P424 Assignment 5

Due Monday, Feb. 23

1) Fun with Dirac delta functions

The Dirac delta function is a convenient way to enforce constraints. Work through the following simple example. The quantity x is distributed according to $f(x) = 1$ for $0 < x < 1$ and is elsewhere zero, and the quantity y is distributed according to $g(y) = 2y$ for $0 < y < 1$ and is elsewhere zero. How is the quantity $z = x + y$ distributed? You can determine this by solving the equation

$$h(z) = \int dx\, f(x) \int dy\, g(y)\, \delta\left(z - (x + y)\right)$$

where the integrations are over all values of x and y and the delta function neatly enforces the relation between x, y and z.

2) Consider the elastic scattering reaction $A + B \to A + B$ in the lab frame (B initially at rest) and assume that the initial energy E_1 of the incoming A particle satisfies $E_1 \ll m_B$ so that the recoil of the target can be neglected.

(a) Use the Golden Rule for scattering to show that the differential cross section is given by

$$\frac{d\sigma}{d\Omega} = \frac{|\mathcal{M}|^2}{(8\pi m_B)^2}$$

(b) Write down the lowest order diagram(s) for this scattering process in ABC theory

(c) Calculate the decay amplitude using the Feynman rules for ABC theory (express your result using the Mandelstam variables s, t and/or u as relevant).

(d) Combine the results from (a) and (c) to obtain the differential cross section (in the limit $E_1 \ll m_B$ and assuming that m_A and m_C are tiny compared to m_B).

(e) Show that the total cross-section is

$$\sigma = \frac{g^4}{4\pi m_B^6}$$

under the conditions stated in part (d).