Form factors of $B \to D/D^*/D^{**}\ell\nu$ decays

1 Angular momentum of D, D^* and D^{**}

Form factors depend on angular momentum, so let us first consider angular momentums of mesons. In atomic spectroscopy, there are two useful schemes to add angular momentums: LS coupling and jj coupling. For equal-mass meson system, it is traditional to couple the orbital angular momentum $\vec{L}$ with the total spin of the quark and antiquark, $\vec{S}$

$$\vec{S} = \vec{s}_1 + \vec{s}_2$$
$$\vec{J} = \vec{L} + \vec{S}$$

This is the standard practice for light mesons and heavy quarkonium ($c\bar{c}$ and $b\bar{b}$). The good quantum nubers are then S, L and J.
There are spin-singlet and spin-triplet levels because S is always $S = 0, 1$.
For $L = 0$ and $S = 0$, $J = 0$: ${}^1S_0$
For $L = 0$ and $S = 1$, $J = 1$: ${}^3S_1$
For $L = 1$ and $S = 0$, $J = 1$: ${}^1P_1$
For $L = 1$ and $S = 1$, $J = 0, 1, 2$: ${}^3P_0$, ${}^3P_1$ or ${}^3P_2$
In general, for $S = 0$ and $S = 1$

$${}^1L_L \quad \text{and} \quad {}^3L_{L-1,L,L+1}$$

For the mesons consist of heavy and light quarks, we can apply jj coupling scheme because of the spin symmetry in HQET, which suggests that the spin of havy quark is conserved. First, we can add the angular momentum (L) and the spin of the light qaurk (s_l) to get the angular momentum of lihgt constituent (j_l). Then we can add the spin of the heavy quark (s_h) to j_l.

$$\vec{j}_l = \vec{L} + \vec{s}_l$$
$$\vec{J} = \vec{j}_l + \vec{s}_h$$

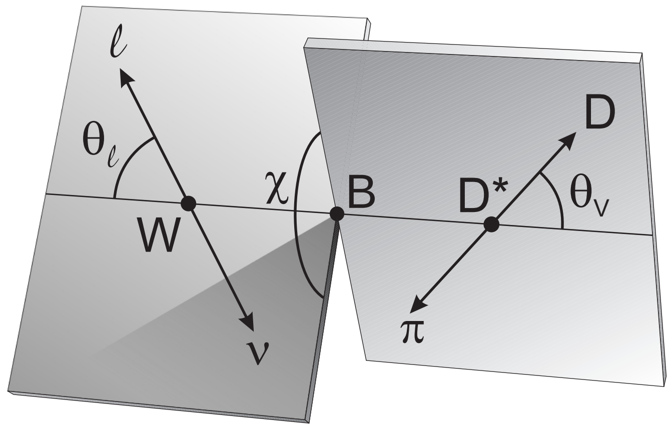
For $L = 0$, $j_l = 1/2$, and $J = 0$ (D) or 1 (D^*).
For $L = 1$, $j_l = 1/2$ or $3/2$. For $j_l = 1/2$, $J = 0$ (D_0^*) or 1 (D_1'), and for $j_l = 3/2$, $J = 1$ (D_1) or 2 (D_2^*).
Parity of mesons is $(-1)^L$. Thus, for $L = 0$, $P = +$ and for $L = 1$, $P = -$.
These are summarized in the table.

j_l^P	Particles	J^P	mass (GeV)
$\frac{1}{2}^+$	D, D^*	0^-, 1^-	1.971
$\frac{1}{2}^+$	D_0^*, D_1'	0^+, 1^+	2.40
$\frac{3}{2}^+$	D_1, D_2^*	1^+, 2^+	2.445

This classification implies that the $j = 3/2$ states will decay only through the d-wave and so will be narrow. The $j = 1/2$ states can decay through the s-wave and in general will be broad.

2 Kinematic variables

In the case of $B \to D^*/D^{**}\ell\nu$ decays



These $\cos\theta_\ell$, $\cos\theta_V$ and χ are the kinematic variables we need.

3 Matrix elements, form factors and decay rates

The amplitude of $B \to D^{(*,**)}\ell\nu$ is given by

$$\mathcal{M} = -i\frac{G_F}{\sqrt{2}}V_{cb}L^\mu H_\mu$$

where the leptonic current can be written in terms of Dirac spinors u_l and v_ν

$$L^\mu = \bar{u}_l\gamma^\mu(1-\gamma_5)v_\nu$$

The hadronic current H_μ can be written in terms of quark spinors b and c

$$H^\mu =< D|\bar{c}\gamma^\mu(1-\gamma_5)b|B >=< D|V^\mu|B > + < D|A^\mu|B >$$

where

$$V^\mu = \bar{c}\gamma^\mu b$$
$$A^\mu = \bar{c}\gamma^\mu\gamma_5 b$$

3.1 $B \to D\ell\nu$

In this case

$$< D|A^\mu|B >= 0$$

and

$$< D(p')|V^\mu|B(p) >= F_1(q^2)\left[(p+p')^\mu - \frac{m_B^2 - m_D^2}{q^2}q^\mu\right] + F_0(q^2)\frac{m_B^2 - m_D^2}{q^2}q^\mu$$

or

$$< D(p')|V^\mu|B(p) >= f_+(q^2)(p+p')^\mu + f_-(q^2)(p-p')^\mu$$

where

$$\begin{array}{l} F_1(q^2) = f_+(q^2) \\ F_0(q^2) = f_+(q^2) + \frac{q^2}{m_B^2 - m_D^2} f_-(q^2) \\ q^\mu = p^\mu - p'^\mu \end{array}$$

The form factors $F_1(q^2)$ and $F_0(q^2)$ can be associated with the excahnge of particles with quantum numbers $J^P = 1^-$ and $J^P = 0^+$, respectively. There is no singular behaviour at $q^2 = 0$ because $F_1(0) = F_0(0)$. In practice the terms proportional to q^μ are nearly always negligible. Thus

$$< D(p')|V^\mu|B(p) >= F_1(q^2)(p+p')^\mu$$

or

$$< D(p')|V^\mu|B(p) >= f_+(q^2)(p+p')^\mu$$

The differential decay rate is given by

$$\frac{d\Gamma(B \to D\ell\nu)}{dq^2} = \frac{G_F^2|V_{cb}|^2}{24\pi^3}|\vec{P}_D|^3|f_+(q^2)|^2 = \frac{G_F^2|V_{cb}|^2}{192\pi^3 m_B^3}\left[(q^2 - m_B^2 - m_D^2)^2 - 4m_B^2 m_D^2\right]^{3/2}|f_+(q^2)|^2$$

where we used the following calculation. In the B rest frame,

$$p = (m_B, \vec{0}), \quad p' = (\sqrt{m_D^2 + |\vec{P}_D|^2}, \vec{p}')$$

Thus

$$\begin{array}{l} q^2 = (p-p')^2 = p^2 + p'^2 - 2p\cdot p' = m_B^2 + m_D^2 - 2m_B\sqrt{m_D^2 + |\vec{P}_D|^2} \\ \Rightarrow 4m_B^2\left(m_D^2 + |\vec{P}_D|^2\right) = (q^2 - m_B^2 - m_D^2)^2 \\ \Rightarrow |\vec{P}_D| = \frac{1}{2m_B}\left[(q^2 - m_B^2 - m_D^2)^2 - 4m_B^2 m_D^2\right]^{1/2} \end{array}$$

3.2 $B \to D^*\ell\nu$

In this case

$$< D^*(p',\varepsilon)|V^\mu|B(p) >= \frac{2i\epsilon^{\mu\nu\alpha\beta}}{m_B + m_{D^*}}\varepsilon^*_\nu p'_\alpha p_\beta V(q^2)$$

and

$$\begin{array}{rcl} < D^*(p',\varepsilon)|A^\mu|B(p) > & = & -(m_B + m_{D^*})\varepsilon^{*\mu}A_1(q^2) + \frac{\varepsilon^*\cdot q}{m_B + m_{D^*}}(p+p')^\mu A_2(q^2) \\ & & +2m_{D^*}\frac{\varepsilon^*\cdot q}{q^2}q^\mu A_3(q^2) - 2m_{D^*}\frac{\varepsilon^*\cdot q}{q^2}q^\mu A_0(q^2) \end{array}$$

where

$$A_3(q^2) = \frac{m_B + m_{D^*}}{2m_{D^*}}A_1(q^2) - \frac{m_B - m_{D^*}}{2m_{D^*}}A_2(q^2)$$

with $A_0(0) = A_3(0)$. Or

$$< D^*(p',\varepsilon)|V^\mu|B(p) >= i\epsilon^{\mu\nu\alpha\beta}\varepsilon^*_\nu(p+p')_\alpha(p-p')_\beta g(q^2)$$

and

$$< D^*(p',\varepsilon)|A^\mu|B(p) >= \varepsilon^{*\mu} f(q^2) + \varepsilon^* \cdot q(p+p')^\mu a_+(q^2) + \varepsilon^* \cdot q(p-p')^\mu a_-(q^2)$$

where

$$\begin{array}{l} V(q^2) = (m_B - m_{D^*})g(q^2) \\ A_1(q^2) = \frac{1}{m_B - m_{D^*}} f(q^2) \\ A_2(q^2) = (m_B - m_{D^*})a_+(q^2) \\ A_3(q^2) - A_0(q^2) = \frac{q^2}{2m_{D^*}} a_-(q^2) \end{array}$$

Here, we used

$$\begin{array}{l} \epsilon^{\mu\nu\alpha\beta} p_\alpha p_\beta = \epsilon^{\mu\nu\alpha\beta} p'_\alpha p'_\beta = 0 \\ \epsilon^{\mu\nu\alpha\beta} p_\alpha p'_\beta = -\epsilon^{\mu\nu\alpha\beta} p'_\alpha p_\beta \end{array}$$

Again, terms proportional to q^μ only play an important role for the case $l = \tau$

$$< D^*(p',\varepsilon)|A^\mu|B(p) >= -(m_B + m_{D^*})\varepsilon^{*\mu} A_1(q^2) + \frac{\varepsilon^* \cdot q}{m_B + m_{D^*}}(p+p')^\mu A_2(q^2)$$

or

$$< D^*(p',\varepsilon)|A^\mu|B(p) >= \varepsilon^{*\mu} f(q^2) + \varepsilon^* \cdot q(p+p')^\mu a_+(q^2)$$

The form factors $A_1(q^2)$ and $A_2(q^2)$ can be associated with the excahnge of particles eith quantum numbers $J^P = 1^+$, whereas $V(q^2)$ is associated with $J^P = 1^-$. Thse form factors are related to helicity form factors $H_0(q^2)$, $H_+(q^2)$ and $H_-(q^2)$, wich correspond to the three allowed polalization state of D^* (one longitudinal and two transverse). A_1 contributes to all three helicity components, A_2 contributes only to the helicity-zeo componemt and V contributes only to the helicity ± 1 componemts

$$\begin{array}{l} H_+(q^2) = (m_B + m_{D^*})A_1(q^2) - \frac{\sqrt{(q^2 - m_B^2 - m_{D^*}^2)^2 - 4m_B^2 m_{D^*}^2}}{m_B + m_{D^*}} V(q^2) \\ H_-(q^2) = (m_B + m_{D^*})A_1(q^2) + \frac{\sqrt{(q^2 - m_B^2 - m_{D^*}^2)^2 - 4m_B^2 m_{D^*}^2}}{m_B + m_{D^*}} V(q^2) \\ H_0(q^2) = \frac{1}{2m_{D^*}\sqrt{q^2}} \left[(m_B^2 - m_{D^*}^2 - q^2)(m_B + m_{D^*})A_1(q^2) - \frac{(q^2 - m_B^2 - m_{D^*}^2)^2 - 4m_B^2 m_{D^*}^2}{m_B + m_{D^*}} A_2(q^2) \right] \end{array}$$

Differential decay rate is given in terms of the helicity form factors,

$$\begin{array}{rcl} \frac{d\Gamma(B \to D^* \ell\nu)}{dq^2 d\cos\theta_V d\cos\theta_\ell d\chi} & = & \frac{3G_F^2}{8(4\pi)^4} |V_{cb}|^2 \frac{p_{D^*} q^2}{m_B^2} [(1 - \cos\theta_\ell)^2 \sin^2\theta_V |H_+(q^2)|^2 \\ & & +(1 + \cos\theta_\ell)^2 \sin^2\theta_V |H_-(q^2)|^2 \\ & & +4\sin^2\theta_\ell \cos^2\theta_V |H_0(q^2)|^2 \\ & & -4\sin\theta_\ell (1 - \cos\theta_\ell) \sin\theta_V \cos\theta_V \cos\chi H_+(q^2) H_0(q^2) \\ & & +4\sin\theta_\ell (1 + \cos\theta_\ell) \sin\theta_V \cos\theta_V \cos\chi H_-(q^2) H_0(q^2) \\ & & -2\sin^2\theta_\ell \sin^2\theta_V \cos 2\chi H_+(q^2) H_-(q^2)] \end{array}$$

3.3 $B \to D_0^* \ell\nu$

In this case

$$< D_0^*(p',\varepsilon)|V^\mu|B(p) >= 0$$

and

$$< D_0^*(p',\varepsilon)|A^\mu|B(p) >= u_+(q^2)(p+p')^\mu + u_-(q^2)(p-p')^\mu$$

3.4 $B \to D_1'\ell\nu$

In this case

$$< D_1'(p',\varepsilon)|V^\mu|B(p) >= \varepsilon^{*\mu} l_{1/2}(q^2) + \varepsilon^* \cdot q(p+p')^\mu c_{1/2+}(q^2) + \varepsilon^* \cdot q(p-p')^\mu c_{1/2-}(q^2)$$

and

$$< D_1'(p',\varepsilon)|A^\mu|B(p) >= i\epsilon^{\mu\nu\alpha\beta}\varepsilon_\nu^*(p+p')_\alpha(p-p')_\beta q_{1/2}(q^2)$$

3.5 $B \to D_1\ell\nu$

In this case

$$< D_1(p',\varepsilon)|V^\mu|B(p) >= \varepsilon^{*\mu} l_{3/2}(q^2) + \varepsilon^* \cdot q(p+p')^\mu c_{3/2+}(q^2) + \varepsilon^* \cdot q(p-p')^\mu c_{3/2-}(q^2)$$

and

$$< D_1(p',\varepsilon)|A^\mu|B(p) >= i\epsilon^{\mu\nu\alpha\beta}\varepsilon_\nu^*(p+p')_\alpha(p-p')_\beta q_{3/2}(q^2)$$

3.6 $B \to D_2^*\ell\nu$

In this case

$$< D_2^*(p',\varepsilon)|V^\mu|B(p) >= i\epsilon^{\mu\nu\alpha\beta}\varepsilon_\nu^*(p+p')_\alpha(p-p')_\beta h(q^2)$$

and

$$< D_2^*(p',\varepsilon)|A^\mu|B(p) >= \varepsilon^{*\mu} k(q^2) + \varepsilon^* \cdot q(p+p')^\mu b_+(q^2) + \varepsilon^* \cdot q(p-p')^\mu b_-(q^2)$$

4 ISGW2 model

The quark-model B and D masses are

$$\begin{aligned} \tilde{m}_B &= m_b + m_u = m_b + m_d \\ \tilde{m}_D &= m_c + m_u = m_c + m_d \end{aligned}$$

wher m_u, m_d, m_c and m_b are u, d, c and b quark masses, respectively. In ISGW2 model,

$$\begin{aligned} m_d &= m_u = 0.33\text{GeV} \\ m_c &= 1.82\text{GeV} \\ m_b &= 5.20\text{GeV} \end{aligned}$$

Using, these quark masses, ISGW2 define

$$\mu_\pm = \left(\frac{1}{m_c} \pm \frac{1}{m_b}\right)^{-1}$$

The hyperfine-averaged physicsl mass of a HQS multiplet with light degrees of freedom having angular momentum j_l

$$\bar{m}_{j_l} = \frac{j_l+1}{2j_l+1} m_{J=j_l+\frac{1}{2}} + \frac{j_l}{2j_l+1} m_{J=j_l-\frac{1}{2}}$$

For example

$$\bar{m}_B = \tfrac{3}{4}m_B + \tfrac{1}{4}m_{B^*} = 5.31$$
$$\bar{m}_D = \bar{m}_{D^*} = \tfrac{3}{4}m_D + \tfrac{1}{4}m_{D^*}$$
$$\bar{m}_{D_0^*} = \bar{m}_{D_1{}'} = \tfrac{3}{4}m_{D_1'} + \tfrac{1}{4}m_{D_0^*}$$
$$\bar{m}_{D_1} = \bar{m}_{D_2^*} = \tfrac{5}{8}m_{D_2^*} + \tfrac{3}{8}m_{D_1}$$

where

$$m_B = 5.27\text{GeV}, \quad m_{B^*} = 5.33\text{GeV}$$
$$m_D = 1.87\text{GeV}, \quad m_{D^*} = 2.01\text{GeV}$$
$$m_{D_0^*} = 2.40\text{GeV}, \quad m_{D_1'} = 2.49\text{GeV}$$
$$m_{D_1} = 2.42\text{GeV}, \quad m_{D_2^*} = 2.46\text{GeV}$$

There are also parameters β in the wave functions, and those values in the ISGW2 model are

$$\beta_B = 0.431\text{GeV}$$
$$\beta_D = 0.45\text{GeV}$$
$$\beta_{D^*} = 0.38\text{GeV}$$
$$\beta_{D^{**}} = 0.33\text{GeV}$$

Also, β_{BX}^2 ($X = D$, D^* or D^{**}) is defined as

$$\beta_{BX}^2 \equiv \frac{1}{2}(\beta_B^2 + \beta_X^2)$$

The momentum transfer is

$$t = (p_B - p_D)^2 = q^2$$

and the maximum momentur transfer is

$$t_m = (m_B - m_D)^2$$

where m_D can be m_{D^*} or $m_{D^{**}}$.
In ISGW2 model, the factor $F_n^{(\alpha)}$ is defined as

$$F_n^{(\alpha)} = \left(\frac{\bar{m}_B}{\tilde{m}_B}\right)^{n_B(\alpha)} \left(\frac{\bar{m}_D}{\tilde{m}_D}\right)^{n_X(\alpha)} \left(\frac{\tilde{m}_D}{\tilde{m}_B}\right)^{1/2} \left(\frac{\beta_B\beta_D}{\beta_{BD}^2}\right)^{n/2} \left[1 + \frac{1}{6N}r^2(t_m - t)\right]^{-N}$$

where $N = 2 + n + n'$. n and n' are the harmonic oscillator quantum numbers of the initial and final wavefunctions. Therefore, $N = 2$ for S wave to S wave ($B \to D$ or D^*), and $N = 3$ for S wave to P wave ($B \to D^{**}$). $n_B(\alpha)$ and $n_X(\alpha)$ is given in the table

Form factors (α)	$n_B(\alpha)$	$n_X(\alpha)$
$f_+ + f_-$	-1/2	+1/2
$f_+ - f_-$	+1/2	-1/2
g	-1/2	-1/2
f	+1/2	+1/2
$a_+ + a_-$	-3/2	+1/2
$a_+ - a_-$	-1/2	-1/2
$u_+ + u_-$	-1/2	+1/2
$u_+ - u_-$	+1/2	-1/2
l	+1/2	+1/2
$c_+ + c_-$	-3/2	+1/2
$c_+ - c_-$	-1/2	-1/2
q	-1/2	-1/2
h	-3/2	-1/2
k	-1/2	+1/2
$b_+ + b_-$	-5/2	+1/2
$b_+ - b_-$	-3/2	-1/2

and r^2 is

$$r^2 = \frac{3}{4m_b m_c} + \frac{3m_d^2}{2\bar{m}_B\bar{m}_D\beta_{BD}^2} + \frac{1}{\bar{m}_B\bar{m}_D}\left(\frac{16}{33-2N_f}\right)\ln\left[\frac{\alpha_s(\mu_{QM})}{\alpha_s(m_c)}\right]$$

where, if a scale $\mu < 0.6$ GeV,

$$\alpha_s(\mu) = 0.6$$

and if $\mu > 0.6$ GeV,

$$\alpha_s(\mu) = \frac{12\pi}{33-2N_f}\left[\ln\left(\frac{\mu^2}{\Lambda_{QCD}^2}\right)\right]^{-1}$$

where N_f is the number of active flavors below the scale μ. For $\mu = m_c$, $N_f = 3$, and for $\mu = m_b$, $N_f = 4$. Thus, $N_f = 3$ for $b \to c$ decays. The constants in the above eqations are

$$\mu_{QM} = 0.1\text{GeV}$$
$$\Lambda_{QCD}^2 = 0.04\text{GeV}^2$$

The factor $R^{(\alpha)}$ is defined as

$$R^{(\alpha)} = C_{cb}\left[f^{(\alpha)} + \tilde{\beta}_{cb}^{(\alpha)}(w)\frac{\alpha_s\left(\sqrt{m_b m_c}\right)}{\pi}\right]$$

where $f^{(\alpha)} = 0$ for $a_+ + a_-$ and $f^{(\alpha)} = 1$ for others.
Here

$$C_{cb} = \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{a_I}\left[\frac{\alpha_s(m_c)}{\alpha_s(\mu_{QM})}\right]^{a_L(w)}$$

is independent of α and has

$$a_I = -\frac{6}{33 - 2N_f}$$

and

$$a_L(w) = -\frac{8}{33 - 2N_f'}[wr(w) - 1]$$

with

$$r(w) = \frac{1}{\sqrt{w^2 - 1}} \ln\left(w + \sqrt{w^2 - 1}\right)$$

and

$$w = 1 + \frac{t_m - t}{2m_B m_D}$$

where m_D may be m_D^* or m_D^{**} depending on the decay modes. N_f is the number of active flavours below the scale m_b and N_f' is the number of active flavours below the scale m_c. Thus, $N_f = 4$ and $N_f' = 3$. In IGWS2, $\tilde{w}$ is defined as

$$\tilde{w} = 1 + \frac{t_m - t}{2\bar{m}_B \bar{m}_D}$$

For w near 1

$$\left[\frac{\alpha_s(m_c)}{\alpha_s(\mu_{QM})}\right]^{a_L(w)} = 1 - \frac{2}{3}\left(\frac{8}{33 - 2N_f'}\right) \ln\left[\frac{\alpha_s(\mu_{QM})}{\alpha_s(m_c)}\right](w - 1)$$

In SP5/SP6, $\left[\frac{\alpha_s(m_c)}{\alpha_s(\mu_{QM})}\right]^{a_L(w)} = 1$ was used.

At $w = 1$, $\tilde{\beta}_{cb}$ are

$$\begin{array}{l}
\tilde{\beta}_{cb}^{(f_+ + f_-)}(1) = \gamma_{cb} - \frac{2}{3}\chi_{cb} \\
\tilde{\beta}_{cb}^{(f_+ - f_-)}(1) = \gamma_{cb} + \frac{2}{3}\chi_{cb} \\
\tilde{\beta}_{cb}^{(g)}(1) = \gamma_{cb} + \frac{2}{3} \\
\tilde{\beta}_{cb}^{(f)}(1) = \gamma_{cb} - \frac{2}{3} \\
\tilde{\beta}_{cb}^{(a_+ + a_-)}(1) = -1 - \chi_{cb} + \frac{4}{3}\frac{1}{(1-z_{cb})} + \frac{2}{3}\frac{1+z_{cb}}{(1-z_{cb})^2}\gamma_{cb} \\
\tilde{\beta}_{cb}^{(a_+ - a_-)}(1) = -\frac{1}{3} - \chi_{cb} + \frac{4}{3}\frac{1}{(1-z_{cb})} + \left[1 - \frac{2}{3}\frac{1+z_{cb}}{(1-z_{cb})^2}\right]\gamma_{cb}
\end{array}$$

where

$$\gamma_{cb} = \frac{2z_{cb}}{1 - z_{cb}} \ln\left(\frac{1}{z_{cb}}\right) - 2$$

and

$$\chi_{cb} = -1 - \frac{\gamma_{cb}}{1 - z_{cb}}$$

with

$$z_{cb} = \frac{m_c}{m_b}$$

w dependence of $\tilde{\beta}$ is weak and negligible.

In ISGW, LL-coupling was used, and ISGW2 also follows the convention. But, ISGW2 aware that jj-coupling is a better representaion of havy-light mesons. Thus, ISGW2 gives jj-coupling type of form factors for D_1 and D_1'.

4.1 $B \to D\ell\nu$ (1^2S_0)

Matrix element is

$$< D(p')|V^\mu|B(p) >= f_+(q^2)(p+p')^\mu + f_-(q^2)(p-p')^\mu$$

The differential decay rate is given by

$$\frac{d\Gamma(B \to D\ell\nu)}{dq^2} = \frac{G_F^2|V_{cb}|^2}{192\pi^3 m_B^3}\left[(q^2-m_B^2-m_D^2)^2 - 4m_B^2 m_D^2\right]^{3/2}|f_+(q^2)|^2$$

where f_+ is the form factor.

$$f_+ + f_- = \left[2 - \frac{\tilde{m}_D}{m_c}\left(1 - \frac{m_d m_c}{2\mu_+ \tilde{m}_D}\frac{\beta_B^2}{\beta_{BD}^2}\right)\right] F_3^{(f_+ + f_-)} R^{(f_+ + f_-)}$$

and

$$f_+ - f_- = \frac{\tilde{m}_B}{m_c}\left(1 - \frac{m_d m_c}{2\mu_+ \tilde{m}_D}\frac{\beta_B^2}{\beta_{BD}^2}\right) F_3^{(f_+ - f_-)} R^{(f_+ - f_-)}$$

Thus, we get f_+

$$f_+ = \frac{(f_+ + f_-) + (f_+ - f_-)}{2}$$

Form factors for the differential decay rate below is given by

$$\alpha = \gamma = 0, \quad \beta_{++} = f_+^2$$

4.2 $B \to D^*\ell\nu$ (1^3S_0)

Matrix element is

$$< D^*(p',\varepsilon)|V^\mu|B(p) >= i\epsilon^{\mu\nu\alpha\beta}\varepsilon_\nu^*(p+p')_\alpha(p-p')_\beta g(q^2)$$

and

$$< D^*(p',\varepsilon)|A^\mu|B(p) >= \varepsilon^{*\mu} f(q^2) + \varepsilon^* \cdot q(p+p')^\mu a_+(q^2) + \varepsilon^* \cdot q(p-p')^\mu a_-(q^2)$$

Form factors are

$$f = C_f \tilde{m}_B\left[(1 + tildew) + \frac{m_d(\tilde{w}-1)}{2\mu_+}\right] F_3^{(f)} R^{(f)}$$

where $C_f = 0.989$ for $B \to D^*$.

$$g = \frac{1}{2}\left[\frac{1}{m_c} - \frac{m_d \beta_B^2}{2\mu_- \tilde{m}_D \beta_{BD}^2}\right] F_3^{(g)} R^{(g)}$$

$$a_+ + a_- = C_{cb}\left[\frac{m_d}{(1+\tilde{w})m_c m_b}\frac{\beta_D^2}{\beta_{BD}^2}\left(1 - \frac{m_d}{2\tilde{m}_B}\frac{\beta_D^2}{\beta_{BD}^2}\right) + \tilde{\beta}^{(a_+ + a_-)}\frac{\alpha_s}{\pi}\tilde{m}_B\right] F_3^{(a_+ + a_-)}$$

$$a_+ - a_- = -\frac{1}{\tilde{m}_D}\left[\frac{\tilde{m}_B}{m_b} - \frac{m_d}{2\mu_+}\frac{\beta_D^2}{\beta_{BD}^2} + \frac{\tilde{w} m_d \tilde{m}_B}{(1+\tilde{w})m_c m_b}\frac{\beta_D^2}{\beta_{BD}^2}\left(1 - \frac{m_d}{2\tilde{m}_B}\frac{\beta_D^2}{\beta_{BD}^2}\right)\right] F_3^{(a_+ - a_-)} R^{(a_+ - a_-)}$$

Form factors for the differential decay rate below is given by

$$\begin{aligned}
&\alpha = f^2 + 4m_B^2 g^2 p_D^2 \\
&\beta_{++} = \tfrac{f^2}{4m_D^2} - m_B^2 g^2 y + \tfrac{1}{2}\left[(1-y)\tfrac{m_B^2}{m_D^2} - 1\right] f a_+ + \tfrac{m_B^2 p_D^2}{m_D^2} a_+^2 \\
&\gamma = 2gf
\end{aligned}$$

where

$$p_D^2 = \frac{[m_B^2(1-y) + m_D^2]^2}{4m_B^2} - m_D^2$$

is the square of the recoil three-momentum of the D.

4.3 $B \to D_0^* \ell \nu$ (1^3P_0)

Matrix elemsnt is

$$< D_0^*(p', \varepsilon)|V^\mu|B(p) >= 0$$

and

$$< D_0^*(p', \varepsilon)|A^\mu|B(p) >= u_+(q^2)(p+p')^\mu + u_-(q^2)(p-p')^\mu$$

Form factors are

$$u_+ + u_- = -\sqrt{\frac{2}{3}}\frac{m_d}{\beta_B} F_5^{(u_+ + u_-)}$$

$$u_+ - u_- = \sqrt{\frac{2}{3}}\frac{m_d \tilde{m}_B}{\beta_B \tilde{m}_D} F_5^{(u_+ - u_-)}$$

Form factors for the differential decay rate below is given by

$$\alpha = \gamma = 0, \quad \beta_{++} = u_+^2$$

4.4 $B \to D_1' \ell \nu$ (1^3P_1)

Matrix elemsnt is

$$< D_1'(p',\varepsilon)|V^\mu|B(p) >= \varepsilon^{*\mu} l(q^2) + \varepsilon^* \cdot q(p+p')^\mu c_+(q^2) + \varepsilon^* \cdot q(p-p')^\mu c_-(q^2)$$

and

$$< D_1'(p',\varepsilon)|A^\mu|B(p) >= i\epsilon^{\mu\nu\alpha\beta}\varepsilon^*_\nu (p+p')_\alpha (p-p')_\beta q(q^2)$$

Form factors are

$$q = -\frac{m_d}{2\tilde{m}_D \beta_B}\left(\frac{5+\tilde{w}}{6}\right) F_5^{(q)}$$

$$l = -\tilde{m}_B \beta_B \left[\frac{1}{\mu_-} + \frac{m_d \tilde{m}_D (\tilde{w}-1)}{\beta_B^2}\left(\frac{5+\tilde{w}}{6m_c} - \frac{1}{2\mu_-}\frac{m_d}{\tilde{m}_D}\frac{\beta_B^2}{\beta_{BD}^2}\right)\right] F_5^{(l)}$$

$$c_+ + c_- = -\frac{m_d \tilde{m}_D}{2m_c \tilde{m}_B \beta_B}\left(1 - \frac{m_d m_c}{2\tilde{m}_D \mu_-}\frac{\beta_B^2}{\beta_{BD}^2}\right) F_5^{(c_+ + c_-)}$$

$$c_+ - c_- = -\frac{m_d \tilde{m}_D}{2m_c \tilde{m}_B \beta_B}\left(\frac{\tilde{w}+2}{3} - \frac{m_d m_c}{2\tilde{m}_D \mu_-}\frac{\beta_B^2}{\beta_{BD}^2}\right) F_5^{(c_+ - c_-)}$$

Form factors for the differential decay rate below is given by

$$\begin{aligned}
&\alpha = l^2 + 4m_B^2 q^2 p_D^2 \\
&\beta_{++} = \tfrac{l^2}{4m_D^2} - m_B^2 q^2 y + \tfrac{1}{2}\left[(1-y)\tfrac{m_B^2}{m_D^2} - 1\right] l c_+ + \tfrac{m_B^2 p_D^2}{m_D^2} c_+^2 \\
&\gamma = 2lq
\end{aligned}$$

4.5 $B \to D_1 \ell \nu$ (1^1P_1)

Matrix elemsnt is Matrix elemsnt is

$$< D_1'(p',\varepsilon)|V^\mu|B(p) >= \varepsilon^{*\mu} r(q^2) + \varepsilon^* \cdot q(p+p')^\mu s_+(q^2) + \varepsilon^* \cdot q(p-p')^\mu s_-(q^2)$$

and

$$< D_1'(p',\varepsilon)|A^\mu|B(p) >= i\epsilon^{\mu\nu\alpha\beta}\varepsilon^*_\nu (p+p')_\alpha (p-p')_\beta v(q^2)$$

Form factors are

$$v = \left[\frac{\tilde{m}_B \beta_B}{4\sqrt{2} m_b m_c \tilde{m}_D} + \frac{(\tilde{w}-1)}{6\sqrt{2}}\frac{m_d}{\tilde{m}_D \beta_B}\right] F_5^{(v)}$$

$$r = \frac{\tilde{m}_B \beta_B}{\sqrt{2}}\left[\frac{1}{\mu_+} + \frac{m_d \tilde{m}_D}{3m_c \beta_B^2}(\tilde{w}-1)^2\right] F_5^{(r)}$$

$$s_+ + s_- = \frac{m_d}{\sqrt{2}\tilde{m}_B \beta_B}\left(1 - \frac{m_d}{m_c} + \frac{m_d}{2\mu_+}\frac{\beta_B^2}{\beta_{BD}^2}\right) F_5^{(s_+ + s_-)}$$

$$s_+ - s_- = \frac{m_d}{\sqrt{2} m_c \beta_B}\left(\frac{4-\tilde{w}}{3} - \frac{m_d m_c}{2\tilde{m}_D \mu_+}\frac{\beta_B^2}{\beta_{BD}^2}\right) F_5^{(s_+ - s_-)}$$

Form factors for the differential decay rate below is given by

$$\begin{aligned}
&\alpha = r^2 + 4m_B^2 v^2 p_D^2 \\
&\beta_{++} = \tfrac{r^2}{4m_D^2} - m_B^2 v^2 y + \tfrac{1}{2}\left[(1-y)\tfrac{m_B^2}{m_D^2} - 1\right] r s_+ + \tfrac{m_B^2 p_D^2}{m_D^2} s_+^2 \\
&\gamma = 2rv
\end{aligned}$$

4.6 $B \to D_2^* \ell\nu$ (1^3P_2)

Matrix elemsnt is

$$< D_2^*(p',\varepsilon)|V^\mu|B(p) >= i\epsilon^{\mu\nu\alpha\beta}\varepsilon_\nu^*(p+p')_\alpha(p-p')_\beta h(q^2)$$

and

$$< D_2^*(p',\varepsilon)|A^\mu|B(p) >= \varepsilon^{*\mu}k(q^2) + \varepsilon^*\cdot q(p+p')^\mu b_+(q^2) + \varepsilon^*\cdot q(p-p')^\mu b_-(q^2)$$

Form factors are

$$h = \frac{m_d}{2\sqrt{2}\tilde{m}_B\beta_B}\left[\frac{1}{m_c} - \frac{m_d\beta_B^2}{2\mu_-\tilde{m}_D\beta_{BD}^2}\right]F_5^{(h)}$$

$$k = \frac{m_d}{\sqrt{2}\beta_B}(1+\tilde{w})F_5^{(k)}$$

$$b_+ + b_- = \frac{m_d^2}{4\sqrt{2}m_c m_b\tilde{m}_B\beta_B}\frac{\beta_D^2}{\beta_{BD}^2}\left(1 - \frac{m_d}{2\tilde{m}_B}\frac{\beta_D^2}{\beta_{BD}^2}\right)F_5^{(b_+ + b_-)}$$

$$b_+ - b_- = -\frac{m_d}{\sqrt{2}m_b\tilde{m}_D\beta_B}\left[1 - \frac{m_d m_b}{2\mu_+\tilde{m}_B}\frac{\beta_D^2}{\beta_{BD}^2} + \frac{m_d}{4m_c}\frac{\beta_D^2}{\beta_{BD}^2}\left(1 - \frac{m_d}{2\tilde{m}_B}\frac{\beta_D^2}{\beta_{BD}^2}\right)\right]F_5^{(b_+ - b_-)}$$

Form factors for the differential decay rate below is given by

$$\begin{aligned}
&\alpha = \tfrac{m_B^2 p_D^2}{2m_D^2}(k^2 + 4m_B^2 h^2 p_D^2) \\
&\beta_{++} = -\tfrac{1}{2}y\tfrac{m_B^4 p_D^2}{m_D^2}h^2 + \tfrac{m_B^2}{24m_D^2}\left(y + \tfrac{4p_D^2}{m_D^2}\right)k^2 + \tfrac{1}{3}\tfrac{m_B^2 p_D^2}{m_D^2}\left[(1-y)\tfrac{m_B^2}{m_D^2} - 1\right]kb_+ + \tfrac{2}{3}\left(\tfrac{m_B^2 p_D^2}{m_D^2}\right)^2 b_+^2 \\
&\gamma = \tfrac{m_B^2 p_D^2}{m_D^2}kh
\end{aligned}$$

4.7 $B \to D_1^* \ell\nu$ ($\frac{1}{2}^+$)

Form factors are

$$q_{1/2} = \sqrt{\frac{1}{6}}\frac{m_d}{\tilde{m}_D\beta_B}\left(1 - \frac{\beta_B^2\tilde{m}_B}{4m_d m_c m_b}\right)F_5^{(q_{1/2})}$$

$$l_{1/2} = \sqrt{\frac{2}{3}}\tilde{m}_B\beta_B\left[\frac{1}{2m_c} - \frac{3}{2m_b} + \frac{m_d\tilde{m}_D(\tilde{w}-1)}{\beta_B^2}\left(\frac{1}{m_c} - \frac{1}{2\mu_-}\frac{m_d}{\tilde{m}_D}\frac{\beta_B^2}{\beta_{BD}^2}\right)\right]F_5^{(l_{1/2})}$$

$$c_{1/2+} + c_{1/2-} = \frac{m_d^2\beta_D^2}{\sqrt{6}m_c\tilde{m}_B\beta_B\beta_{BD}^2}F_5^{(c_{1/2+}+c_{1/2-})}$$

$$c_{1/2+} - c_{1/2-} = -\sqrt{\frac{2}{3}}\frac{m_d}{2\tilde{m}_D\beta_B}\left(1 + \frac{m_d}{2m_c}\frac{\beta_D^2}{\beta_{BD}^2}\right)F_5^{(c_{1/2+}-c_{1/2-})}$$

4.8 $B \to D_1 \ell\nu$ $(\frac{1}{2}^+)$

Form factors are

$$q_{3/2} = -\frac{1}{2\sqrt{3}}\frac{m_d}{\tilde{m}_D\beta_B}\left(\frac{1+\tilde{w}}{2} + \frac{\beta_B^2\tilde{m}_B}{2m_dm_cm_b}\right)F_5^{(q_{3/2})}$$

$$l_{3/2} = -\frac{2}{\sqrt{3}}\tilde{m}_B\beta_B\left[\frac{1}{m_c} + \frac{m_d\tilde{m}_D(\tilde{w}-1)}{2\beta_B^2}\left(\frac{\tilde{w}+1}{2m_c} - \frac{1}{2\mu_-}\frac{m_d}{\tilde{m}_D}\frac{\beta_B^2}{\beta_{BD}^2}\right)\right]F_5^{(l_{3/2})}$$

$$c_{3/2+} + c_{3/2-} = -\frac{\sqrt{3}m_d}{2\tilde{m}_B\beta_B}\left[1 - \frac{m_d}{2m_c} - \frac{m_d}{3}\frac{\beta_B^2}{\beta_{BD}^2}\left(\frac{1}{2\mu_-} - \frac{1}{\mu_+}\right)\right]F_5^{(c_{3/2+}+c_{3/2-})}$$

$$c_{3/2+} - c_{3/2-} = -\frac{m_d}{2\sqrt{3}\tilde{m}_D\beta_B}\left[\frac{(2-\tilde{w})\tilde{m}_D}{m_c} + \frac{m_d\beta_B^2}{\beta_{BD}^2}\left(\frac{1}{2\mu_-} - \frac{1}{\mu_+}\right)\right]F_5^{(c_{3/2+}-c_{3/2-})}$$

4.9 Differential decay rate

Differential decay rate is given by

$$\frac{d^2\Gamma}{dxdy} = \Gamma_0\left[\frac{y}{m_B^2}\alpha + 2\left(2x\left(1+y-\frac{m_D^2}{m_B^2}\right) - 4x^2 - y\right)\beta_{++} - y\left(1-4x+y-\frac{m_D^2}{m_B^2}\right)\gamma\right]$$

where

$$\Gamma_0 = \frac{G_F^2|V_{cb}|^2m_B^5}{32\pi^3}$$

and

$$x = \frac{E_e}{m_B}, \quad y = \frac{t}{m_B^2} = \frac{(p_B-p_D)^2}{m_B^2}$$

5 HQET model

5.1 Velocity transfer w

The velocity transfer is related to q^2

$$w = \frac{M_B^2 + M_D^2 - q^2}{2M_BM_D} = v_B \cdot v_D = \frac{p_B \cdot p_D}{M_BM_D}$$

where, D may be D^* or D^{**}. In the context of HQET, it is more natural to use w instead of q^2

5.2 $B \to D\ell\nu$

The differential decay rate is given by

$$\frac{d\Gamma(B\to D\ell\nu)}{dq^2} = \frac{G_F^2|V_{cb}|^2}{192\pi^3m_B^3}\left[(q^2-m_B^2-m_D^2)^2 - 4m_B^2m_D^2\right]^{3/2}|F_1(q^2)|^2$$

where F_1 is the form factor. In HQET, the form factor is related to heavy quark form factors $h_+(w)$ and $h_-(w)$

$$RF_1(q^2) = h_+(w) - \frac{m_B - m_D}{m_B + m_D} h_-(w)$$

where R is

$$R = \frac{\sqrt{m_B M_D}}{m_B + m_D} \simeq 0.88$$

Hence

$$\frac{d\Gamma(B \to D\ell\nu)}{dw} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} (m_B + m_D)^2 m_D^3 (w^2 - 1)^{3/2} \left| h_+(w) - \frac{m_B - m_D}{m_B + m_D} h_-(w) \right|^2$$

In haevy quark limit, the form factors $h_+(w)$ and $h_-(w)$ are

$$h_+(w) = \xi(w), \;\; h_-(w) = 0$$

where $\xi(w)$ is the Isgur-Wise Function.

$$\xi(w) = 1 - \rho_D^2 (w - 1)$$

5.3 $B \to D^* \ell\nu$

The differential decay rate is given by

$$\begin{aligned}
\frac{d\Gamma(B \to D^* \ell\nu)}{dw d\cos\theta_V d\cos\theta_\ell d\chi} \;\; = \;\; & \frac{3G_F^2}{4(4\pi)^4} |V_{cb}|^2 m_B m_{D^*}^2 r \sqrt{w^2 - 1} (1 - 2wr + r^2) \times \\
& [(1 - \cos\theta_\ell)^2 \sin^2\theta_V |H_+(w)|^2 \\
& + (1 + \cos\theta_\ell)^2 \sin^2\theta_V |H_-(w)|^2 \\
& + 4\sin^2\theta_\ell \cos^2\theta_V |H_0(w)|^2 \\
& - 4\sin\theta_\ell (1 - \cos\theta_\ell) \sin\theta_V \cos\theta_V \cos\chi H_+(w) H_0(w) \\
& + 4\sin\theta_\ell (1 + \cos\theta_\ell) \sin\theta_V \cos\theta_V \cos\chi H_-(w) H_0(w) \\
& - 2\sin^2\theta_\ell \sin^2\theta_V \cos 2\chi H_+(w) H_-(w)]
\end{aligned}$$

where r is the mass ratio

$$r = \frac{m_{D^*}}{m_B}$$

$H_i(w)$ are

$$H_i = -m_B \frac{R^*(1 - r^2)(w + 1)}{2\sqrt{1 - 2wr + r^2}} h_{A_1}(w) \widetilde{H}_i(w)$$

where R^* is

$$R^* = \frac{2\sqrt{m_B m_{D^*}}}{m_B + m_D}$$

$h_{A_1}(w)$ is

$$h_{A_1}(w) = h_{A_1}(0) \left(1 - \rho^2 (w - 1)\right)$$

where ρ^2 is the form factor slope, and $\widetilde{H}_i(w)$ are

$$\begin{aligned}
\widetilde{H}_\pm(w) &= \tfrac{\sqrt{1 - 2wr + r^2}}{1 - r} \left(1 \mp \sqrt{\tfrac{w - 1}{w + 1}} R_1(w)\right) \\
\widetilde{H}_0(w) &= 1 + \tfrac{w - 1}{1 - r} (1 - R_2(w))
\end{aligned}$$

where R_1 and R_2 are the form factor ratios.

6 LLSW model

Kinematic variables are w and θ. This *teta* is the angle between the charged lepton and the charmed meson in the rest frame of the virtual W boson. Thus $\theta = \pi - \chi$,

$$\cos\theta = \cos(\pi - \chi) = -\cos\chi$$

6.1 $B \to D_1 \ell \nu$

The matrix elements are

$$\frac{< D_1(v', \varepsilon)|V^\mu|B(v) >}{\sqrt{m_{D_1} m_B}} = f_{V_1} \varepsilon^{*\mu} + (f_{V_2} v^\mu + f_{V_3} v'^\mu)(\varepsilon^* \cdot v)$$

$$\frac{< D_1(v', \varepsilon)|A^\mu|B(v) >}{\sqrt{m_{D_1} m_B}} = i f_A \epsilon^{\mu\alpha\beta\gamma} \varepsilon^*_\alpha v_\beta v'_\gamma$$

The differential decay rate is

$$\begin{aligned}
\tfrac{d^2\Gamma_{D_1}}{dw d\cos\theta} &= 3\Gamma_0 r_1^3 \sqrt{w^2-1}\{\sin^2\theta[(w-r_1)f_{V_1} + (w^2-1)(f_{V_3} + r_1 f_{V_2})]^2 \\
&+(1-2r_1 w + r_1^2)[(1+\cos^2\theta)(f_{V_1}^2 + (w^2-1)f_A^2) - 4\cos\theta\sqrt{w^2-1} f_{V_1} f_A]\}
\end{aligned}$$

where

$$\Gamma_0 = \frac{G_F^2 |V_{cb}|^2 m_B^5}{192\pi^3}$$

and

$$r_1 = \frac{m_{D_1}}{m_B}$$

The form factors are

$$\begin{aligned}
\sqrt{6} f_A &= -(w+1)\tau - \varepsilon_b\{(w-1)[(\bar\Lambda' + \bar\Lambda)\tau - (2w+1)\tau_1 - \tau_2] + (w+1)\eta_b\} \\
&-\varepsilon_c[4(w\bar\Lambda' - \bar\Lambda)\tau - 3(w-1)(\tau_1 - \tau_2) + (w+1)(\eta_{\rm ke} - 2\eta_1 - 3\eta_3)] \\
\sqrt{6} f_{V_1} &= (1-w^2)\tau - \varepsilon_b(w^2-1)[(\bar\Lambda' + \bar\Lambda)\tau - (2w+1)\tau_1 - \tau_2 + \eta_b] \\
&-\varepsilon_c[4(w+1)(w\bar\Lambda' - \bar\Lambda)\tau - (w^2-1)(3\tau_1 - 3\tau_2 - \eta_{\rm ke} + 2\eta_1 + 3\eta_3)] \\
\sqrt{6} f_{V_2} &= -3\tau - 3\varepsilon_b[(\bar\Lambda' + \bar\Lambda)\tau - (2w+1)\tau_1 - \tau_2 + \eta_b] \\
&-\varepsilon_c[(4w-1)\tau_1 + 5\tau_2 + 3\eta_{\rm ke} + 10\eta_1 + 4(w-1)\eta_2 - 5\eta_3)] \\
\sqrt{6} f_{V_3} &= (w-2)\tau + \varepsilon_b\{(2+w)[(\bar\Lambda' + \bar\Lambda)\tau - (2w+1)\tau_1 - \tau_2] - (2-w)\eta_b\} \\
&+\varepsilon_c[4(w\bar\Lambda' - \bar\Lambda)\tau + (2+w)\tau_1 + (2+3w)\tau_2 \\
&+(w-2)\eta_{\rm ke} - 2(6+w)\eta_1 - 4(w-1)\eta_2 - (3w-2)\eta_3]
\end{aligned}$$

The differential decay rate can be expanded in powers of $w-1$

$$\begin{aligned}
\tfrac{d^2\Gamma_{D_1}}{dw d\cos\theta} &= \Gamma_0 \tau^2(1) r_1^3 \sqrt{w^2-1} \textstyle\sum_n (w-1)^n \\
&\{\sin^2\theta s_1^{(n)} + (1-2r_1 w + r_1^2)[(1+\cos^2\theta)t_1^{(n)} - 4\cos\theta\sqrt{w^2-1} u_1^{(n)}]\}
\end{aligned}$$

where

$$\begin{aligned}
s_1^{(0)} &= 32\varepsilon_c^2(1-r_1)^2(\bar\Lambda'-\bar\Lambda)^2+...\\
s_1^{(1)} &= 32\varepsilon_c(1-r_1^2)(\bar\Lambda'-\bar\Lambda)+...\\
s_1^{(2)} &= 8(1+r_1)^2+...\\
t_1^{(0)} &= 32\varepsilon_c^2(\bar\Lambda'-\bar\Lambda)^2+...\\
t_1^{(1)} &= 4+8\varepsilon_c[4(\bar\Lambda'-\bar\Lambda)+\hat\eta_{\rm ke}-2\hat\eta_1-3\hat\eta_3]+8\varepsilon_b\hat\eta_b+...\\
t_1^{(2)} &= 8(1+\hat\tau')+...\\
u_1^{(0)} &= 8\varepsilon_c(\bar\Lambda'-\bar\Lambda)+...\\
u_1^{(1)} &= 2+...
\end{aligned}$$

where

$$\varepsilon_c=\frac{1}{2m_c},\qquad \varepsilon_b=\frac{1}{2m_b}$$

and

$$\hat\eta_i=\frac{\eta_i}{\tau(1)},\qquad \tau'=\frac{d\tau}{dt},\qquad \hat{\tau}'=\frac{\tau'}{\tau(1)}$$

are all evaluated at $w=1$.

$$\begin{aligned}
&\bar\Lambda'-\bar\Lambda=0.39\text{GeV}\\
&\bar\Lambda=0.4\text{GeV}\\
&m_c=1.4\text{GeV}\\
&m_b=4.8\text{GeV}\\
&\tau_B=1.6\text{ps}\\
&\hat{\tau}'=-1.5\\
&|V_{cb}|^2 0.04
\end{aligned}$$

6.2 $B\to D_2^*\ell\nu$

The matrix elements are

$$\frac{<D_2^*(v',\varepsilon)|V^\mu|B(v)>}{\sqrt{m_{D_2^*}m_B}}=ik_V\epsilon^{\mu\alpha\beta\gamma}\varepsilon^*_{\alpha\sigma}v^\sigma v_\beta v'_\gamma$$

$$\frac{<D_2^*(v',\varepsilon)|A^\mu|B(v)>}{\sqrt{m_{D_2^*}m_B}}=k_{A_1}\varepsilon^{*\mu\alpha}v_\alpha+(k_{A_2}v^\mu+k_{A_3}v'^\mu)\varepsilon^*_{\alpha\beta}v^\alpha v^\beta$$

The differential decay rate is

$$\begin{aligned}
\frac{d^2\Gamma_{D_2^*}}{dw\,d\cos\theta} &= \tfrac{3}{2}\Gamma_0 r_2^3(w^2-1)^{3/2}\{\tfrac{4}{3}\sin^2\theta[(w-r_2)k_{A_1}+(w^2-1)(k_{A_3}+r_2k_{A_2})]^2\\
&+(1-2r_2w+r_2^2)[(1+\cos^2\theta)(k_{A_1}^2+(w^2-1)k_V^2)-4\cos\theta\sqrt{w^2-1}k_{A_1}k_V]\}
\end{aligned}$$

where

$$r_2=\frac{m_{D_2^*}}{m_B}$$

Form factors are

$$\begin{aligned}
k_V &= -\tau - \varepsilon_b[(\bar\Lambda' + \bar\Lambda)\tau - (2w+1)\tau_1 - \tau_2 + \eta_b] - \varepsilon_c(\tau_1 - \tau_2 + \eta_{\rm ke} - 2\eta_1 + \eta_3) \\
k_{A_1} &= -(1+w)\tau - \varepsilon_b\{(w-1)[(\bar\Lambda' + \bar\Lambda)\tau - (2w+1)\tau_1 - \tau_2] + (1+w)\eta_b\} \\
&\quad -\varepsilon_c[(w-1)(\tau_1 - \tau_2) + (w+1)(\eta_{\rm ke} - 2\eta_1 + \eta_3)] \\
k_{A_2} &= -2\varepsilon_c(\tau_1 + \eta_2) \\
k_{A_3} &= \tau + \varepsilon_b[(\bar\Lambda' + \bar\Lambda)\tau - (2w+1)\tau_1 - \tau_2 + \eta_b] - \varepsilon_c(\tau_1 + \tau_2 - \eta_{\rm ke} + 2\eta_1 - 2\eta_2 - \eta_3)
\end{aligned}$$

The differential decay rate can be expanded in powers of $w-1$

$$\begin{aligned}
\frac{d^2\Gamma_{D_2^*}}{dw\,d\cos\theta} &= \tfrac{3}{2}\Gamma_0\tau^2(1)r_2^3(w^2-1)^{3/2}\textstyle\sum_n(w-1)^n \\
&\quad \{\tfrac{4}{3}\sin^2\theta s_2^{(n)} + (1-2r_2w+r_2^2)[(1+\cos^2\theta)t_2^{(n)} - 4\cos\theta\sqrt{w^2-1}u_2^{(n)}]\}
\end{aligned}$$

where

$$\begin{aligned}
s_2^{(0)} &= 4(1-r_2)^2[1 + 2\varepsilon_b\hat\eta_b + 2\varepsilon_c(\hat\eta_{\rm ke} - 2\hat\eta_1 + \hat\eta_3)] + ... \\
s_2^{(1)} &= 4(1-r_2)^2(1+2\hat\tau') + ... \\
t_2^{(0)} &= 4 + 8\varepsilon_b\hat\eta_b + 8\varepsilon_c(\hat\eta_{\rm ke} - 2\hat\eta_1 + \hat\eta_3) + ... \\
t_2^{(1)} &= 2(3+4\hat\tau') + ... \\
u_2^{(0)} &= 2 + ...
\end{aligned}$$

6.3 $B \to D_0^*\ell\nu$

The matrix elements are

$$\frac{< D_0^*(v',\varepsilon)|V^\mu|B(v) >}{\sqrt{m_{D_0^*}m_B}} = 0$$

$$\frac{< D_0^*(v',\varepsilon)|A^\mu|B(v) >}{\sqrt{m_{D_0^*}m_B}} = g_+(v^\mu + v'^\mu) + g_-(v^\mu - v'^\mu)$$

The differential decay rate is

$$\frac{d^2\Gamma_{D_0^*}}{dw\,d\cos\theta} = 3\Gamma_0 r_0^{*3}(w^2-1)^{3/2}\sin^2\theta[(1+r_0^*)g_+ - (1-r_0^*)g_-]^2$$

where

$$r_0^* = \frac{m_{D_0^*}}{m_B}$$

Form factors are

$$\begin{aligned}
g_+ &= \varepsilon_c\left[2(w-1)\zeta_1 - 3\zeta\tfrac{w\bar\Lambda^* - \bar\Lambda}{w+1}\right] - \varepsilon_b\left[\tfrac{\bar\Lambda^*(2w+1) - \bar\Lambda(w+2)}{w+1}\zeta - 2(w-1)\zeta_1\right] \\
g_- &= \zeta + \varepsilon_c\left[\chi_{\rm ke} + 6\chi_1 - 2(w+1)\chi_2\right] + \varepsilon_b\chi_b
\end{aligned}$$

The differential decay rate can be expanded in powers of $w-1$

$$\frac{d^2\Gamma_{D_0^*}}{dw\,d\cos\theta} = 3\Gamma_0\zeta^2(1)r_0^{*3}(w^2-1)^{3/2}\sin^2\theta\sum_n(w-1)^n s_0^{(n)}$$

where where

$$\begin{aligned}
s_0^{(0)} &= (1-r_0^*)^2[1 + 2\varepsilon_c(\hat\chi_{\rm ke} + 6\hat\chi_1 - 4\hat\chi_2) + 4\varepsilon_b\hat\chi_b] + 3(\varepsilon_c + \varepsilon_b)(1 - r_0^{*2})(\bar\Lambda^* - \bar\Lambda) + ... \\
s_0^{(1)} &= 2(1-r_0^*)^2\hat\zeta' + ...
\end{aligned}$$

6.4 $B \to D_1^* \ell \nu$

The matrix elements are

$$\frac{< D_1^*(v',\varepsilon)|V^\mu|B(v) >}{\sqrt{m_{D_1^*} m_B}} = g_{V_1}\varepsilon^{*\mu} + (g_{V_2}v^\mu + g_{V_3}v'^\mu)(\varepsilon^* \cdot v)$$

$$\frac{< D_1^*(v',\varepsilon)|A^\mu|B(v) >}{\sqrt{m_{D_1^*} m_B}} = i g_A \epsilon^{\mu\alpha\beta\gamma}\varepsilon^*_\alpha v_\beta v'_\gamma$$

The differential decay rate is

$$\begin{aligned}\frac{d^2\Gamma_{D_1^*}}{dw d\cos\theta} &= 3\Gamma_0 r_1^{*3}\sqrt{w^2-1}\{\sin^2\theta[(w-r_1^*)g_{V_1} + (w^2-1)(g_{V_3}+r_1^* g_{V_2})]^2 \\ &\quad +(1-2r_1^* w + r_1^{*2})[(1+\cos^2\theta)(g_{V_1}^2 + (w^2-1)g_A^2) - 4\cos\theta\sqrt{w^2-1}g_{V_1}g_A]\}\end{aligned}$$

where

$$r_1^* = \frac{m_{D_1^*}}{m_B}$$

Form factors are

$$\begin{aligned}g_A &= \zeta + \varepsilon_c\left[\tfrac{w\bar\Lambda^*-\bar\Lambda}{w+1}\zeta + \chi_{\rm ke} - 2\chi_1\right] - \varepsilon_b\left[\tfrac{\bar\Lambda^*(2w+1)-\bar\Lambda(w+2)}{w+1}\zeta - 2(w-1)\zeta_1 - \chi_b\right] \\ g_{V_1} &= (w-1)\zeta + \varepsilon_c\left[(w\bar\Lambda^* - \bar\Lambda)\zeta + (w-1)(\chi_{\rm ke} - 2\chi_1)\right] \\ &\quad -\varepsilon_b\left[(\bar\Lambda^*(2w+1) - \bar\Lambda(w+2))\zeta - 2(w^2-1)\zeta_1 - (w-1)\chi_b\right] \\ g_{V_2} &= 2\varepsilon_c(\zeta_1 - \chi_2) \\ g_{V_3} &= -\zeta - \varepsilon_c\left[\tfrac{w\bar\Lambda^*-\bar\Lambda}{w+1}\zeta + 2\zeta_1 + \chi_{\rm ke} - 2\chi_1 + 2\chi_2\right] + \varepsilon_b\left[\tfrac{\bar\Lambda^*(2w+1)-\bar\Lambda(w+2)}{w+1}\zeta - 2(w-1)\zeta_1 - \chi_b\right]\end{aligned}$$

The differential decay rate can be expanded in powers of $w-1$

$$\begin{aligned}\frac{d^2\Gamma_{D_1^*}}{dw d\cos\theta} &= 3\Gamma_0\zeta^2(1) r_1^{*3}\sqrt{w^2-1}\textstyle\sum_n (w-1)^n \\ &\quad \{\sin^2\theta s_1^{(n)} + (1-2r_1^* w + r_1^{*2})[(1+\cos^2\theta)t_1^{(n)} - 4\cos\theta\sqrt{w^2-1}u_1^{(n)}]\}\end{aligned}$$

where

$$\begin{aligned}s_1^{(0)} &= (\varepsilon_c - 3\varepsilon_b)^2(1-r_1^*)^2(\bar\Lambda^* - \bar\Lambda)^2 + ... \\ s_1^{(1)} &= -2(\varepsilon_c - 3\varepsilon_b)(1-r_1^{*2})(\bar\Lambda^* - \bar\Lambda) + ... \\ s_1^{(2)} &= (1+r_1^{*2}) + ... \\ t_1^{(0)} &= (\varepsilon_c - 3\varepsilon_b)^2(\bar\Lambda^* - \bar\Lambda)^2 + ... \\ t_1^{(1)} &= 2 + 4(\varepsilon_c - 3\varepsilon_b)(\bar\Lambda^* - \bar\Lambda) + 4\varepsilon_c(\hat\chi_{\rm ke} - 2\hat\chi_1) + 4\varepsilon_b\hat\chi_b + ... \\ t_1^{(2)} &= 2(1+2\hat\zeta') + ... \\ u_1^{(0)} &= (\varepsilon_c - 3\varepsilon_b)(\bar\Lambda^* - \bar\Lambda) + ... \\ u_1^{(1)} &= 1 + ...\end{aligned}$$

where

$$\zeta(1) = \frac{2}{\sqrt{3}}\tau(1), \quad \hat\zeta' = \frac{1}{2} + \hat\tau'$$

6.5 Differential decay rate using x, y

In ISGW2 model differential decay rate fo the form $\frac{d^2\Gamma}{dxdy}$ is used. Here $x = \frac{E_e}{m_B}$ and $y = \frac{q^2}{m_B^2}$. Using

$$w = \frac{m_B^2 + m_D^2 - q^2}{2m_B m_D} = \frac{m_B^2 + m_D^2 - m_B^2 y}{2m_B m_D}$$

The rescaled lepton energy is given by

$$2x = \frac{2E_e}{m_B} = 1 - rw - r\sqrt{w^2-1}\cos\theta$$

Hence

$$\frac{\partial x}{\partial w} = -\frac{r}{2}\left(1 - \frac{2w}{\sqrt{w^2-1}}\right), \qquad \frac{\partial y}{\partial w} = -\frac{2m_D}{m_B} = -2r$$

and

$$\frac{\partial x}{\partial \cos\theta} = -\frac{r}{2}\sqrt{w^2-1}, \qquad \frac{\partial y}{\partial \cos\theta} = 0$$

Thus

$$dxdy = \left(\frac{\partial x}{\partial w}\frac{\partial y}{\partial \cos\theta} - \frac{\partial y}{\partial w}\frac{\partial x}{\partial \cos\theta}\right) dw d\cos\theta$$

$$\Rightarrow \qquad dxdy = -r^2\sqrt{w^2-1} dw d\cos\theta$$

7 GoityRoberts model

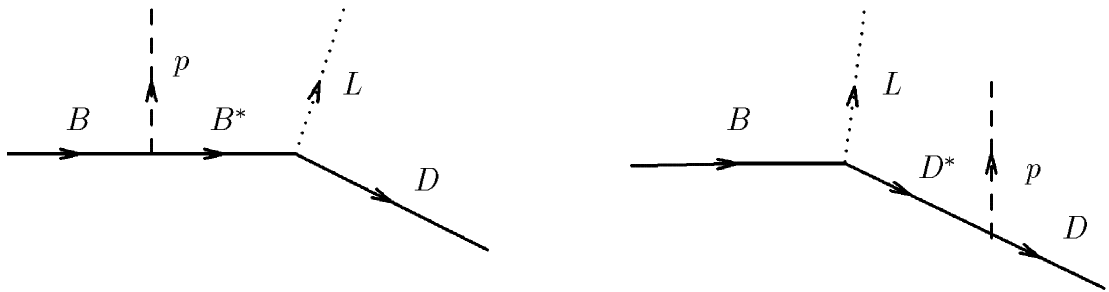
In GoiryRoberts model, the "soft-pion" domain was studied in the framework of the chiral Lagrangian approach combined with the $1/m_Q$ expansion.

7.1 $B \to D\pi\ell\nu$

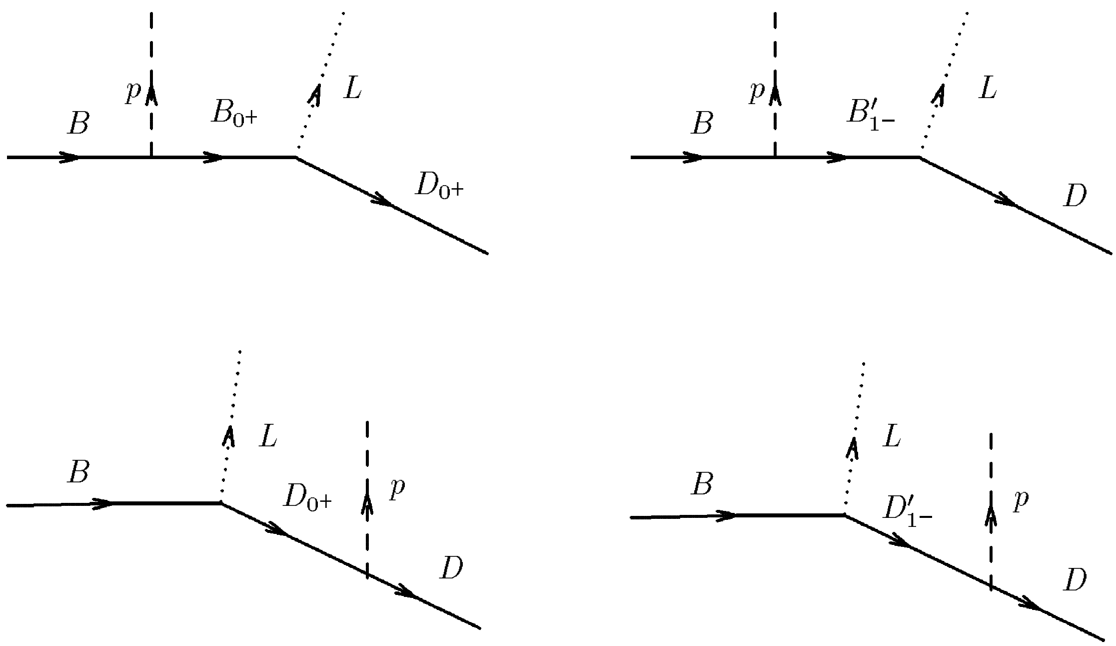
The amplitude for this process has the general form

$$T = \kappa j_\mu \Omega^\mu$$

Here j_μ is the V-A charged leptonic current and the Ω^μ is the hadronic part of the amplitude. To evaluate Ω^μ, these six diagrams are considered to get the amplitude (matrix element)



(a)



(b)

(a) shows two non-resonant decays , (b) shows four resonalt decays. The non-resonant portion of Ω^μ is

$$\begin{aligned}\Omega_\mu^{NR} \;\; = \;\; & \tfrac{g}{F_0}\xi(\nu)p_\nu\Big\{\tfrac{\Theta^{\nu\rho}(v)}{-2(p\cdot v-\delta m_B)+i\varepsilon}\left[iv^\alpha v'^\beta\epsilon_{\mu\alpha\beta\rho}+g_{\mu\rho}(1+\nu)-v_\mu v'_\rho\right] \\ & +\tfrac{\Theta^{\nu\rho}(v')}{2(p\cdot v'-\delta m_D)+i\varepsilon}\left[iv^\alpha v'^\beta\epsilon_{\mu\alpha\beta\rho}+g_{\mu\rho}(1+\nu)-v_\rho v'_\mu\right]\Big\}\end{aligned}$$

where $\delta m_D = m_{D^*} - m_D$ and $\delta m_B = m_{B^*} - m_B$, and

$$\Theta^{\mu\nu}(v) = g^{\mu\nu} - v^\mu v^\nu$$

The resonant portion of Ω^μ is

$$\begin{aligned}
\Omega_\mu^R \;\; = \;\; & \tfrac{\alpha_1}{F_0}\rho_1(\nu)(v-v')_\mu\left[-\tfrac{p\cdot v}{2(-p\cdot v-\delta\tilde{m}_1)}+\tfrac{p\cdot v'}{2(p\cdot v'-\delta\tilde{m}_1)}\right] \\
& +\tfrac{\alpha_2}{3F_0}\rho_2(\nu)p_\rho\Big\{\tfrac{\Theta^{\nu\rho}(v)}{2(-p\cdot v-\delta\tilde{m}_2)}\left[i\epsilon_{\mu\nu\alpha\beta}v^\alpha v'^\beta(\nu-1)+g_{\mu\nu}(\nu^2-1)-v'_\nu\left(v_\mu(2+\nu)-3v'_\mu\right)\right] \\
& +\tfrac{\Theta^{\nu\rho}(v')}{2(p\cdot v'-\delta\tilde{m}_2)}\left[i\epsilon_{\mu\nu\alpha\beta}v^\alpha v'^\beta(\nu-1)+g_{\mu\nu}(\nu^2-1)-v_\nu\left(v'_\mu(2+\nu)-3v_\mu\right)\right]\Big\} \\
& +\tfrac{\alpha_3}{F_0}\xi^{(1)}(\nu)p_\nu\Big\{\tfrac{\Theta^{\nu\rho}(v)}{-2p\cdot v-\delta\tilde{m}_3}\left[i\epsilon_{\mu\alpha\beta\rho}v^\alpha v'^\beta+g_{\mu\rho}(\nu+1)-v_\mu v'_\rho\right] \\
& +\tfrac{\Theta^{\nu\rho}(v')}{2p\cdot v'-\delta\tilde{m}_3}\left[i\epsilon_{\mu\alpha\beta\rho}v^\alpha v'^\beta+g_{\mu\rho}(\nu+1)-v_\rho v'_\mu\right]\Big\}
\end{aligned}$$

Here we denote $\delta\tilde{m}-j=\delta m_j-i\Gamma_j/2$, where Γ_j is the total width of the resonance. Contribution from other resonances are small. Isgur-Wise functions are given by

$$\begin{aligned}
&\xi(\nu)=\exp\left[\tfrac{\bar{\Lambda}^2}{4\beta^2}(\nu^2-1)\right] \\
&\xi^{(1)}(\nu)=-\sqrt{\tfrac{2}{3}}\left[\tfrac{\bar{\Lambda}^2}{4\beta^2}(\nu^2-1)\right]\exp\left[\tfrac{\bar{\Lambda}^2}{4\beta^2}(\nu^2-1)\right] \\
&\rho_1(\nu)=\tfrac{1}{\sqrt{2}}\tfrac{\bar{\Lambda}}{\beta}\left(\tfrac{2\beta\beta'}{\beta^2+\beta'^2}\right)^{5/2}\exp\left[\tfrac{\bar{\Lambda}^2}{2(\beta^2+\beta'^2)}(\nu^2-1)\right] \\
&\rho_2(\nu)=\tfrac{1}{2\sqrt{2}}\left(\tfrac{\bar{\Lambda}}{\beta}\right)^2\left(\tfrac{2\beta\beta'}{\beta^2+\beta'^2}\right)^{7/2}\exp\left[\tfrac{\bar{\Lambda}^2}{2(\beta^2+\beta'^2)}(\nu^2-1)\right]
\end{aligned}$$

wehre β and β' are the harmonic oscillator parameters of the ground and excited states, respectively. $\bar{\Lambda}$ is defined by

$$M_{(0^-,1^-)}=m_Q+\bar{\Lambda}$$

Ω_μ can be, in general, expressed as

$$\Omega_\mu=-ihm_Bm_D\epsilon_{\mu\nu\rho\sigma}v^\nu v'^\rho p_\pi^\sigma+A_1p_{\pi\mu}+A_2m_Bv_\mu+A_3m_Dv'_{mu}$$

where h, A_1, A_2 and A_3 are the form factors and p_π is the 4-momentum of the pion. The non-resonant contribution to the fromm factors are

$$\begin{aligned}
&h_{NR}=\tfrac{g}{2F_0}\tfrac{\xi(\nu)}{m_Bm_D}\left(\tfrac{1}{p_\pi\cdot v+\delta m_B-i\varepsilon}-\tfrac{1}{p_\pi\cdot v'-\delta m_D+i\varepsilon}\right) \\
&A_{1NR}=-\tfrac{g}{2F_0}\xi(\nu)(1+\nu)\left(\tfrac{1}{p_\pi\cdot v+\delta m_B-i\varepsilon}-\tfrac{1}{p_\pi\cdot v'-\delta m_D+i\varepsilon}\right) \\
&A_{2NR}=\tfrac{g}{2F_0}\tfrac{\xi(\nu)}{m_B}\left(\tfrac{p_\pi\cdot v+p_\pi\cdot v'}{p_\pi\cdot v+\delta m_B-i\varepsilon}\right) \\
&A_{3NR}=-\tfrac{g}{2F_0}\tfrac{\xi(\nu)}{m_D}\left(\tfrac{p_\pi\cdot v+p_\pi\cdot v'}{p_\pi\cdot v'-\delta m_D+i\varepsilon}\right)
\end{aligned}$$

In SP5/SP6, the contribution from $B\to D^*\ell\nu\to D\pi\ell\nu$ was removed. Hence

$$\begin{aligned}
&h_{NR}=\tfrac{g}{2F_0}\tfrac{\xi(\nu)}{m_Bm_D}\left(\tfrac{1}{p_\pi\cdot v+\delta m_B-i\varepsilon}\right) \\
&A_{1NR}=-\tfrac{g}{2F_0}\xi(\nu)(1+\nu)\left(\tfrac{1}{p_\pi\cdot v+\delta m_B-i\varepsilon}\right) \\
&A_{2NR}=\tfrac{g}{2F_0}\tfrac{\xi(\nu)}{m_B}\left(\tfrac{p_\pi\cdot v+p_\pi\cdot v'}{p_\pi\cdot v+\delta m_B-i\varepsilon}\right) \\
&A_{3NR}=0
\end{aligned}$$

The resonant contribution to the form factors are

$$
\begin{aligned}
h_R &= \frac{\alpha_2\rho_2(\nu)}{6F_0 m_B m_D}(\nu-1)\left(\frac{1}{p_\pi\cdot v+\delta\tilde{m}_2}-\frac{1}{p_\pi\cdot v'-\delta\tilde{m}_2}\right)+\frac{\alpha_3\xi^{(1)}(\nu)}{2F_0 m_B m_D}\left(\frac{1}{p_\pi\cdot v+\delta\tilde{m}_3}-\frac{1}{p_\pi\cdot v'-\delta\tilde{m}_3}\right)\\
A_{1R} &= -\frac{\alpha_2\rho_2(\nu)}{6F_0}(\nu^2-1)\left(\frac{1}{p_\pi\cdot v+\delta\tilde{m}_2}-\frac{1}{p_\pi\cdot v'-\delta\tilde{m}_2}\right)-\frac{\alpha_3\xi^{(1)}(\nu)}{2F_0}(1+\nu)\left(\frac{1}{p_\pi\cdot v+\delta\tilde{m}_3}-\frac{1}{p_\pi\cdot v'-\delta\tilde{m}_3}\right)\\
A_{2R} &= \frac{\alpha_1\rho_1(\nu)}{2F_0 m_B}\left(\frac{p_\pi\cdot v}{p_\pi\cdot v+\delta\tilde{m}_1}+\frac{p_\pi\cdot v'}{p_\pi\cdot v'-\delta\tilde{m}_2}\right)\\
&\quad+\frac{\alpha_2\rho_2(\nu)}{F_0 m_B}\left\{\frac{1}{p_\pi\cdot v+\delta\tilde{m}_2}\left[\frac{1}{6}(\nu p_\pi\cdot v'-p_\pi\cdot v)+\frac{1}{3}(p_\pi\cdot v'-\nu p_\pi\cdot v)\right]+\frac{1}{2}\frac{1}{p_\pi\cdot v'-\delta\tilde{m}_2}(p_\pi\cdot v-\nu p_\pi\cdot v')\right\}\\
&\quad+\frac{\alpha_3\xi^{(1)}(\nu)}{2F_0 m_B}\left(\frac{p_\pi\cdot(v+v')}{p_\pi\cdot v+\delta\tilde{m}_3}\right)\\
A_{3R} &= -\frac{\alpha_1\rho_1(\nu)}{2F_0 m_D}\left(\frac{p_\pi\cdot v}{p_\pi\cdot v+\delta\tilde{m}_1}+\frac{p_\pi\cdot v'}{p_\pi\cdot v'-\delta\tilde{m}_1}\right)\\
&\quad-\frac{\alpha_2\rho_2(\nu)}{F_0 m_D}\left\{\frac{1}{p_\pi\cdot v'-\delta\tilde{m}_2}\left[\frac{1}{6}(\nu p_\pi\cdot v-p_\pi\cdot v')+\frac{1}{3}(p_\pi\cdot v-\nu p_\pi\cdot v')\right]+\frac{1}{2}\frac{1}{p_\pi\cdot v+\delta\tilde{m}_2}(p_\pi\cdot v'-\nu p_\pi\cdot v)\right\}\\
&\quad-\frac{\alpha_3\xi^{(1)}(\nu)}{2F_0 m_D}\left(\frac{p_\pi\cdot(v+v')}{p_\pi\cdot v'-\delta\tilde{m}_3}\right)
\end{aligned}
$$

In SP5/SP6 contributios from D^{**} was removed

$$
\begin{aligned}
h_R &= \frac{\alpha_2\rho_2(\nu)}{6F_0 m_B m_D}(\nu-1)\left(\frac{1}{p_\pi\cdot v+\delta\tilde{m}_2}\right)+\frac{\alpha_3\xi^{(1)}(\nu)}{2F_0 m_B m_D}\left(\frac{1}{p_\pi\cdot v+\delta\tilde{m}_3}\right)\\
A_{1R} &= -\frac{\alpha_2\rho_2(\nu)}{6F_0}(\nu^2-1)\left(\frac{1}{p_\pi\cdot v+\delta\tilde{m}_2}\right)-\frac{\alpha_3\xi^{(1)}(\nu)}{2F_0}(1+\nu)\left(\frac{1}{p_\pi\cdot v+\delta\tilde{m}_3}\right)\\
A_{2R} &= \frac{\alpha_1\rho_1(\nu)}{2F_0 m_B}\left(\frac{p_\pi\cdot v}{p_\pi\cdot v+\delta\tilde{m}_1}\right)\\
&\quad+\frac{\alpha_2\rho_2(\nu)}{F_0 m_B}\left\{\frac{1}{p_\pi\cdot v+\delta\tilde{m}_2}\left[\frac{1}{6}(\nu p_\pi\cdot v'-p_\pi\cdot v)+\frac{1}{3}(p_\pi\cdot v'-\nu p_\pi\cdot v)\right]\right\}\\
&\quad+\frac{\alpha_3\xi^{(1)}(\nu)}{2F_0 m_B}\left(\frac{p_\pi\cdot(v+v')}{p_\pi\cdot v+\delta\tilde{m}_3}\right)\\
A_{3R} &= -\frac{\alpha_1\rho_1(\nu)}{2F_0 m_D}\left(\frac{p_\pi\cdot v}{p_\pi\cdot v+\delta\tilde{m}_1}\right)\\
&\quad-\frac{\alpha_2\rho_2(\nu)}{F_0 m_D}\left\{\frac{1}{2}\frac{1}{p_\pi\cdot v+\delta\tilde{m}_2}(p_\pi\cdot v'-\nu p_\pi\cdot v)\right\}
\end{aligned}
$$

Numerical values used in SP5/SP6 are

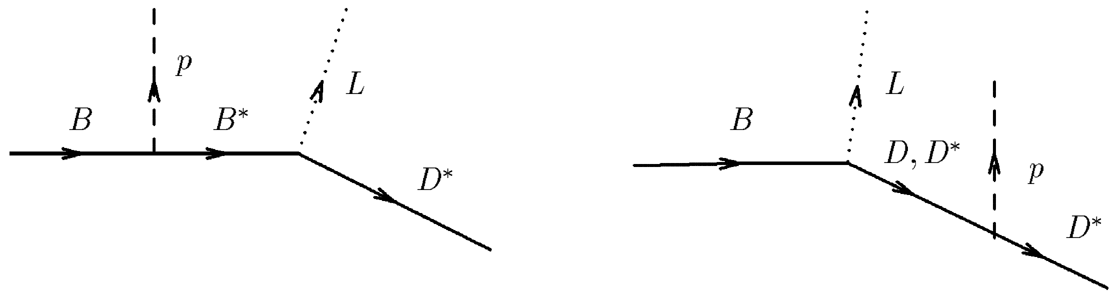
$$
\begin{aligned}
&g = 0.5\\
&\alpha_1 = -1.430\\
&\alpha_2 = -0.140\\
&\alpha_3 = 0.690\\
&F_0 = 0.093\\
&\delta m_B = 0.0460 - \frac{0.00001}{2}i\\
&\delta m_D = 0.1421 - \frac{0.00006}{2}i\\
&\delta\tilde{m}_1 = 0.392 - \frac{1.040}{2}i\\
&\delta\tilde{m}_2 = 0.709 - \frac{0.405}{2}i\\
&\delta\tilde{m}_3 = 0.563 - \frac{0.191}{2}i\\
&\beta = 0.285\\
&\beta' = 0.280 \quad \text{for} \quad \rho_1(\nu)\\
&\beta' = 0.260 \quad \text{for} \quad \rho_2(\nu)\\
&\bar{\Lambda} = 0.750
\end{aligned}
$$

7.2 $B \to D^*\pi\ell\nu$

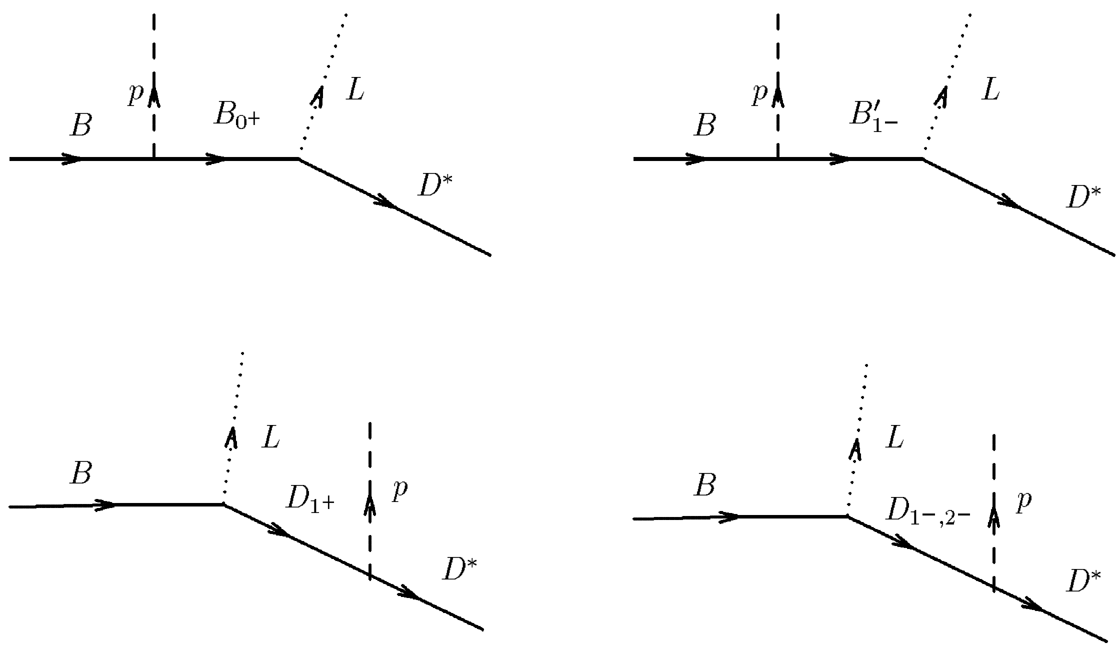
The amplitude for this process has the general form

$$T = \kappa j_\mu \Omega^{\mu\nu}\varepsilon_{D^*\nu}$$

where $\varepsilon_{D^*\nu}$ is the polarization vector of the D^*.



(a)



(b)

(a) shows two non-resonant decays , (b) shows four resonalt decays. The non-resonant portion of $\Omega_{\mu\nu}$ is

$$\begin{aligned}
\Omega^{NR}_{\mu\nu} \;=\;& \tfrac{g}{2F_0}\xi(\nu)\Big\{(v+v')_\mu \tfrac{p_\nu}{p\cdot v'+\delta m_D+i\varepsilon} \\
& -p^\rho \tfrac{\Theta_{\rho\sigma}(v)}{-(p\cdot v+\delta m_B)+i\varepsilon}\left[g_{\nu\sigma}(v+v')_\mu - g_{\mu\sigma}v_\nu - g_{\mu\nu}v'_\sigma + i\epsilon_{\mu\alpha\sigma\nu}(v+v')^\alpha\right] \\
& +\tfrac{\Theta_{\rho\delta}(v')}{p\cdot v'+i\varepsilon}\left[-i\epsilon_{\mu\rho\alpha\beta}v'^\alpha v^\beta + g_{\mu\rho}(1+\nu) - v_\rho v'_\mu\right] i\epsilon_{\omega\nu\gamma\delta}p^\gamma v'^\omega\Big\}
\end{aligned}$$

The resonant portion of $\Omega_{\mu\nu}$ is

$$
\begin{aligned}
\Omega^R_{\mu\nu} \;=\; & \tfrac{\alpha_1}{F_0}\rho_1(\nu)\left[-g_{\mu\nu}(\nu-1)+v'_\mu v_\nu - i\epsilon_{\mu\nu\alpha\beta}v^\alpha v'^\beta\right]\left[-\tfrac{p\cdot v}{2(-p\cdot v-\delta\tilde m_1)}+\tfrac{p\cdot v'}{2(p\cdot v'-\delta\tilde m_1)}\right] \\
& -\tfrac{\alpha_2}{3F_0}\rho_2(\nu)\Big\{p_\rho\tfrac{\Theta^{\rho\sigma}(v)}{2(-p\cdot v-\delta\tilde m_2)}\left[-g_{\nu\sigma}(v+v')_\mu(\nu-1)+3v_\nu v'_\mu v'_\sigma-2g_{\mu\nu}v'_\sigma(\nu-1)\right. \\
& \left.+g_{\mu\sigma}v_\nu(\nu-1)+i\epsilon_{\mu\sigma\alpha\nu}(v-v')^\alpha(1+\nu)+2i\epsilon_{\sigma\nu\alpha\beta}v'^\alpha v^\beta v'_\mu+i\epsilon_{\mu\nu\alpha\beta}v'_\sigma v'^\alpha v^\beta\right] \\
& +3p^\gamma\tfrac{\Theta^{\sigma\rho}_{\gamma\nu}(v')}{2(p\cdot v'-\delta\tilde m_2)+i\varepsilon}\left[v_\delta g_{\rho\mu}(\nu-1)-v_\delta v_\rho v'_\mu+i\epsilon_{\mu\delta\alpha\beta}v_\rho v^\alpha v'^\beta\right] \\
& +\tfrac{i}{2}\tfrac{\Theta^{\sigma\rho}(v)}{2(p\cdot v'-\delta\tilde m_2)}\epsilon_{\rho\delta\gamma\nu}p^\delta_\pi v'^\gamma\left[i\epsilon_{\mu\sigma\alpha\beta}v^\alpha v'^\beta(\nu-1)+g_{\mu\sigma}(\nu^2-1)-v_\alpha\left(v'_\mu(2+\nu)-3v_\mu\right)\right]\Big\} \\
& +\tfrac{\alpha_3}{F_0}\xi^{(1)}(\nu)\Big\{(v+v')_\mu\tfrac{p_\nu}{2(p\cdot v'-\delta\tilde m_3)} \\
& +p^\rho\tfrac{\Theta_{\rho\sigma}(v)}{-2(-p\cdot v-\delta\tilde m_3)}\left[g_{\nu\sigma}(v+v')_\mu-g_{\mu\sigma}v_\nu-g_{\mu\nu}v'_\sigma+i\epsilon_{\mu\alpha\sigma\nu}(v+v')^\alpha\right] \\
& +\tfrac{\Theta_{\rho\delta}(v')}{2(p\cdot v'-\delta\tilde m_3)}\left[-i\epsilon_{\mu\rho\alpha\beta}v'^\alpha v^\beta+g_{\mu\rho}(1+\nu)-v_\rho v'_\mu\right]i\epsilon_{\omega\nu\gamma\delta}p^\gamma v'^\omega\Big\}
\end{aligned}
$$

The tensor $\Omega_{\mu\nu}$ can be expressed as

$$
\begin{aligned}
\Omega_{\mu\nu} \;=\; & \tfrac{i}{2}\epsilon_{\mu\nu\rho\sigma}\left[h_1 m_B m_D v^\rho v'^\sigma + h_2 m_B v^\rho p^\sigma_\pi + h_3 m_D v'^\rho p^\sigma_\pi\right] \\
& +f_1 m_B v_\mu p_{\pi\nu} + f_2 m_D v'_\mu p_{\pi\nu} + f_3 p_{\pi\mu}p_{\pi\nu} + f_4 m_B^2 v_\mu v_\nu + f_5 m_B m_D v'_\mu v_\nu + f_6 m_B p_{\pi\mu} v_\nu \\
& +k g_{\mu\nu} \\
& +\tfrac{i}{2}\epsilon_{\mu\delta\rho\sigma}v^\delta v'^\rho p^\sigma_\pi(g_1 p_{\pi\nu}+g_2 m_B v_\nu)+\tfrac{i}{2}\epsilon_{\nu\delta\rho\sigma}v^\delta v'^\rho p^\sigma_\pi(g_3 m_B v_\mu + g_4 m_D v'_\mu + g_5 p_{\pi\mu})
\end{aligned}
$$

The non-resonant contribution to the form factors are

$$
\begin{aligned}
h_{1NR} &= -\tfrac{g\xi(\nu)}{F_0 m_B m_D}\tfrac{p_\pi\cdot v}{p_\pi\cdot v+\delta m_B-i\varepsilon} \\
h_{2NR} &= -\tfrac{g\xi(\nu)}{F_0 m_B}\tfrac{1}{p_\pi\cdot v+\delta m_B-i\varepsilon} \\
h_{3NR} &= -\tfrac{g\xi(\nu)}{F_0 m_D}\left(\tfrac{1}{p_\pi\cdot v+\delta m_B-i\varepsilon}-\tfrac{1+\nu}{p_\pi\cdot v'+i\varepsilon}\right) \\
f_{1NR} &= -\tfrac{g\xi(\nu)}{2F_0 m_B}\left(\tfrac{1}{p_\pi\cdot v+\delta m_B-i\varepsilon}-\tfrac{1}{p_\pi\cdot v'+\delta m_D+i\varepsilon}\right) \\
f_{2NR} &= \tfrac{m_B}{m_D}f_{1NR} \\
f_{3NR} &= 0 \\
f_{4NR} &= 0 \\
f_{5NR} &= -\tfrac{g\xi(\nu)}{2F_0 m_B m_D}\left(1+\tfrac{p_\pi\cdot v}{p_\pi\cdot v+\delta m_B-i\varepsilon}\right) \\
f_{6NR} &= -\tfrac{g\xi(\nu)}{2F_0 m_B}\left(\tfrac{1}{p_\pi\cdot v+\delta m_B-i\varepsilon}-\tfrac{1}{p_\pi\cdot v'+i\varepsilon}\right) \\
k_{NR} &= \tfrac{g\xi(\nu)}{2F_0}\left(\tfrac{p_\pi\cdot v'-\nu p_\pi\cdot v}{p_\pi\cdot v+\delta m_B-i\varepsilon}+\tfrac{p_\pi\cdot v-\nu p_\pi\cdot v'}{p_\pi\cdot v'+i\varepsilon}\right) \\
g_{1NR} &= 0 \\
g_{2NR} &= 0 \\
g_{3NR} &= 0 \\
g_{4NR} &= \tfrac{g\xi(\nu)}{F_0 m_D}\tfrac{1}{p_\pi\cdot v'+i\varepsilon} \\
g_{5NR} &= 0
\end{aligned}
$$

The resonant contribution to the form factors are

$$
\begin{aligned}
h_{1R} &= \frac{\alpha_1\rho_1(\nu)}{F_0 m_B m_D}\left(\frac{p_\pi\cdot v}{-p_\pi\cdot v-\delta\tilde{m}_1} - \frac{p_\pi\cdot v'}{p_\pi\cdot v'-\delta\tilde{m}_1}\right) + \frac{\alpha_2\rho_2(\nu)}{3F_0 m_B m_D}\left(\frac{p_\pi\cdot v(1+2\nu)-p_\pi\cdot v'}{p_\pi\cdot v+\delta\tilde{m}_2} + \frac{3}{2}\frac{\nu p_\pi\cdot v'-p_\pi\cdot v}{p_\pi\cdot v'-\delta\tilde{m}_2}\right) \\
&\quad -\frac{\alpha_3\xi^{(1)}(\nu)}{F_0 m_B m_D}\frac{p_\pi\cdot v}{p_\pi\cdot v+\delta\tilde{m}_3} \\
h_{2R} &= \frac{\alpha_2(1+\nu)\rho_2(\nu)}{3F_0 m_B}\frac{1}{-p_\pi\cdot v-\delta\tilde{m}_2} - \frac{\alpha_3\xi^{(1)}(\nu)}{F_0 m_B}\frac{1}{p_\pi\cdot v+\delta\tilde{m}_3} \\
h_{3R} &= \frac{\alpha_2\rho_2(\nu)}{3F_0 m_D}\left(\frac{1+\nu}{p_\pi\cdot v+\delta\tilde{m}_2} - \frac{\nu^2-1}{2(p_\pi\cdot v'-\delta\tilde{m}_2)}\right) - \frac{\alpha_3\xi^{(1)}(\nu)}{F_0 m_D}\left(\frac{1}{p_\pi\cdot v+\delta\tilde{m}_3} - \frac{1+\nu}{p_\pi\cdot v'-\delta\tilde{m}_3}\right) \\
f_{1R} &= -\frac{\alpha_2\rho_2(\nu)(\nu-1)}{6F_0 m_B}\left(\frac{1}{p_\pi\cdot v+\delta\tilde{m}_2} - \frac{1}{p_\pi\cdot v'-\delta\tilde{m}_2}\right) - \frac{\alpha_3\xi^{(1)}(\nu)}{2F_0 m_B}\left(\frac{1}{p_\pi\cdot v+\delta\tilde{m}_3} - \frac{1}{p_\pi\cdot v'-\delta\tilde{m}_3}\right) \\
f_{2R} &= \frac{m_B}{m_D} f_{1R} \\
f_{3R} &= 0 \\
f_{4R} &= 0 \\
f_{5R} &= -\frac{\alpha_1\rho_1(\nu)}{2F_0 m_B m_D}\left(\frac{p_\pi\cdot v}{p_\pi\cdot v+\delta\tilde{m}_1} + \frac{p_\pi\cdot v'}{p_\pi\cdot v'-\delta\tilde{m}_1}\right) + \frac{\alpha_2\rho_2(\nu)}{2F_0 m_B m_D}\left(\frac{p_\pi\cdot v'-\frac{1}{3}p_\pi\cdot v(1+2\nu)}{p_\pi\cdot v+\delta\tilde{m}_2} + \frac{p_\pi\cdot v-\frac{1}{3}p_\pi\cdot v'(1+2\nu)}{p_\pi\cdot v'-\delta\tilde{m}_2}\right) \\
&\quad +\frac{\alpha_3\xi^{(1)}(\nu)}{F_0 m_B m_D}\left(\frac{p_\pi\cdot v}{p_\pi\cdot v+\delta\tilde{m}_3} + \frac{p_\pi\cdot v'}{p_\pi\cdot v'-\delta\tilde{m}_3}\right) \\
f_{6R} &= \frac{\alpha_2\rho_2(\nu)(\nu-1)}{6F_0 m_B}\left(\frac{1}{p_\pi\cdot v+\delta\tilde{m}_2} - \frac{1}{p_\pi\cdot v'-\delta\tilde{m}_2}\right) + \frac{\alpha_3\xi^{(1)}(\nu)}{2F_0 m_B}\left(\frac{1}{p_\pi\cdot v+\delta\tilde{m}_3} - \frac{1}{p_\pi\cdot v'-\delta\tilde{m}_3}\right) \\
k_R &= -\frac{\alpha_1\rho_1(\nu)(\nu-1)}{2F_0}\left(\frac{p_\pi\cdot v}{p_\pi\cdot v+\delta\tilde{m}_1} + \frac{p_\pi\cdot v'}{p_\pi\cdot v'-\delta\tilde{m}_1}\right) - \frac{\alpha_2\rho_2(\nu)(\nu-1)}{3F_0}\left(\frac{p_\pi\cdot v'-\nu p_\pi\cdot v}{p_\pi\cdot v+\delta\tilde{m}_2} + \frac{p_\pi\cdot v-\nu p_\pi\cdot v'}{p_\pi\cdot v'-\delta\tilde{m}_2}\right) \\
&\quad +\frac{\alpha_3\xi^{(1)}(\nu)}{2F_0}\left(\frac{p_\pi\cdot v'-\nu p_\pi\cdot v}{p_\pi\cdot v+\delta\tilde{m}_3} + \frac{p_\pi\cdot v-\nu p_\pi\cdot v'}{p_\pi\cdot v'-\delta\tilde{m}_3}\right) \\
g_{1R} &= 0 \\
g_{2R} &= -\frac{\alpha_2\rho_2(\nu)}{2F_0 m_B}\frac{1}{p_\pi\cdot v'-\delta\tilde{m}_2} \\
g_{3R} &= -g_{2R} \\
g_{4R} &= \frac{\alpha_2\rho_2(\nu)}{3F_0 m_D}\left(\frac{2}{p_\pi\cdot v+\delta\tilde{m}_2} - \frac{1+\frac{1}{2}\nu}{p_\pi\cdot v'-\delta\tilde{m}_2}\right) + \frac{\alpha_3\xi^{(1)}(\nu)}{F_0}\frac{1}{p_\pi\cdot v'-\delta\tilde{m}_3} \\
g_{5R} &= 0
\end{aligned}
$$

In SP5/SP6 contributios from D^{**} was removed

$$
\begin{aligned}
h_{1R} &= \frac{\alpha_1\rho_1(\nu)}{F_0 m_B m_D}\left(\frac{p_\pi\cdot v}{-p_\pi\cdot v-\delta\tilde{m}_1}\right) + \frac{\alpha_2\rho_2(\nu)}{3F_0 m_B m_D}\left(\frac{p_\pi\cdot v(1+2\nu)-p_\pi\cdot v'}{p_\pi\cdot v+\delta\tilde{m}_2}\right) \\
&\quad -\frac{\alpha_3\xi^{(1)}(\nu)}{F_0 m_B m_D}\frac{p_\pi\cdot v}{p_\pi\cdot v+\delta\tilde{m}_3} \\
h_{2R} &= \frac{\alpha_2(1+\nu)\rho_2(\nu)}{3F_0 m_B}\frac{1}{-p_\pi\cdot v-\delta\tilde{m}_2} - \frac{\alpha_3\xi^{(1)}(\nu)}{F_0 m_B}\frac{1}{p_\pi\cdot v+\delta\tilde{m}_3} \\
h_{3R} &= \frac{\alpha_2\rho_2(\nu)}{3F_0 m_D}\left(\frac{1+\nu}{p_\pi\cdot v+\delta\tilde{m}_2}\right) - \frac{\alpha_3\xi^{(1)}(\nu)}{F_0 m_D}\left(\frac{1}{p_\pi\cdot v+\delta\tilde{m}_3}\right) \\
f_{1R} &= -\frac{\alpha_2\rho_2(\nu)(\nu-1)}{6F_0 m_B}\left(\frac{1}{p_\pi\cdot v+\delta\tilde{m}_2}\right) - \frac{\alpha_3\xi^{(1)}(\nu)}{2F_0 m_B}\left(\frac{1}{p_\pi\cdot v+\delta\tilde{m}_3}\right) \\
f_{2R} &= \frac{m_B}{m_D} f_{1R} \\
f_{3R} &= 0 \\
f_{4R} &= 0 \\
f_{5R} &= -\frac{\alpha_1\rho_1(\nu)}{2F_0 m_B m_D}\left(\frac{p_\pi\cdot v}{p_\pi\cdot v+\delta\tilde{m}_1}\right) + \frac{\alpha_2\rho_2(\nu)}{2F_0 m_B m_D}\left(\frac{p_\pi\cdot v'-\frac{1}{3}p_\pi\cdot v(1+2\nu)}{p_\pi\cdot v+\delta\tilde{m}_2}\right) \\
&\quad +\frac{\alpha_3\xi^{(1)}(\nu)}{F_0 m_B m_D}\left(\frac{p_\pi\cdot v}{p_\pi\cdot v+\delta\tilde{m}_3}\right) \\
f_{6R} &= \frac{\alpha_2\rho_2(\nu)(\nu-1)}{6F_0 m_B}\left(\frac{1}{p_\pi\cdot v+\delta\tilde{m}_2}\right) + \frac{\alpha_3\xi^{(1)}(\nu)}{2F_0 m_B}\left(\frac{1}{p_\pi\cdot v+\delta\tilde{m}_3}\right) \\
k_R &= -\frac{\alpha_1\rho_1(\nu)(\nu-1)}{2F_0}\left(\frac{p_\pi\cdot v}{p_\pi\cdot v+\delta\tilde{m}_1}\right) - \frac{\alpha_2\rho_2(\nu)(\nu-1)}{3F_0}\left(\frac{p_\pi\cdot v'-\nu p_\pi\cdot v}{p_\pi\cdot v+\delta\tilde{m}_2}\right) \\
&\quad +\frac{\alpha_3\xi^{(1)}(\nu)}{2F_0}\left(\frac{p_\pi\cdot v'-\nu p_\pi\cdot v}{p_\pi\cdot v+\delta\tilde{m}_3}\right) \\
g_{1R} &= 0 \\
g_{2R} &= 0 \\
g_{3R} &= -g_{2R} \\
g_{4R} &= \frac{\alpha_2\rho_2(\nu)}{3F_0 m_D}\left(\frac{2}{p_\pi\cdot v+\delta\tilde{m}_2}\right) \\
g_{5R} &= 0
\end{aligned}
$$

Numerical values used in SP5/SP6 are

$$
\begin{aligned}
& g = 0.5 \\
& \alpha_1 = -1.430 \\
& \alpha_2 = -0.140 \\
& \alpha_3 = 0.690 \\
& F_0 = 0.093 \\
& \delta m_B = 0.0460 - \frac{0.00001}{2} i \\
& \delta m_D = 0.1421 - \frac{0.00006}{2} i \\
& \delta\tilde{m}_1 = 0.392 - \frac{1.040}{2} i \\
& \delta\tilde{m}_2 = 0.709 - \frac{0.405}{2} i \\
& \delta\tilde{m}_3 = 0.563 - \frac{0.191}{2} i \\
& \beta = 0.285 \\
& \beta' = 0.280 \quad \text{for} \quad \rho_1(\nu) \\
& \beta' = 0.260 \quad \text{for} \quad \rho_2(\nu) \\
& \bar{\Lambda} = 0.750
\end{aligned}
$$