dt}$. Swapping sides and integrating, we have $V_2\left(1 + \frac{1}{G}\right) = -\frac{1}{CR}\int\left(V_{in} + \frac{V_2}{G}\right)dt$.

ii) Neglecting terms of order $\frac{1}{G}$, the above is $V_2 = -\frac{1}{CR}\int_0^t V_{in}\,dt$
$= -\frac{1}{10^{-7} \times 10^6}\int_0^t 0.1\,dt = -10 \times 0.1t = -t$ volts. And $V_{out}\left(1 + \frac{1}{G}\right) =$

iii) Using the trial substitution mentioned, we have
$(Ae^{\beta t} + B)\left(1 + \frac{1}{G}\right) = -\frac{1}{CRG}\int_0^t (Ae^{\beta t} + B + GV_{in})\,dt$. Performing the integral, we have $(Ae^{\beta t} + B)\left(1 + \frac{1}{G}\right) = -\frac{1}{CRG}\left[\frac{A}{\beta}e^{\beta t} + Bt + GV_{in}t\right]_0^t \Rightarrow$
$(Ae^{\beta t} + B)\left(1 + \frac{1}{G}\right) = \frac{A}{CRG\beta}(1 - e^{\beta t}) - \frac{t}{CRG}(B + GV_{in})$. Now we need to solve for A, B, and β in terms of C, R, G, and V_{in}. Since the terms linear in t must cancel, B must equal $-GV_{in}$. We then know that the constant terms must be equal and the exponential terms must also be equal, thus $B\left(1 + \frac{1}{G}\right) = \frac{A}{CRG\beta}$ and $A\left(1 + \frac{1}{G}\right) = \frac{-A}{CRG\beta}$. From the second of these equalities, $\beta = \frac{-1}{CR(G+1)}$, and from the first, $\frac{A}{CRG\beta} = -V_{in}(G+1)$
$\Rightarrow A = -V_{in}CRG\beta(G+1) = GV_{in}$.

iv) After $t = 1$ second, V_{in} returns to 0 volts, and we have that $V_2\left(1 + \frac{1}{G}\right) = -\frac{1}{CR}\left(\int_0^t \frac{V_2\,dt}{G} + \int_0^1 V_{in}\,dt\right)$
$= \left(-\frac{1}{CR}\int_0^t \frac{V_2\,dt}{G}\right) - \left(\frac{1}{10^{-7} \times 10^6}\int_0^1 0.1\,dt\right) = \left(-\frac{1}{CR}\int_0^t \frac{V_2\,dt}{G}\right) - 1$. Thus, ignoring terms of order $\frac{1}{G}$, we have that V_2 settles down to ≈ -1 volt. $V_{out}\left(1 + \frac{1}{G}\right) \approx -\frac{1}{CR}\left(\int_0^t \frac{V_{out}\,dt}{G} + \int_0^t V_2\,dt\right) \approx$ (ignoring terms of order $\frac{1}{G}$) $-\frac{1}{CR}\int_0^t -1\,dt = 10t$ volts, and thus will grow linearly to saturation.

ii) $-\frac{1}{CR}\int\left(V_2 + \frac{V_{out}}{G}\right)dt =$ (ignoring terms of order $\frac{1}{G}$) $-\frac{1}{CR}\int^t -t\,dt = -10\left(-\frac{t^2}{2}\right) = 5t^2$ volts.

9.9.5 If the applied voltage is 250 V RMS, the peak applied voltage will be $250\sqrt{2} = 353.6$ V, thus the peak voltage across the output and load resistors will be $353.6 - 0.6 =$ 353 V. The peak current will be $\frac{V}{R_{total}} = \frac{353}{1k\Omega + 200\Omega} = \frac{353}{1200} =$ 0.294 A.

9.9.6 From the hyperphysics webpage (sent via e-mail), the peak-to-peak ripple will be $V_{r(pp)} = \frac{V_{p(in)}T}{R_L C}$, where T is the time between peaks. For 50 Hz input, T will be $\frac{1}{100}$ of a second, since there will be two positive peaks per cycle for full-wave rectified output as in this circuit. Thus, $V_{r(pp)} = \frac{0.01 V_{p(in)}}{R_L C}$. ~~$V_{r(dc)}$ will equal $V_{p(in)}\left(1 - \frac{1}{200 R_L C}\right) = \left(1 - \frac{0.005}{RC}\right)V_{p(in)}$~~. Thus, the magnitude of the ripple ~~will equal~~ as a fraction of the peak voltage will equal $V_{r(pp)}/V_{p(in)} = \frac{0.01}{R_L C}$, and we want to find the minimum value of R_L so that this is less than 5%. Thus, we have $0.05 = \frac{0.01}{R_L C}$. $C = 500\mu F = 0.0005$ F, thus $0.05 = \frac{0.01}{0.0005 R_L}$ $\Rightarrow 0.05 = \frac{20}{R_L} \Rightarrow R_L = 400\Omega$. Note that this is different than the book's answer of 800Ω; this is likely because Bugg was not accounting for the fact that 50 Hz AC input implies that the time between full-wave rectified peaks is $\frac{1}{100}$ second, not $\frac{1}{50}$ second. (If one uses the incorrect latter value, one gets the book's incorrect answer of 800Ω.) Given the correct answer of 400Ω, the DC current through the load will be approximately $\frac{250\sqrt{2}}{400} = 0.88$ A, twice the book's value of 0.44 A.