Global Fit for Branching Fractions and FF Parameters of $B \to D^{(*)} l \nu$ Decays

- Our fitting method in a nutshell

Essence of our fitting (1)

- Consider a simple case
  - Only two signal mode : mode1 (BF1) and mode2 (BF2)
  - Only one background category : bkg
- We construct 3D binned histograms from data and MC.
  - One histogram for data
  - Three MC histograms (using MC truth info):
    - One mode1 histogram
    - One mode 2 histogram
    - One bkg histogram
- If the MC is perfect and N^{MC} are luminosity normalized to data,

$$\chi^2 = \sum_{\text{bin}} \frac{\left(N^{data} - N_1^{MC} - N_2^{MC} - N_b^{MC}\right)^2}{\sigma_d^2 + \sigma_1^2 + \sigma_2^2 + \sigma_b^2} \cong \text{number of bins}$$

Essence of our fitting (2)

- What if BF1 and BF2 are different from the one used in MC?
  - We use coefficients to adjust the yield

$$\chi^2 = \sum_{\text{bin}} \frac{\left(N^{data} - \frac{BF1}{BF1_{SP6}} N_1^{MC} - \frac{BF2}{BF2_{SP6}} N_2^{MC} - N_b^{MC} \right)^2}{\sigma_d^2 + \sigma_1^2 + \sigma_2^2 + \sigma_b^2}$$

  - These BF1 and BF2 can be a free parameter in the fit.
  - If we can adjust with coefficients, it is easier because we don't have to remake 3D histograms.
- What if the BF of one of the (many) decay processes in bkg is different from the one used in MC?
  - We cannot just use a coefficient to adjust the difference.
  - We have to go back and remake the 3D histogram of bkg.
  - This is BF re-weighting of background MC.

Essence of our fitting (3)

- What if one of FF parameters of mode1 is different from the one used in MC?
  - Again, a coefficient cannot adjust the difference.
  - We have to go back and remake the 3D histogram of mode1.
  - This is FF-reweighting.
- So, if we want to measure FF parameters, we have to go back and recreate 3D histograms in the iteration of the fitting?
  - This is an option but this is too slow....
  - There is a better way!

Essence of FF re-weighting

- Suppose decay rate of mode1 is given by
  - $\Gamma = A(b - p1 * c)$
  - *p1* is a FF parameter and *A*, *b* and *c* are constants.
- Weight is given by
  - $W = (b - p1 * c) / (b - p1_{SP6} * c)$
  - $p1_{SP6}$ is the value used in MC and *p1* is a new value.
- *W* differ event by event because *b* and *c* depends on kinematic variables of the event.
  - *W* cannot go into coefficients.
  - We have to remake 3D histograms for different *p1*

$$N_1^{MC} = \sum_{\text{event}} 1 \Rightarrow N_1^{MC} = \sum_{\text{event}} W$$

Essence of FF parameter fitting

- Weight can be

$$W = W_b - p_1 W_c = \frac{b}{b - p_{1SP6} c} - p_1 \frac{c}{b - p_{1SP6} c}$$

- Instead of making one N_1^{MC}, we can make two histograms which are independent of *p1*

$$N_a^{MC} = \sum_{\text{event}} W_a \, , \quad N_c^{MC} = \sum_{\text{event}} W_c$$

- Then, chi-square is given by

$$\chi^2 = \sum_{\text{bin}} \frac{\left(N^{data} - \frac{BF1}{BF1_{SP6}} \left(N_a^{MC} - p_1 N_b^{MC} \right) - \frac{BF2}{BF2_{SP6}} N_2^{MC} - N_b^{MC} \right)^2}{\sigma_d^2 + \sigma_1^2 + \sigma_2^2 + \sigma_b^2}$$

- Now, we can fir for *p1* without re-making histograms.

Essence of our fitting (4)

- What if background is not well modeled?
  - We do BF reweighting of some processes.
    - This can change the shape of 3D histogram.
    - But, still may need some adjustments?
  - We use coefficients to adjust background.

$$\chi^2 = \sum_{\text{bin}} \frac{\left(N^{data} - \frac{BF1}{BF1_{SP6}} \left(N_a^{MC} - p_1 N_b^{MC} \right) - \frac{BF2}{BF2_{SP6}} N_2^{MC} - C_b N_b^{MC} \right)^2}{\sigma_d^2 + \sigma_1^2 + \sigma_2^2 + \sigma_b^2}$$

  - This C_b can adjust the data-MC difference of the overall number of background candidates.

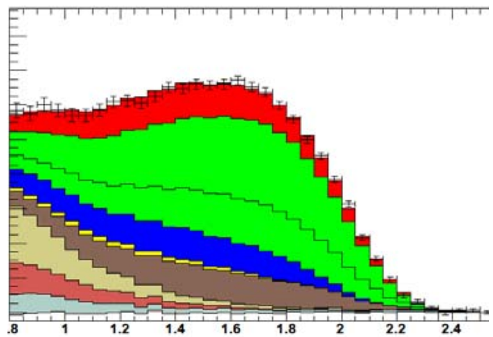
Fitting strategy – Binning(1)

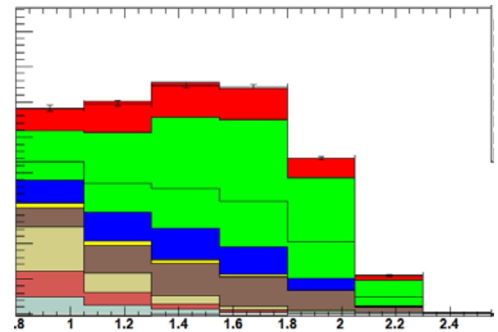
- 3D binning
  - lepton momentum, *D* momentum and cosBY
  - The size of each bin is decided based on
    - As much number of candidates per bin as possible.
    - Keep original shape or dependence to the variable.
    - Maximum separation between signal modes and backgrounds.
- cosBY (Y=*Dl*)
  - B->Dlnu mode has sharp edge at cosBY = ±1.
  - Because of beam energy fluctuation, this edge is not well simulated.
  - Thus, we avoid to have a bin boundary at cosBY = ±1.

Fitting strategy – Binning(2)

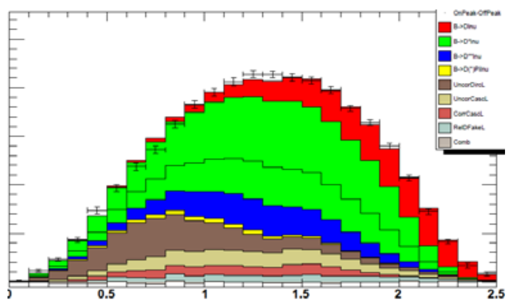
Binning for fitting

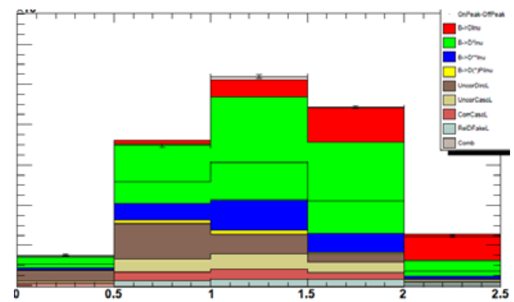
Lepton momentum



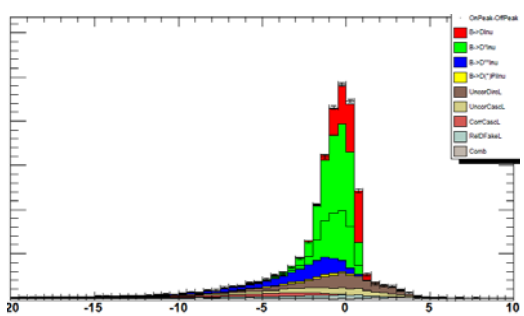


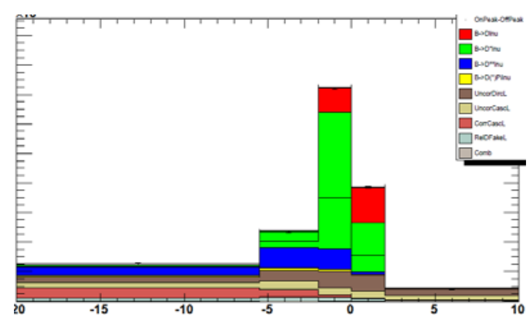
D momentum





cosBY





Fitting strategy – Reweighting

- MC/FF re-weighting is done in two stages
- Histogram making stage
  - BF re-weighting of background processes :
    - Inclusive *B*->*D* decay BF (*D* momentum dependent)
    - Inclusive *B*->lepton decay BF
    - Semileptonic *D* decay BF (Cascade background)
    - $B \to D^{(*)} \tau \nu$ decay BF
  - cosBY correction to adjust beam energy effect.
  - $B \to D^{**} l \nu$ FF re-weighting (ISGW2 -> LLSW)
- Fitting stage
  - The items goes into coefficients :
    - Charm decay BF ($D^{**}/D^{*}/D$ decay BF)
    - B^{+}/B^{0} production ratio
  - FF parameters as explained on page 7
    - Slope of $B \to D l \nu$; Slope, R_1 and R_2 of $B \to D^{*} l \nu$

Fitting strategy – Free parameters

- Isospin symmetry on *B* decays.
- 14 free parameters in the fitting
  - 3 BF
    - $B^+ \to Dl\nu$
    - $B^+ \to D^*l\nu$
    - Sum of $B^+ \to D^{**}l\nu$ and non-resonant $D^{(*)}\pi l\nu$
  - 4 FF parameters
    - Slope of $B \to Dl\nu$
    - Slope, R_1 and R_2 of $B \to D^*l\nu$
  - B^+/B^0 production ratio
  - 6 background coefficients (3 each for D^0 and D^+)
    - Uncorrelated
    - Uncorrelated cascade
    - Correlated cascade

$B^+ \to D^{**} l \nu$ and $D^{(*)} \pi l \nu$

- We fix ratios between these decay modes :

$$\mathcal{B}(B^+ \to \bar{D}_0^{*0} \ell^+ \nu) = 0.0045 \pm 0.0009$$
$$\mathcal{B}(B^+ \to \bar{D}_1^{0} \ell^+ \nu) = 0.0054 \pm 0.0006$$
$$\mathcal{B}(B^+ \to \bar{D}_1^{\prime 0} \ell^+ \nu) = 0.0085 \pm 0.0020$$
$$\mathcal{B}(B^+ \to \bar{D}_2^{*0} \ell^+ \nu) = 0.0042 \pm 0.0008$$
$$\mathcal{B}(B^+ \to D^- \pi^+ \ell^+ \nu) = 0.0019 \pm 0.0012$$
$$\mathcal{B}(B^+ \to \bar{D}^0 \pi^0 \ell^+ \nu) = 0.0010 \pm 0.0006$$
$$\mathcal{B}(B^+ \to D^{*-} \pi^+ \ell^+ \nu) = 0.0006 \pm 0.0004$$
$$\mathcal{B}(B^+ \to \bar{D}^{*0} \pi^0 \ell^+ \nu) = 0.0003 \pm 0.0002$$

- Then float the sum of these branching fractions.

BF renormalization

- BF renormalization does not have to be a necessary piece of our analysis.
- If all systematic uncertainty of fitted BFs are well understood and small, we don't need this.
- But, in reality...
  - Sum of BFs is 9% smaller than inclusive BF.
  - Some systematic uncertainties are much smaller after renormalization.