P424 Assignment 1

1) Natural units

(a) The neutral B mesons oscillate between particle and anti-particle states. The probability of a particle initially produced as a B^0 to be detected as a $\overline{B}^0$ is usually written as (using "natural" units)

$$P(B^0 \to \overline{B}^0) = \frac{e^{-t/\tau}}{2}\left(1 - \cos(\Delta m\, t)\right)$$

and Δm is quoted to be $0.502\,\mathrm{ps}^{-1}$, i.e. is treated as a frequency. Express Δm in units of mass and give the numerical value in an appropriate system of units, like $\mathrm{GeV/c^2}$.

(b) A pion has invariant mass $0.1395\,\mathrm{GeV/c^2}$. What is its Compton wavelength in Fermi (fm)?

2) Scattering kinematics

A 2 GeV electron beam is incident on a thin Carbon target. Assuming the electrons scatter dominantly from individual protons, calculate

(a) the invariant mass of an e^--p pair in the CM frame

(b) the energy of the scattered (final state) electron as a function of its angle w.r.t. the electron beam for elastic collisions (i.e. when the proton does not get broken apart).

(c) Does your answer to (b) change if you scatter polarized electrons from a polarized target?

3) Decay kinematics

Consider the decay $K^+ \to \pi^+\pi^0\pi^0$, where $m_K^+ = 495\,\mathrm{MeV}$, $m_{\pi^+} = 139\,\mathrm{MeV}$ and $m_{\pi^0} = 135\,\mathrm{MeV}$.

(a) What is the maximum momentum of the π^+ in the K^+ rest frame?

(b) Suppose the experimental apparatus cannot record decays where the invariant mass of the π^0s, $m_{\pi^0\pi^0}$, is less than 320 MeV. What are the allowed momenta for the π^+ under this restriction?

(c) What is the maximum allowed $m_{\pi^+\pi^0}$ under the conditions of part (b)?

(d) One often finds that random combinatorial background (combinations of particles that did not come from the decay of a single parent particle) populates the area near the boundary of a Dalitz plot. What characterstic do the decays that lie near the boundary of the plot share?

4) Interaction of particles in matter

(a) The amount of energy a 1.5 GeV particle loses when traversing material depends on the particle type. Assume the particles e, μ, π, K and p, each with 1.5 GeV momentum, traverse a medium of thickness $0.01X_0$. Which, on average, loses the most energy (order them from largest to smallest energy loss)?

(b) The particles from (a) enter a quartz bar with index of refraction $n = 1.5$. Which particles will emit Cherenkov radiation?

(c) Suppose you measure K^- and π^- in your detector. You measure the track momentum with high precision. In addition, you measure the amount of time required for the particle to travel the 1 m between the production vertex and a time-of-flight (TOF) detector. What time resolution σ_t must the TOF detector have in order to achieve a 3 standard deviation separation between π^- and K^- at 1 GeV momentum?

(d) Consider an electromagnetic shower initiated by a 1 GeV positron in Lead. If we model the shower process as a series of bremsstrahlung events and subsequent pair production from the photons, estimate (crudely) how many positrons we expect to produce. (Hint: the critical energy in Lead is about 1 MeV.)