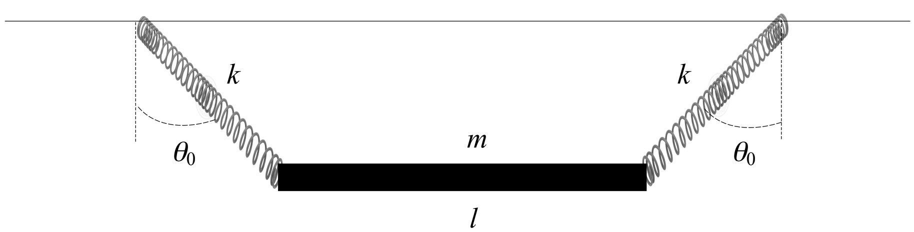
P321b Final Practice Problems
(note that these are a little bit harder than the problems on the actual final, but are good practice for them)

1. Consider a force law of the form
$$F(r) = -k/r^2 - k'/r^4$$
with k and k' being positive constants. Show that if $\rho^2 k > k'$, then a particle can move in a stable circular orbit at $r = \rho$.

2. (F&W 1.13) A rocket with initial velocity v_∞ and impact parameter b approaches a planet of radius R_0 and mass m. What is the condition that the rocket will strike the planet? If it just misses, what is its angle of deflection?

3. (F&W 1.14) The cross section to strike the nuclear surface is of interest when considering nuclear reactions during heavy-ion scattering. By integrating over appropriate impact parameters, show that the cross section to strike a nucleus of radius R in Rutherford scattering is given by $\sigma_r = \pi R^2(1 - V_c/E)$, where $V_c = zZe^2/R$ is the repulsive Coulomb barrier at the nuclear surface, Z is the atomic number of (i.e. number of protons in) the nucleus, z is the charge (in units of the electron/proton charge) of the incoming particle, and it is assumed that $E \geq V_c > 0$.

4. A uniform bar of length l and mass m is suspended by two equal springs of equilibrium length b and force constant k, as shown below:



Find the (three) normal modes of small oscillation in the plane.

5. Obtain a solution to the differential equation:
$$\ddot{x} + \omega_0^2 x - \lambda x^2 = 0$$
correct to second order by writing $x(t) = x_0(t) + \lambda x_1(t) + \lambda^2 x_2(t)$, and using the method of perturbations.