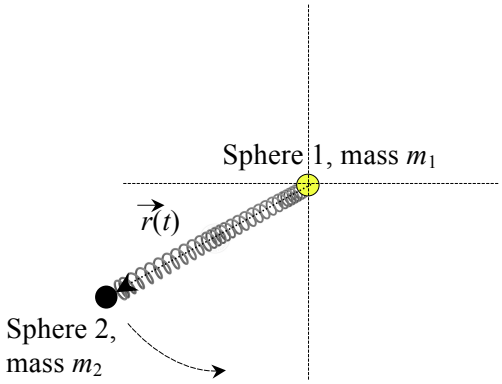
P321b Midterm Practice Problems

1. Two spheres connected by a spring



Two spheres, of masses m_1 and m_2 respectively, are connected by a spring with spring constant k (and with zero length when unextended, so that the potential energy of the spring when stretched to length r is $+\frac{1}{2}kr^2$). The spheres are orbiting around each other in a vacuum, so neither one of the spheres has a fixed position in space. Let us, however, consider a coordinate system which is non-rotating, but is always centred on Sphere 1 (i.e., rather than a coordinate system centred at the centre of mass), and thus we will only consider the position $\vec{r}(t)$ of Sphere 2 *relative* to that of Sphere 1.

(a) What is the Lagrangian for the full system, in terms of m_2, $\vec{r}(t)$, $\dot{\vec{r}}(t)$, and the spring constant k?

(b) Write down the resulting Lagrange's equations of motion, in terms of the polar coordinates r and ϕ (in the plane of motion of the system).

(c) What are the solutions to these equations of motion?

(d) What is the total energy of the system?

2. Two gravitating masses m_1 and m_2 are separated by a distance r_0 and released from rest. Write down the Lagrangian and solve Lagrange's equations of motion to show that when the separation is r ($< r_0$), the speeds are: $v_1 = m_2 \text{ sqrt}[(2G/(m_1+m_2))(1/r - 1/r_0)]$ and $v_2 = m_1 \text{ sqrt}[(2G/(m_1+m_2))(1/r - 1/r_0)]$.