Standard Model Lagrangian Density

1. QCD (color $SU(3)$)

The color $SU(3)$ triplet of quarks

$$\Psi = \begin{pmatrix} \psi^1 \\ \psi^2 \\ \psi^3 \end{pmatrix} = \begin{pmatrix} \psi^{\text{red}} \\ \psi^{\text{green}} \\ \psi^{\text{blue}} \end{pmatrix}$$

Note that Ψ may be u, d, c, s, t, or b.
The eight $SU(3)$ gauge fields = gluon fields

$$A_\mu^a(x)$$

$SU(n)$ gauge transformation

$$\begin{array}{l} \Psi \to \Psi' = U\Psi = \exp\left(-iT^a\varepsilon^a(x)\right)\Psi \\ A_\mu^a T^a \to A'^a_\mu T^a = A_\mu^a U T^a U^{-1} + \frac{1}{g}T^a\partial_\mu\varepsilon^a \end{array}$$

For $SU(3)$

$$T^a = \frac{1}{2}\lambda^a \qquad \left[\frac{1}{2}\lambda^a, \frac{1}{2}\lambda^b\right] = if^{abc}\frac{1}{2}\lambda^c$$

where, λ^a ($a = 1, 2, ..., 8$) are Gell-Mann matrices, and f^{abc} are the SU(3) group structure constants.

The QCD Lagrangian

$$\mathcal{L} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$$

where

$$\begin{aligned} D_\mu &= \partial_\mu + ig_S T^a A^a_\mu = \partial_\mu + ig_S \tfrac{1}{2}\lambda^a A^a_\mu \\ F^a_{\mu\nu} &= A^a_{\mu\nu} - g_S f^{abc} A^b_\mu A^c_\nu \\ A^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu \end{aligned}$$

Hence,

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_A + \mathcal{L}_{\text{int}}$$

whre

$$\begin{aligned} \mathcal{L}_0 &= \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi \\ \mathcal{L}_A &= -\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} \\ &= -\tfrac{1}{4}A^a_{\mu\nu}A^{a\mu\nu} + \tfrac{1}{2}g_S f^{abc} A^a_{\mu\nu} A^{b\mu} A^{c\nu} - \tfrac{1}{4}g_S^2 f^{abc} f^{ars} A^b_\mu A^c_\nu A^{r\mu} A^{s\nu} \\ \mathcal{L}_{\text{int}} &= -g_S \bar{\Psi}\gamma^\mu T^a \Psi A^a_\mu = -g_S \bar{\Psi}\gamma^\mu \tfrac{1}{2}\lambda^a \Psi A^a_\mu \end{aligned}$$

Conserved color current

$$j^{a\mu}_{SU(3)} = g_S \bar{\Psi}\gamma^\mu T^a \Psi = g_S \bar{\Psi}\gamma^\mu \frac{1}{2}\lambda^a \Psi$$

and color charge

$$Q^a = \int dV\, j^{a0}_{SU(3)}$$

2. Electroweak ($U(1)_Y \otimes SU(2)_L$): before symmetry hiding

The weak hypercharge $U(1)$ singlets

neutrinos	$\nu(x) \in \{\nu_e, \nu_\mu, \mu_\tau\}$	$(= \nu_L(x) + \nu_R(x))$
charged leptons	$e(x) \in \{e, \mu, \tau\}$	$(= e_L(x) + e_R(x))$
up-type quarks	$u(x) \in \{u, c, t\}$	$(= u_L(x) + u_R(x))$
down-type quarks	$d(x) \in \{d, s, b\}$	$(= d_L(x) + d_R(x))$

The negative chiral weak isospin $SU(2)_L$ singlets

leptons	$l_R(x) \in \{\nu_R(x), e_R(x)\}$
quarks	$q_R(x) \in \{u_R(x), d_R(x)\}$

The negative chiral weak isospin $SU(2)_L$ doublets

leptons	$l_L(x) = \begin{pmatrix} \nu_L(x) \\ e_L(x) \end{pmatrix}$
quarks	$q_L(x) = \begin{pmatrix} u_L(x) \\ d_L(x) \end{pmatrix}$
complex scalar fields	$\phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}$
and conjugate fields	$\widehat{\phi}(x) = i\sigma_2\phi^* = \begin{pmatrix} \phi^{0*}(x) \\ -\phi^-(x) \end{pmatrix}$

The gauge fields

one $U(1)_Y$ gauge field	: $W^0_\mu(x)$
three $SU(2)_L$ gauge fields	: $W^a_\mu(x)$

Electroweak $U(1)_Y \otimes SU(2)_L$ gauge transformation

$$U = U_1 \otimes U_2$$
$$U_1 = \exp\left[-it^0\varepsilon^0(x)\right]$$
$$U_2 = \begin{cases} \exp\left[-iT^a\varepsilon^a(x)\right] & \text{on } SU(2)_L \text{ doublet} \\ I & \text{on } SU(2)_L \text{ singlet} \end{cases}$$

and

$$l \to l' = Ul, \quad q \to q' = Uq$$
$$W^0_\mu \to W'^0_\mu = W^0_\mu + \tfrac{1}{g'}\partial_\mu\varepsilon^0(x)$$
$$T^aW^a_\mu \to T^aW'^a_\mu = UT^aW^a_\mu U^{-1} + \tfrac{1}{g}\partial_\mu T^a\varepsilon^a(x)$$

For $SU(2)$

$$T^a = \frac{1}{2}\sigma^a \qquad \left[\frac{1}{2}\sigma^a, \frac{1}{2}\sigma^b\right] = i\varepsilon^{abc}\frac{1}{2}\sigma^c$$

where σ^a $(a = 1, 2, 3)$ are Pauli matrices.

The Electroweak Lagrangian before symmetry hiding

$$\mathcal{L} = \bar{l}_L i\gamma^\mu D_\mu l_L + \sum_l \bar{l}_R i\gamma^\mu D_\mu l_R + \bar{q}_L i\gamma^\mu D_\mu q_L + \sum_q \bar{q}_R i\gamma^\mu D_\mu q_R \\ + (D_\mu\phi)^\dagger (D^\mu\phi) - \mathcal{V}(\phi) - \tfrac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \tfrac{1}{4}H_{\mu\nu}H^{\mu\nu} + \mathcal{L}_{\text{Yukawa}}$$

where

potential $\mathcal{V}(\phi) = -\mu^2\phi^\dagger\phi + \lambda\left(\phi^\dagger\phi\right)^2$

Yukawa term $\mathcal{L}_{\text{Yukawa}} = c_e\left[\bar{l}_L\phi e_R + \bar{e}_R\phi^\dagger l_L\right] + c_\nu\left[\bar{l}_L\widehat{\phi}\nu_R + \bar{\nu}_R\widehat{\phi}^\dagger l_L\right] + c_u\left[\bar{q}_L\widehat{\phi}u_R + \bar{u}_R\widehat{\phi}^\dagger q_L\right] + c_d\left[\bar{q}_L\phi d_R + \bar{d}_R\phi^\dagger q_L\right]$

and

$$D_\mu = \begin{cases} \partial_\mu + ig't^0W^0_\mu & \text{for } SU(2)_L \text{ singlets} \\ \partial_\mu + ig't^0W^0_\mu + igT^aW^a_\mu & \text{for } SU(2)_L \text{ doublets} \end{cases}$$

g', g : real coupling constants
t^0 : weak hypercharge
$H_{\mu\nu} = \partial_\mu W^0_\nu - \partial_\nu W^0_\mu$
$G^a_{\mu\nu} = W^a_{\mu\nu} - g\varepsilon^{abc}W^b_\mu W^c_\nu$
$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu$

After some manipulations

$$\mathcal{L} = \mathcal{L}_D + \mathcal{L}_V + \mathcal{L}_S + \mathcal{L}_{DV} + \mathcal{L}_{SV} + \mathcal{L}_{SD}$$

Free Dirac spinor matter fields:

$$\mathcal{L}_D = \bar{e}i\gamma^\mu\partial_\mu e + \bar{\nu}i\gamma^\mu\partial_\mu\nu + \bar{u}i\gamma^\mu\partial_\mu u + \bar{d}i\gamma^\mu\partial_\mu d$$

Pure gauge fields:

$$\mathcal{L}_V = -\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}H_{\mu\nu}H^{\mu\nu} + \mathcal{L}_{VVV} + \mathcal{L}_{VVVV}$$

where

$$\mathcal{L}_{VVV} = \tfrac{1}{2}g\varepsilon^{abc}W^{a\mu\nu}W^b_\mu W^c_\nu$$
$$\mathcal{L}_{VVVV} = -\tfrac{1}{4}g^2\varepsilon^{abc}\varepsilon^{ars}W^b_\mu W^c_\nu W^{r\mu}W^{s\nu}$$

Interaction between matter fields and gauge fields:

$$\mathcal{L}_{DV} = \mathcal{L}_Y + \mathcal{L}_T$$

where

$$\mathcal{L}_Y = -W^0_\mu j^\mu_Y = -W^0_\mu\left(g't^0\bar{l}\gamma^\mu l + g't^0\bar{q}\gamma^\mu q\right)$$
$$\mathcal{L}_T = -W^a_\mu j^{a\mu}_T = -W^0_\mu\left(g\bar{l}_L\gamma^\mu T^a l_L + g\bar{q}_L\gamma^\mu T^a q_L\right)$$

Interaction between scalar fields and gauge fields:

$$\mathcal{L}_{\text{SV}} = ig't^0(\partial_\mu\phi)^\dagger\phi W^{0\mu} - ig't^0\phi^\dagger(\partial_\mu\phi)W^{0\mu*} + ig(\partial_\mu\phi)^\dagger T^a\phi W^{a\mu} - ig\phi^\dagger T^{a\dagger}(\partial_\mu\phi)W^{a\mu*} \\ + g^2t^{02}\phi^\dagger\phi W^{0*}_\mu W^{0\mu} + \tfrac{1}{4}g^2\phi^\dagger\phi W^{a*}_\mu W^{a\mu} - g'gt^o\phi^\dagger T^a\phi W^{0*}_\mu W^{a\mu} - g'gt^o\phi^\dagger T^{a\dagger}\phi W^0_\mu W^{a\mu*}$$

Interaction between scalar fields and matter fields:

$$\mathcal{L}_{\text{SD}} = \mathcal{L}_{\text{Yukawa}}$$

3. Electro ($U(1)_Q$) and weak : after symmetry hiding

Spontaneous symmetry breaking: chosen equilibrium (vacuum) point

$$\phi_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

where $v^2 = \mu^2/\lambda$.

Scalar field in unitary gauge

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \sigma(x) \end{pmatrix}, \quad \widehat{\phi}(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} v + \sigma(x) \\ 0 \end{pmatrix}$$

Re-arrangement of gauge fields:

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W \cos\theta_W & \end{pmatrix} \begin{pmatrix} W_\mu^0 \\ W_\mu^3 \end{pmatrix}$$

where θ_W is a real angle (Weinberg angle).

$$W_\mu^\pm = \frac{1}{\sqrt{2}} \left(W_\mu^1 \mp i W_\mu^2 \right) \quad \text{and} \quad T^\pm = T^1 \pm i T^2$$

and covariant derivative:

$$D_\mu = \partial_\mu + \frac{ig}{\sqrt{2}} \left(T^+ W_\mu^+ + T^- W_\mu^- \right) + ieQA_\mu + ieQ' Z_\mu$$

where

electric charge matrix : $Q = T^3 + t^0$

neutral charge matrix : $Q' = T^3 \cot\theta_W - t^0 \tan\theta_W$

Note that Q and Q' are matrices in $SU(2)$ space.

The Elecroweak ($U(1)_Q$) Lagrangian

$$\mathcal{L} = \mathcal{L}_D + \mathcal{L}_V + \mathcal{L}_H + \mathcal{L}_{DV} + \mathcal{L}_{HD} + \mathcal{L}_{HV}$$

Free Dirac spinor matter fields:

$$\begin{aligned} \mathcal{L}_D = {} & \bar{e} \left[i\gamma^\mu \partial_\mu - M_e \right] e + \bar{\nu} \left[i\gamma^\mu \partial_\mu - M_\nu \right] \nu \\ & + \bar{u} \left[i\gamma^\mu \partial_\mu - M_u \right] u + \bar{d} \left[i\gamma^\mu \partial_\mu - M_d \right] d \end{aligned}$$

where M_e, M_ν, M_u, and M_D are mass matrices in family space.

Pure gauge fields:

$$\begin{aligned} \mathcal{L}_V = {} & -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - \tfrac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + \tfrac{1}{2} M_Z^2 Z_\mu Z^\mu \\ & -\tfrac{1}{2} W_{\mu\nu}^+ W^{-\mu\nu} + M_W^2 W_\mu^+ W^{-\mu} \\ & + \mathcal{L}_{VVV} + \mathcal{L}_{VVVV} \end{aligned}$$

where

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \\ Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu \\ W^\pm_{\mu\nu} &= \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu \end{aligned}$$

and

$$\begin{aligned} \mathcal{L}_{VVV} = &-\tfrac{ig}{4}\cos\theta_W \left[2Z_{\mu\nu}\left(W^{+\mu}W^{-\nu} - W^{+\nu}W^{-\mu}\right)\right. \\ &\qquad \left. -W^+_{\mu\nu}\left(Z^\mu W^{-\nu} - Z^\nu W^{-\mu}\right) + W^-_{\mu\nu}\left(Z^\mu W^{+\nu} - Z^\nu W^{+\mu}\right)\right] \\ &-\tfrac{ig}{4}\sin\theta_W \left[2F_{\mu\nu}\left(W^{+\mu}W^{-\nu} - W^{+\nu}W^{-\mu}\right)\right. \\ &\qquad \left. -W^+_{\mu\nu}\left(A^\mu W^{-\nu} - A^\nu W^{-\mu}\right) + W^-_{\mu\nu}\left(A^\mu W^{+\nu} - A^\nu W^{+\mu}\right)\right] \\ \mathcal{L}_{VVVV} = &-\tfrac{g^2}{4}\left[\left(W_{+\mu}W_{-\nu} - W_{+\nu}W_{-\mu}\right)\left(W^{+\mu}W^{-\nu} - W^{+\nu}W^{-\mu}\right)\right. \\ &+2\cos^2\theta_W \left(Z_\mu W^+_\nu - Z_\nu W^+_\mu\right)\left(Z^\mu W^{-\nu} - Z^\nu W^{-\mu}\right) \\ &+2\sin^2\theta_W \left(A_\mu W^+_\nu - A_\nu W^+_\mu\right)\left(A^\mu W^{-\nu} - A^\nu W^{-\mu}\right) \\ &+2\sin\theta_W\cos\theta_W \left\{\left(Z_\mu W^+_\nu - Z_\nu W^+_\mu\right)\left(A^\mu W^{-\nu} - A^\nu W^{-\mu}\right)\right. \\ &\qquad \left.\left. +\left(Z^\mu W^{-\nu} - Z^\nu W^{-\mu}\right)\left(A_\mu W^+_\nu - A_\nu W^+_\mu\right)\right\}\right] \end{aligned}$$

Pure Higgs field:

$$\mathcal{L}_H = \frac{1}{2}\left(\partial_\mu\sigma\right)\left(\partial^\mu\sigma\right) - \frac{1}{2}M_H^2\sigma^2 - \frac{1}{2v}M_H^2\sigma^3\left(1 + \frac{1}{4v}\sigma\right)$$

Interaction between matter fields and gauge fields:

$$\mathcal{L}_{DV} = \mathcal{L}_{\rm em} + \mathcal{L}_{\rm neutral} + \mathcal{L}_{\rm charged}$$

$$\begin{aligned} \mathcal{L}_{\rm em} &= -A_\mu j^\mu_{\rm em} = -A_\mu\left(e\bar{l}\gamma^\mu Q_l l + e\bar{q}\gamma^\mu Q_q q\right) \\ \mathcal{L}_{\rm neutral} &= -Z_\mu j^\mu_{\rm neutral} = -Z_\mu\left(e\bar{l}\gamma^\mu Q'_l l + e\bar{q}\gamma^\mu Q'_q q\right) \\ \mathcal{L}_{\rm charged} &= -W^+_\mu j^{+\mu}_{\rm charged} - W^-_\mu j^{-\mu}_{\rm charged} \\ &= -\tfrac{e}{\sqrt{2}\sin\theta_W}W^+_\mu\left(\bar{\nu}_L\gamma^\mu V' e_L + \bar{u}_L\gamma^\mu V d_L\right) \\ &= -\tfrac{e}{\sqrt{2}\sin\theta_W}W^-_\mu\left(\bar{e}_L V'^\dagger\gamma^\mu\nu_L + \bar{d}_L V^\dagger\gamma^\mu u_L\right) \end{aligned}$$

where V and V' are the CKM matrix and the MNS matrix respectively.
Interaction between matter fields and Higgs field:

$$\mathcal{L}_{HD} = -\frac{1}{v}\sigma\left(\bar{e}M_e e + \bar{\nu}M_\nu\nu + \bar{u}M_u u + \bar{d}M_d d\right)$$

Interaction between Higgs field and gauge fields:

$$\mathcal{L}_{HV} = \left(M_W^2 W^+_\mu W^{-\mu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu\right)\frac{2}{v}\sigma\left(1 + \frac{1}{2v}\sigma\right)$$