Slow Pi Momentum

1 D^* rest frame

In te decay $D^* \to D\pi$

- Initial state
  $D^* : (m_{D^*}, \mathbf{0})$

- Final state
  $D : (E_D, \text{-}\mathbf{p})$ and $\pi : (E_\pi, \mathbf{p})$

where

$$p^2 = E_\pi^2 - m_\pi^2 \tag{1}$$

Hence,

$$E_D^2 = m_D^2 + p^2 = m_D^2 + E_\pi^2 - m_\pi^2 \tag{2}$$

By conservation

$$\begin{aligned} &m_{D^*} = E_D + E_\pi \\ &\Rightarrow E_D^2 = (m_{D^*} - E_\pi)^2 = m_{D^*}^2 - 2m_{D^*}E_\pi + E_\pi^2 \end{aligned} \tag{3}$$

Thus

$$\begin{aligned} &m_D^2 + E_\pi^2 - m_\pi^2 = m_{D^*}^2 - 2m_{D^*}E_\pi + E_\pi^2 \\ &\Rightarrow 2m_{D^*}E_\pi = m_{D^*}^2 - m_D^2 + m_\pi^2 \end{aligned} \tag{4}$$

Hence

$$E_\pi = \frac{m_{D^*}^2 - m_D^2 + m_\pi^2}{2m_{D^*}} \tag{5}$$

By substituting this

$$p = \sqrt{E_\pi^2 - m_\pi^2} \tag{6}$$

2 B rest frame

The velocity tranfer is

$$w = v_B \cdot v_{D^*} = \frac{E_B E_{D^*} - \mathbf{p}_B \cdot \mathbf{p}_{D^*}}{m_B m_{D^*}} \tag{7}$$

In the B rest frame, this becomes

$$w = \frac{m_B E_{D^*}}{m_B m_{D^*}} = \frac{E_{D^*}}{m_{D^*}} = \gamma \tag{8}$$

The transverse and logitudinal components of the π momentum relative to the velocity of D^* are $(p\sin\theta_V, p\cos\theta_V)$ in the D^* rest frame. The longitudinal component is boosted back to the B rest frame,

$$p'_x = \gamma p\cos\theta_V + \gamma\beta E_\pi = wp\cos\theta_V + w\sqrt{1 - \frac{1}{\gamma}}E_\pi \tag{9}$$

Hence

$$p' = \sqrt{w^2 \left(p\cos\theta_V + E_\pi \sqrt{1 - \frac{1}{\gamma}}\right)^2 + \left(p\sin\theta_V\right)^2} \tag{10}$$

Thus

$$p' = \sqrt{w^2 \left(p\cos\theta_V + E_\pi \sqrt{1 - \frac{1}{\gamma}}\right)^2 + p^2\left(1 - \cos^2\theta_V\right)} \tag{11}$$