Fitting method - Building PDFs

1 χ^2 fitting

We use χ^2 fitting.

1.1 χ^2

The χ^2 is

$$\begin{aligned}\chi^2 = \ & \textstyle\sum_{i=\text{bin}}^{D^0} \frac{(N_i^{data} - \sum_j C_j^{MCBkgd} N_{ij}^{MCBkgd} - \sum_j C_j^{D^0} P_{ij}^{MC})^2}{(\sigma_i^{data})^2 + \sum_j (\sigma_{ij}^{MCBkgd})^2 + \sum_j (C_j^{D^0} \sigma_{ij}^{P})^2} \\ & + \textstyle\sum_{i=\text{bin}}^{D^+} \frac{(N_i^{data} - \sum_j C_j^{MCBkgd} N_{ij}^{MCBkgd} - \sum_j C_j^{D^+} P_{ij}^{MC})^2}{(\sigma_i^{data})^2 + \sum_j (\sigma_{ij}^{MCBkgd})^2 + \sum_j (C_j^{D^+} \sigma_{ij}^{P})^2}\end{aligned}$$

where, i represents each bin and j represents each MC mode. We may have following MC modes.
Background MC are separated into two categories based on D mass plots : peaking background and non-peaking background (combinatorial or continuum). We need this distinction because we perform D mass sideband subtraction. Non-peaking backgrounds are subtracted away by the sideband subtraction, whereas peaking backgrounds stay almost same. Hence, main part of background MC are peaking backgrounds.
For both D^0 mode and D^+ mode:

- Peaking background from B
  - $j = 1$, D and ℓ came from different B^+
  - $j = 2$, D and ℓ came from different B^0
  - $j = 3$, ℓ did not come directry from B^+ (cascade lepton)
  - $j = 4$, ℓ did not come directry from B^0 (cascade lepton)
    For example, $B \to D^* + D_s$ and $D_s \to X_u \ell \nu$
  - $j = 5$, Lepton from B^+ miss ID
  - $j = 6$, Lepton from B^0 miss ID
- Combinatrial background from B
  - $j = 7$, D from B^+ miss reconstruction
  - $j = 8$, D from B^0 miss reconstruction
- Peaking backgroud from ccbar
  - $j = 9$, ccbar with real D
- Continuum background
  - $j = 10$, D from ccbar miss reconstruction

- $j = 11$, uds
- $j = 12$, tautau

Signal MC are separated into four categories : $B \to D/D^*/D^{**}/(D\pi)\ell\nu$.
For signal MC D^0 mode:

- $B \to D\ell\nu$
  - $j = 1 : B^+ \to D\ell\nu$
- $B \to D^*\ell\nu$
  - $j = 2 : B^+ \to D^*\ell\nu$
  - $j = 3 : B^0 \to D^*\ell\nu$
- $B \to D^{**}\ell\nu$
  - $j = 4 : B^+ \to D_0^*\ell\nu$
  - $j = 5 : B^0 \to D_0^*\ell\nu$
  - $j = 6 : B^+ \to D_1\ell\nu$
  - $j = 7 : B^0 \to D_1\ell\nu$
  - $j = 8 : B^+ \to D_1'\ell\nu$
  - $j = 9 : B^0 \to D_1'\ell\nu$
  - $j = 10 : B^+ \to D_2^*\ell\nu$
  - $j = 11 : B^0 \to D_2^*\ell\nu$
- $B \to D\pi\ell\nu$
  - $j = 12 : B^0 \to D\pi\ell\nu$
  - $j = 13 : B^+ \to D\pi\ell\nu$
  - $j = 14 : B^0 \to D^*\pi\ell\nu$
  - $j = 15 : B^+ \to D^*\pi\ell\nu$

For signal MC D^+ mode:

- $B \to D\ell\nu$
  - $j = 1 : B^0 \to D\ell\nu$
- $B \to D^*\ell\nu$
  - $j = 2 : B^0 \to D^*\ell\nu$
- $B \to D^{**}\ell\nu$

- $j = 3:\ B^+ \to D_0^* \ell\nu$
- $j = 4:\ B^0 \to D_0^* \ell\nu$
- $j = 5:\ B^+ \to D_1 \ell\nu$
- $j = 6:\ B^0 \to D_1 \ell\nu$
- $j = 7:\ B^+ \to D_1' \ell\nu$
- $j = 8:\ B^0 \to D_1' \ell\nu$
- $j = 9:\ B^+ \to D_2^* \ell\nu$
- $j = 10:\ B^0 \to D_2^* \ell\nu$

- $B \to D\pi\ell\nu$
  - $j = 11:\ B^0 \to D\pi\ell\nu$
  - $j = 12:\ B^+ \to D\pi\ell\nu$
  - $j = 13:\ B^0 \to D^*\pi\ell\nu$
  - $j = 14:\ B^+ \to D^*\pi\ell\nu$

Quantities in the above equation are:

- N_i^{data}
  The number of data events or candidates in the i-th bin
- N_{ij}^{MCBkgd}
  The number of MC background events or candidates in the i-th bin, normalized to data luminocity. The effect of data MC differences from tracking and PID are included. Details are in section 2.
- C_j^{MCBkgd}
  The coefficient of each MC background. These allow us to vary each componemt of background. Thes are initially set to 1.
- P_{ij}^{MC}
  The PDF of each MC mode in each bin. The effect of data MC differences from tracking and PID are included. The effect of form factors is also included. Details are in section 2.
- $C_j^{D^0}$ and $C_j^{D^+}$
  The coefficient of each MC signal mode (j). This consists of branching fractions. Details are in section 4.
- σ_i^{data}
  The statistical uncertainty of data.
- σ_{ij}^{MCBkgd}
  The statistical uncertainty of background MC normalized to data luminosity.

- σ_{ij}^P
  The statistical uncertainty of signal MC PDF.
  In C_j here, at first we use the branching fractions of B decays and D^{**} decays used in SP5/SP6. Then change them to fitted values after first fitting to perform the second fitting. We may want to iterate a few times. Details are in section 4.

Note that the efficiecncy to reconstruct Dl candidates are common in both data and MC, hence it can be omitted from χ^2

2 Building PDF

We build probability density functions (PDF) from MC using MC truth information.We use the following generic MC samples: ($l =$) BpBm, B0B0bar, ccbar, uds and tautau.

2.1 Tracking and PID correctin

Tracking and PID corrections are done in similar way.

- Tracking efficiency correction
  Tracking efficiency correction factors r_k^{Trk} are calculated candidate by candidate (k) and stored in ntuples.

- PID correction
  PID correction fctors (called weights) r_k^{PID} are also calculated candidate by candidate (k) and stored in ntuples.

2.2 Luminosity normalization

All MC are normalized to data by luminosity. Supposing data luminosity is $\mathcal{L}^{data}$ and MC (sample l) luminosity is $\mathcal{L}_l^{MC}$, the normalization factor r_l^{Lumi} is given by

$$r_l^{Lumi} = \frac{\mathcal{L}^{data}}{\mathcal{L}_l^{MC}}$$

2.3 Background N_{ij}^{MCBkgd}

The N_{ij}^{MCBkgd} is given by

$$N_{ij}^{MCBkgd} = \sum_l \left(r_l^{Lumi} \sum_{k \in i,j,l} r_k^{Trk} r_k^{PID} \right)$$

The sum over k is taken in each MC sample (l), in each MC mode (j) and in each bin (i).

2.4 Signal PDF P_{ij}^{MC}

To consrtuct signal PDF, we need to include wights W_{jk} from form factors (FF). These weights are calculated candidate by candidate (k) in each decay mode (j) and stored in ntuples. See section 3 for details.
The number of signal events or candidates N_{ij}^{MC} of MC mode j is given by

$$N_{ij}^{MC} = \sum_l \left(r_l^{Lumi} \sum_{k \in i,j,l} r_k^{Trk} r_k^{PID} W_{jk} \right)$$

The sum over k is taken in each MC sample (l), in each MC mode (j), and in each bin (i). W_{jk} is the form factor weight described in section 3.
The PDF is given by

$$P_{ij}^{MC} = \frac{N_{ij}^{MC}}{\sum_i N_{ij}^{MC}}$$

3 Form factor effect W_{jk}

3.1 Velocity transfer w

The w is given by

$$w = \frac{p_B \cdot p_D}{m_B m_D} = \frac{E_B E_D - \vec{p}_B \cdot \vec{p}_D}{m_B m_D}$$

We can calculate this value using MC truth value of the 4-momenta of B and D.
This D may be D^* or D^{**} depending on the decay mode.

3.2 $B \to D\ell\nu$

The differential decay rate is given by

$$\frac{d\Gamma(B \to D\ell\nu)}{dw} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} (m_B + m_D)^2 m_D^3 (w^2 - 1)^{3/2} \left| h_+(w) - \frac{m_B - m_D}{m_B + m_D} h_-(w) \right|^2$$

In haevy quark limit, the form factors $h_+(w)$ and $h_-(w)$ are

$$h_+(w) = \xi(w), \;\; h_-(w) = 0$$

where $\xi(w)$ is the Isgur-Wise Function. In SP5/SP6 ISGW2 model is used (need to confirm!!!):

$$h_+(w) = \xi(w) = 1$$

Sine we would like to measure the slope of this form factor, we will set

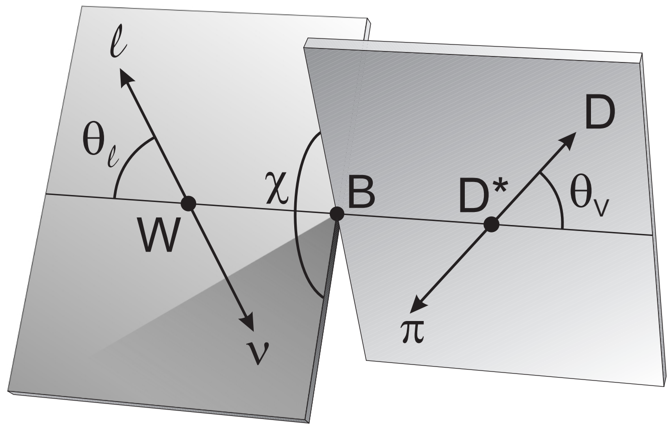
$$h_+(w) = \xi(w) = 1 - \rho_D^2 (w - 1)$$

The weight is given by the ratio of this change and original SP5/SP6

$$W_{jk}(w_k) = [1 - \rho_D^2 (w_k - 1)]^2$$

3.3 $B \to D^*\ell\nu$

Three kinematic variables $\cos\theta_\ell$, $\cos\theta_V$ and χ is defined in the figure



The differential decay rate is given by

$$\begin{aligned}\frac{d\Gamma(B\to D^*\ell\nu)}{dw d\cos\theta_V d\cos\theta_\ell d\chi} \quad = \quad & \frac{3G_F^2}{4(4\pi)^4}|V_{cb}|^2 m_B m_{D^*}^2 r\sqrt{w^2-1}(1-2wr+r^2)\times \\ & [(1-\cos\theta_\ell)^2\sin^2\theta_V|H_+(w)|^2 \\ & +(1+\cos\theta_\ell)^2\sin^2\theta_V|H_-(w)|^2 \\ & +4\sin^2\theta_\ell\cos^2\theta_V|H_0(w)|^2 \\ & -4\sin\theta_\ell(1-\cos\theta_\ell)\sin\theta_V\cos\theta_V\cos\chi H_+(w)H_0(w) \\ & +4\sin\theta_\ell(1+\cos\theta_\ell)\sin\theta_V\cos\theta_V\cos\chi H_-(w)H_0(w) \\ & -2\sin^2\theta_\ell\sin^2\theta_V\cos 2\chi H_+(w)H_-(w)]\end{aligned}$$

where r is the mass ratio

$$r = \frac{m_{D^*}}{m_B}$$

$H_i(w)$ are

$$H_i = -m_B\frac{R^*(1-r^2)(w+1)}{2\sqrt{1-2wr+r^2}}h_{A_1}(w)\widetilde{H}_i(w)$$

where R^* is

$$R^* = \frac{2\sqrt{m_B m_{D^*}}}{m_B+m_D}$$

$h_{A_1}(w)$ is

$$h_{A_1}(w) = h_{A_1}(0)\left(1-\rho^2(w-1)\right)$$

where ρ^2 is the form factor slope, and $\widetilde{H}_i(w)$ are

$$\begin{aligned}\widetilde{H}_\pm(w) &= \tfrac{\sqrt{1-2wr+r^2}}{1-r}\left(1\mp\sqrt{\tfrac{w-1}{w+1}}R_1(w)\right) \\ \widetilde{H}_0(w) &= 1+\tfrac{w-1}{1-r}\left(1-R_2(w)\right)\end{aligned}$$

were R_1 and R_2 are the form factor ratios.
We define

$$
\begin{aligned}
&c_L \equiv \cos\theta_\ell \\
&c_V \equiv \cos\theta_V \\
&c_\chi \equiv \cos\chi \\
&h_\pm(w) \equiv (1-r)\widetilde{H}_\pm(w) = \sqrt{1-2wr+r^2}\left(1 \mp \sqrt{\tfrac{w-1}{w+1}}R_1\right) \\
&h_0(w) \equiv (1-r)\widetilde{H}_0(w) = (1-r) + (w-1)\,(1-R_2)
\end{aligned}
$$

then

$$
\begin{aligned}
\tfrac{d\Gamma(B\to D^*\ell\nu)}{dwd\cos\theta_V d\cos\theta_\ell d\chi} \;=\;& f(w)\,(1-\rho^2(w-1))^2 \times \\
&[(1-c_L)^2(1-c_V^2)h_+^2 + (1+c_L)^2(1-c_V^2)h_-^2 \\
&+4(1-c_L^2)c_V^2h_0^2 - 2(1-c_L^2)(1-c_V^2)(2c_\chi^2-1)h_+h_- \\
&-4\sqrt{1-c_L^2}(1-c_L)\sqrt{1-c_V^2}c_Vc_\chi h_+h_0 \\
&+4\sqrt{1-c_L^2}(1+c_L)\sqrt{1-c_V^2}c_Vc_\chi h_-h_0]
\end{aligned}
$$

The masses are

$$
\begin{aligned}
m_{B^+} &= 5279.0 \pm 0.5\text{MeV} \\
m_{B^0} &= 5279.4 \pm 0.5\text{MeV} \\
m_{D^{*0}} &= 2006.7 \pm 0.5\text{MeV} \\
m_{D^{*+}} &= 2010.5 \pm 0.5\text{MeV}
\end{aligned}
$$

In SP5/SP6 HQET model was used, and the values of parameters are

$$
\begin{aligned}
R_1^0 &= 1.18 \\
R_2^0 &= 0.72 \\
\rho_0^2 &= 0.92
\end{aligned}
$$

The values from a recent Babar measurement are

$$
\begin{aligned}
R_1 &= 1.396 \pm 0.112 \\
R_2 &= 0.885 \pm 0.075 \\
\rho^2 &= 1.145 \pm 0.124
\end{aligned}
$$

In this analysis a different parameterization was used for $h_{A_1}(w)$

$$h_{A_1}(w) = h_{A_1}(0)\left(1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3\right)$$

where

$$z = \frac{\sqrt{w+1}-\sqrt{2}}{\sqrt{w+1}+\sqrt{2}}$$

Thus, the differential decay rate is

$$
\begin{aligned}
\tfrac{d\Gamma(B\to D^*\ell\nu)}{dwd\cos\theta_V d\cos\theta_\ell d\chi} \;=\;& f(w)\,(1-8\rho^2z+(53\rho^2-15)z^2-(231\rho^2-91)z^3)^2 \times \\
&[(1-c_L)^2(1-c_V^2)h_+^2 + (1+c_L)^2(1-c_V^2)h_-^2 \\
&+4(1-c_L^2)c_V^2h_0^2 - 2(1-c_L^2)(1-c_V^2)(2c_\chi^2-1)h_+h_- \\
&-4\sqrt{1-c_L^2}(1-c_L)\sqrt{1-c_V^2}c_Vc_\chi h_+h_0 \\
&+4\sqrt{1-c_L^2}(1+c_L)\sqrt{1-c_V^2}c_Vc_\chi h_-h_0]
\end{aligned}
$$

The weight is given by taking the ratio of decay rates which calculated by using old and new form factor parameters. The factor $f(w)$ is cancelled by taking ratio.

3.4 $B \to D_0^* \ell \nu$

This is similar to $B \to D\ell\nu$ case

$$W_{jk}(w_k) = [1 - \rho^2_{D_0^*}(w_k - 1)]^2$$

3.5 $B \to D_1 \ell \nu$

This is similar to $B \to D^* \ell \nu$ case. Here, the mass ratio is

$$r_1 = \frac{m_{D_1}}{m_B}$$

where

$$m_{D_1^0} = 2422.2 \pm 1.8\text{MeV}$$

We also use the form factor parameters

$$\begin{aligned} R_1^1 &= 1.18 \\ R_2^1 &= 0.72 \\ \rho^2_{D_1} &= 0.92 \end{aligned}$$

Hence

$$W_{jk}(w_k) = \left|1 - \rho^2_{D_1}(w-1)\right|^2 \left[\frac{2(1 - 2wr_1 + r_1^2)}{(1-r_1)^2}\left(1 + \frac{w-1}{w+1}(R_1^1)^2\right) + \left(1 + \frac{w-1}{1-r_1}\left(1 - R_2^1\right)\right)^2\right]$$

3.6 $B \to D_1' \ell \nu$

This is also similar to $B \to D^* \ell \nu$ case. Here, the mass ratio is

$$r_1' = \frac{m_{D_1'}}{m_B}$$

We also use the form factor parameters

$$\begin{aligned} R_1^{1\prime} &= 1.18 \\ R_2^{1\prime} &= 0.72 \\ \rho^2_{D_1'} &= 0.92 \end{aligned}$$

Hence

$$W_{jk}(w_k) = \left|1 - \rho^2_{D_1'}(w-1)\right|^2 \left[\frac{2(1 - 2wr_1 + r_1^2)}{(1-r_1)^2}\left(1 + \frac{w-1}{w+1}(R_1^{1\prime})^2\right) + \left(1 + \frac{w-1}{1-r_1}\left(1 - R_2^{1\prime}\right)\right)^2\right]$$

3.7 $B \to D_2^* \ell \nu$

We set

$$W_{jk}(w_k) = ?$$

3.8 $B \to D\pi\ell\nu$

We set

$$W_{jk}(w_k) = 1$$

3.9 $B \to D^*\pi\ell\nu$

We set

$$W_{jk}(w_k) = 1$$

4 Coefficients $C_j^{D^0}$ and $C_j^{D^+}$

The coefficients consist of Branching fractions.

4.1 Number of produced B

Supposing integrated luminocity of data is $\mathcal{L}^{data}$, the number of produced B pairs N are

$$N = \sigma \mathcal{L}^{data}$$

where σ is the crosssection of B pairs at $\Upsilon(4S)$ energy

$$\sigma = 1.05\text{nb}^{-1} = 1.05 \times 10^6\text{fb}^{-1}$$

Branching fractions of B^+B^- and $B^0\bar{B}^0$ are:

$$\begin{aligned} f_{+-} &= \mathcal{B}(\Upsilon(4S) \to B^+B^-) \\ f_{00} &= \mathcal{B}(\Upsilon(4S) \to B^0\bar{B}^0) \end{aligned}$$

In the first approximation $f_{+-} = f_{00} = 0.5$. We define the ratio of these two branching fraction

$$f_{+0} \equiv \frac{f_{+-}}{f_{00}}$$

The number of produced B for neutral B modes is

$$N_{neut}^{prod} = 2f_{00}N = 2f_{00}\,\sigma\mathcal{L}^{data}$$

and for charged B modes

$$N_{chrg}^{prod} = 2f_{+-}N = 2f_{+-}\,\sigma\mathcal{L}^{data} = f_{+0}N_{neut}^{prod}$$

Since we are interested only in relative branching fractions, this N_{neut}^{prod} is irelevant to our analysis.

4.2 Branching fractions

4.2.1 Isospin symmetry

From isospin symmetry, these dcay rates are identical

$$\Gamma(B^+ \to \bar{D}^0\ell^+\nu) = \Gamma(B^0 \to D^-\ell^+\nu)$$
$$\Gamma(B^+ \to \bar{D}^{*0}\ell^+\nu) = \Gamma(B^0 \to D^{*-}\ell^+\nu)$$
$$\Gamma(B^+ \to \bar{D}^{**0}\ell^+\nu) = \Gamma(B^0 \to D^{**-}\ell^+\nu)$$

Branching fractions are not same because of the difference in total rate or life time of B^+ and B^0

$$t_{+0} \equiv \frac{\tau_{B^+}}{\tau_{B^0}} = 1.086 \pm 0.017 = \frac{\Gamma_{B^0}}{\Gamma_{B^+}}$$

So branching fractions are,

$$\frac{\mathcal{B}(B^+ \to \bar{D}^{*0}\ell^+\nu)}{\mathcal{B}(B^0 \to D^{*-}\ell^+\nu)} = \frac{\Gamma(B^+ \to D^*\ell\nu)}{\Gamma(B^0 \to D^*\ell\nu)}\frac{\Gamma_{B^0}}{\Gamma_{B^+}} = t_{+0}$$

Isospin symmetry may be applied to D^{**} decays and $B \to D\pi\ell\nu$ decays

$$\Gamma(D^{**0} \to D^+\pi^-) = \Gamma(D^{**+} \to D^0\pi^+)$$
$$\Gamma(D^{**0} \to D^0\pi^0) = \Gamma(D^{**+} \to D^+\pi^0)$$
$$\Gamma(D^{**0} \to D^{*+}\pi^-) = \Gamma(D^{**+} \to D^{*0}\pi^+)$$
$$\Gamma(D^{**0} \to D^{*0}\pi^0) = \Gamma(D^{**+} \to D^{*+}\pi^0)$$

$$\Gamma(B^+ \to D^-\pi^+\ell^+\nu) = \Gamma(B^0 \to \bar{D}^0\pi^-\ell^+\nu)$$
$$\Gamma(B^+ \to D^{*-}\pi^+\ell^+\nu) = \Gamma(B^0 \to \bar{D}^{*0}\pi^-\ell^+\nu)$$

And also

$$\Gamma(D^{**0} \to D^+\pi^-) = 2\Gamma(D^{**0} \to D^0\pi^0)$$
$$\Gamma(D^{**+} \to D^0\pi^+) = 2\Gamma(D^{**+} \to D^+\pi^0)$$
$$\Gamma(D^{**0} \to D^{*+}\pi^-) = 2\Gamma(D^{**0} \to D^{*0}\pi^0)$$
$$\Gamma(D^{**+} \to D^{*0}\pi^+) = 2\Gamma(D^{**+} \to D^{*+}\pi^0)$$

$$\Gamma(B^+ \to D^-\pi^+\ell^+\nu) = 2\Gamma(B^+ \to \bar{D}^0\pi^0\ell^+\nu)$$
$$\Gamma(B^0 \to \bar{D}^0\pi^-\ell^+\nu) = 2\Gamma(B^0 \to D^-\pi^0\ell^+\nu)$$
$$\Gamma(B^+ \to D^{*-}\pi^+\ell^+\nu) = 2\Gamma(B^+ \to \bar{D}^{*0}\pi^0\ell^+\nu)$$
$$\Gamma(B^0 \to \bar{D}^{*0}\pi^-\ell^+\nu) = 2\Gamma(B^0 \to D^{*-}\pi^0\ell^+\nu)$$

because, for example,

$$D^{**+} : |1/2, 1/2>$$
$$D^0 : |1/2, -1/2>$$
$$\pi^+ : |1, 1>$$
$$D^0\pi^+ : |1, 1> \times |1/2, -1/2> = \sqrt{\tfrac{1}{3}}|3/2, 1/2> + \sqrt{\tfrac{2}{3}}|1/2, 1/2>$$
$$D^+ : |1/2, 1/2>$$
$$\pi^0 : |1, 0>$$
$$D^+\pi^0 : |1, 0> \times |1/2, 1/2> = \sqrt{\tfrac{2}{3}}|3/2, 1/2> - \sqrt{\tfrac{1}{3}}|1/2, 1/2>$$

hence

$$\Gamma(D^{**+} \to D^0\pi^+) : \Gamma(D^{**+} \to D^+\pi^0) = \frac{2}{3} : \frac{1}{3} = 2 : 1$$

Branching fractions for $B \to D\pi\ell\nu$ decays are

$$\begin{aligned}
&2\mathcal{B}(B^+ \to \bar{D}^0\pi^0\ell^+\nu) = \mathcal{B}(B^+ \to D^-\pi^+\ell^+\nu) \\
&= 2t_{+0}\mathcal{B}(B^0 \to D^-\pi^0\ell^+\nu) = t_{+0}\mathcal{B}(B^0 \to \bar{D}^0\pi^-\ell^+\nu) \\
&2\mathcal{B}(B^+ \to \bar{D}^{*0}\pi^0\ell^+\nu) = \mathcal{B}(B^+ \to D^{*-}\pi^+\ell^+\nu) \\
&= 2t_{+0}\mathcal{B}(B^0 \to D^{*-}\pi^0\ell^+\nu) = t_{+0}\mathcal{B}(B^0 \to \bar{D}^{*0}\pi^-\ell^+\nu)
\end{aligned}$$

Supposing total rates or lifetimes of D^{**0} and D^{**+} are same, branching fractions for D^{**} decays are

$$\begin{aligned}
&\mathcal{B}(D^{**0} \to D^+\pi^-) = 2\mathcal{B}(D^{**0} \to D^0\pi^0) = \mathcal{B}(D^{**+} \to D^0\pi^+) = 2\mathcal{B}(D^{**+} \to D^+\pi^0) \\
&\mathcal{B}(D^{**0} \to D^{*+}\pi^-) = 2\mathcal{B}(D^{**0} \to D^{*0}\pi^0) = \mathcal{B}(D^{**+} \to D^{*0}\pi^+) = 2\mathcal{B}(D^{**+} \to D^{*+}\pi^0)
\end{aligned}$$

4.2.2 Definition of notation

We define branching fractions as follows

$$
\begin{aligned}
B_{B1} &\equiv \mathcal{B}(B^+ \to \bar{D}^0\ell^+\nu) = t_{+0}\mathcal{B}(B^0 \to D^-\ell^+\nu) \\
B_{B2} &\equiv \mathcal{B}(B^+ \to \bar{D}^{*0}\ell^+\nu) = t_{+0}\mathcal{B}(B^0 \to D^{*-}\ell^+\nu) \\
B_{B31} &\equiv \mathcal{B}(B^+ \to \bar{D}_0^{*0}\ell^+\nu) = t_{+0}\mathcal{B}(B^0 \to D_0^{*-}\ell^+\nu) \\
B_{B32} &\equiv \mathcal{B}(B^+ \to \bar{D}_1^{0}\ell^+\nu) = t_{+0}\mathcal{B}(B^0 \to D_1^{-}\ell^+\nu) \\
B_{B33} &\equiv \mathcal{B}(B^+ \to \bar{D}_1^{\prime 0}\ell^+\nu) = t_{+0}\mathcal{B}(B^0 \to D_1^{\prime -}\ell^+\nu) \\
B_{B34} &\equiv \mathcal{B}(B^+ \to \bar{D}_2^{*0}\ell^+\nu) = t_{+0}\mathcal{B}(B^0 \to D_2^{*-}\ell^+\nu) \\
B_{B41} &\equiv \mathcal{B}(B^+ \to D^-\pi^+\ell^+\nu) = 2\mathcal{B}(B^+ \to \bar{D}^0\pi^0\ell^+\nu) \\
&= t_{+0}\mathcal{B}(B^0 \to \bar{D}^0\pi^-\ell^+\nu) = 2t_{+0}\mathcal{B}(B^0 \to D^-\pi^0\ell^+\nu) \\
B_{B42} &\equiv \mathcal{B}(B^+ \to D^{*-}\pi^+\ell^+\nu) = 2\mathcal{B}(B^+ \to \bar{D}^{*0}\pi^0\ell^+\nu) \\
&= t_{+0}\mathcal{B}(B^0 \to \bar{D}^{*0}\pi^-\ell^+\nu) = 2t_{+0}\mathcal{B}(B^0 \to D^{*-}\pi^0\ell^+\nu) \\
B_{D^{**}11} &\equiv \mathcal{B}(D_0^{*0} \to D^+\pi^-) = 2\mathcal{B}(D_0^{*0} \to D^0\pi^0) \\
&= \mathcal{B}(D_0^{*+} \to D^0\pi^+) = 2\mathcal{B}(D_0^{*+} \to D^+\pi^0) \\
B_{D^{**}12} &\equiv \mathcal{B}(D_0^{*0} \to D^{*+}\pi^-) = 2\mathcal{B}(D_0^{*0} \to D^{*0}\pi^0) \\
&= \mathcal{B}(D_0^{*+} \to D^{*0}\pi^+) = 2\mathcal{B}(D_0^{*+} \to D^{*+}\pi^0) \\
B_{D^{**}21} &\equiv \mathcal{B}(D_1^{0} \to D^+\pi^-) = 2\mathcal{B}(D_1^{0} \to D^0\pi^0) \\
&= \mathcal{B}(D_1^{+} \to D^0\pi^+) = 2\mathcal{B}(D_1^{+} \to D^+\pi^0) \\
B_{D^{**}22} &\equiv \mathcal{B}(D_1^{0} \to D^{*+}\pi^-) = 2\mathcal{B}(D_1^{0} \to D^{*0}\pi^0) \\
&= \mathcal{B}(D_1^{+} \to D^{*0}\pi^+) = 2\mathcal{B}(D_1^{+} \to D^{*+}\pi^0) \\
B_{D^{**}31} &\equiv \mathcal{B}(D_1^{\prime 0} \to D^+\pi^-) = 2\mathcal{B}(D_1^{\prime 0} \to D^0\pi^0) \\
&= \mathcal{B}(D_1^{\prime +} \to D^0\pi^+) = 2\mathcal{B}(D_1^{\prime +} \to D^+\pi^0) \\
B_{D^{**}32} &\equiv \mathcal{B}(D_1^{0} \to D^{*+}\pi^-) = 2\mathcal{B}(D_1^{0} \to D^{*0}\pi^0) \\
&= \mathcal{B}(D_1^{+} \to D^{*0}\pi^+) = 2\mathcal{B}(D_1^{+} \to D^{*+}\pi^0) \\
B_{D^{**}41} &\equiv \mathcal{B}(D_2^{*0} \to D^+\pi^-) = 2\mathcal{B}(D_2^{*0} \to D^0\pi^0) \\
&= \mathcal{B}(D_2^{*+} \to D^0\pi^+) = 2\mathcal{B}(D_2^{*+} \to D^+\pi^0) \\
B_{D^{**}42} &\equiv \mathcal{B}(D_2^{*0} \to D^{*+}\pi^-) = 2\mathcal{B}(D_2^{*0} \to D^{*0}\pi^0) \\
&= \mathcal{B}(D_2^{*+} \to D^{*0}\pi^+) = 2\mathcal{B}(D_2^{*+} \to D^{*+}\pi^0) \\
B_{D^{*0}1} &\equiv \mathcal{B}(D^{*0} \to D^0\pi^0) = 61.9 \pm 2.9\% \\
B_{D^{*0}2} &\equiv \mathcal{B}(D^{*0} \to D^0\gamma) = 38.1 \pm 2.9\% \\
B_{D^{*+}1} &\equiv \mathcal{B}(D^{*+} \to D^0\pi^+) = 67.7 \pm 0.5\% \\
B_{D^{*+}2} &\equiv \mathcal{B}(D^{*+} \to D^+\pi^0) = 30.7 \pm 0.5\% \\
B_{D^{*+}3} &\equiv \mathcal{B}(D^{*+} \to D^+\gamma) = 1.6 \pm 0.4\% \\
B_{D^01} &\equiv \mathcal{B}(D^0 \to K^-\pi^+) = 3.80 \pm 0.09\% \\
B_{D^02} &\equiv \mathcal{B}(D^0 \to K^-\pi^+\pi^+\pi^-) = 7.46 \pm 0.31\% \\
B_{D^03} &\equiv \mathcal{B}(D^0 \to K^-\pi^+\pi^0) = 13.0 \pm 0.8\% \\
B_{D^04} &\equiv \mathcal{B}(D^0 \to \bar{K}^0\pi^+\pi^-) = 5.97 \pm 0.35\% \\
B_{D^+1} &\equiv \mathcal{B}(D^+ \to K^-\pi^+\pi^+) = 9.2 \pm 0.6\% \\
B_{D^+2} &\equiv \mathcal{B}(D^+ \to \bar{K}^0\pi^+) = 2.82 \pm 0.19\%
\end{aligned}
$$

The following is the values used in SP5/SP6. These values are used to calculate C_j at the first iteration of fitting.

$$
\begin{aligned}
&B_{B1} = 0.0560\\
&B_{B2} = 0.0210\\
&B_{B31} = 0.0020\\
&B_{B32} = 0.0056\\
&B_{B33} = 0.0037\\
&B_{B34} = 0.0037\\
&B_{B41} = 0.0060\\
&B_{B42} = 0.0020\\
&B_{D^{**}11} = 2/3\\
&B_{D^{**}12} = 0\\
&B_{D^{**}21} = 2/3\\
&B_{D^{**}22} = 0\\
&B_{D^{**}31} = 2/3\\
&B_{D^{**}32} = 0\\
&B_{D^{**}41} = \mathcal{B}(D_2^{*0} \to D^+\pi^-) = 0.4590\\
&\mathcal{B}(D_2^{*0} \to D^0\pi^0) = 0.2290\\
&B_{D^{**}42} = \mathcal{B}(D_2^{*0} \to D^{*+}\pi^-) = 0.2090\\
&\mathcal{B}(D_2^{*0} \to D^{*0}\pi^0) = 0.1030
\end{aligned}
$$

The following numbers are also used in SP5/SP6

$$
\begin{aligned}
&t_{+0} = 1\\
&f_{+0} = 1\\
&B_{D^{*0}1} = 0.6190\\
&B_{D^{*0}2} = 0.3810\\
&B_{D^{*+}1} = 0.6830\\
&B_{D^{*+}2} = 0.3060\\
&B_{D^{*+}3} = 0.0110\\
&B_{D^01} = 0.0383\\
&B_{D^02} = 0.0074\\
&B_{D^03} = 0.1390\\
&B_{D^04} = 0.0540\\
&B_{D^+1} = 0.0900\\
&B_{D^+2} = 0.0289
\end{aligned}
$$

4.2.3 Some assumptions

The branching fractions B_{B1}, B_{B2}, B_{B31}, B_{B32}, B_{B33}, B_{B34}, B_{B41}, and B_{B42} are the unknowns or parameters we want to determine by fitting. However, it may be difficult to determine B_{B32}, B_{B33}, and B_{B34} separately. And it is also difficult to distinguish B_{B41} from B_{B42}. Hence, we need some assumptions to express B_{B32}, B_{B33}, and B_{B34} by one unknown B_{B3} and to express B_{B41} and B_{B42} by one unknown B_{B4}

D^{**} branching fractions are not well known, but our fitting may not have the power to determine $B_{D^{**}11}$ - $B_{D^{**}42}$ on top of B_{B1}, B_{B2}, etc. Thus, we need some assumptions to give numerical values to these D^{**} branching fractions or to express these branching fractions by one or two parameters and let them float in the fitting.

4.3 $D^0 \to K^- \pi^+$ mode

4.3.1 $j = 1:\ B^+ \to D\ell\nu$

The decay contribute to this mode is
$B^+ \to \bar{D}^0 \ell^+ \nu$
Number of produced events are

$$N_1^{D^0} = N_{chrg}^{prod} B_{B1} B_{D^0 1} = N_{neut}^{prod} f_{+0} B_{B1} B_{D^0 1}$$

Since N_{neut}^{prod} and $B_{D^0 1}$ are common to data and MC and are not relevant for relative branching fraction measurements, the coefficient is

$$C_1^{D^0} = f_{+0} B_{B1}$$

4.3.2 $j = 2:\ B^+ \to D^* \ell\nu$

The decay contribute to this mode is
$B^+ \to \bar{D}^{*0} \ell^+ \nu$ and $\bar{D}^{*0} \to \bar{D}^0 + \gamma/\pi^0$
Number of produced events are

$$N_2^{D^0} = N_{chrg}^{prod} B_{B2} (B_{D^{*0}1} + B_{D^{*0}2}) B_{D^0 1} = N_{neut}^{prod} f_{+0} B_{B2} B_{D^0 1}$$

Since N_{neut}^{prod} and $B_{D^0 1}$ are common to data and MC and are not relevant for relative branching fraction measurements, the coefficient is

$$C_2^{D^0} = f_{+0} B_{B2}$$

4.3.3 $j = 3:\ B^0 \to D^* \ell\nu$

The decay contribute to this mode is
$B^0 \to D^{*-} \ell^+ \nu$ and $D^{*-} \to \bar{D}^0 + \pi^-$
Number of produced events are

$$N_3^{D^0} = N_{neut}^{prod} \frac{B_{B2}}{t_{+0}} B_{D^{*+}1} B_{D^0 1} = N_{neut}^{prod} B_{B2} B_{D^0 1} \frac{B_{D^{*+}1}}{t_{+0}}$$

Since N_{neut}^{prod} and $B_{D^0 1}$ are common to data and MC and are not relevant for relative branching fraction measurements, the coefficient is

$$C_3^{D^0} = \frac{B_{D^{*+}1}}{t_{+0}} B_{B2}$$

4.3.4 $j = 4$: $B^+ \to D^{**}\ell\nu$ $(D^{**} = D_0^*)$

Three decay modes contribute to this mode
1. $B^+ \to \bar{D}^{**0}\ell^+\nu$ and $\bar{D}^{**0} \to D^0 + \pi^0$
2. $B^+ \to \bar{D}^{**0}\ell^+\nu$ and $\bar{D}^{**0} \to D^{*-} + \pi^+$ and $D^{*-} \to \bar{D}^0 + \pi^-$
3. $B^+ \to \bar{D}^{**0}\ell^+\nu$ and $\bar{D}^{**0} \to D^{*0} + \pi^0$ and $D^{*0} \to \bar{D}^0 + \pi^0/\gamma$

Hence, number of events is

$$\begin{aligned} N_4^{D^0} &= N_{chrg}^{prod} B_{B31} \tfrac{B_{D^{**}11}}{2} B_{D^01} + N_{chrg}^{prod} B_{B31} B_{D^{**}12} B_{D^{*+}1} B_{D^01} \\ &\quad + N_{chrg}^{prod} B_{B31} \tfrac{B_{D^{**}12}}{2} (B_{D^{*0}1} + B_{D^{*0}2}) B_{D^01} B_{D^{*+}1} B_{D^01} \\ &= N_{neut}^{prod} B_{D^01} B_{B31} f_{+0} \left(\tfrac{B_{D^{**}11}}{2} + B_{D^{**}12} \left(B_{D^{*+}1} + \tfrac{1}{2} \right) \right) \end{aligned}$$

Since N_{neut}^{prod} and B_{D^01} are common to data and MC and are not relevant for relative branching fraction measurements, the coefficient is

$$C_4^{D^0} = B_{B31} f_{+0} \left[\frac{B_{D^{**}11}}{2} + B_{D^{**}12} \left(B_{D^{*+}1} + \frac{1}{2} \right) \right]$$

4.3.5 $j = 5$: $B^0 \to D^{**}\ell\nu$ $(D^{**} = D_0^*)$

Three decay mode contribute to this mode
1. $B^0 \to D^{**-}\ell^+\nu$ and $D^{**-} \to \bar{D}^0 + \pi^-$
2. $B^0 \to D^{**-}\ell^+\nu$ and $D^{**-} \to \bar{D}^{*0} + \pi^-$ and $\bar{D}^{*0} \to \bar{D}^0 + \gamma/\pi^0$
3. $B^0 \to D^{**-}\ell^+\nu$ and $D^{**-} \to D^{*+} + \pi^0$ and $D^{*+} \to \bar{D}^0 + \pi^+$

Hence, number of events is

$$\begin{aligned} N_5^{D^0} &= N_{neut}^{prod} \tfrac{B_{B31}}{t_{+0}} B_{D^{**}11} B_{D^01} + N_{neut}^{prod} \tfrac{B_{B31}}{t_{+0}} B_{D^{**}12} (B_{D^{*0}1} + B_{D^{*0}2}) B_{D^01} \\ &\quad + N_{neut}^{prod} \tfrac{B_{B31}}{t_{+0}} \tfrac{B_{D^{**}12}}{2} B_{D^{*+}1} B_{D^01} \\ &= N_{neut}^{prod} B_{D^01} B_{B31} \left[B_{D^{**}11} + B_{D^{**}12} \left(1 + \tfrac{B_{D^{*+}1}}{2} \right) \right] \end{aligned}$$

Since N_{neut}^{prod} and B_{D^01} are common to data and MC and are not relevant for relative branching fraction measurements, the coefficient is

$$C_5^{D^0} = B_{B31} \left[B_{D^{**}11} + B_{D^{**}12} \left(1 + \frac{B_{D^{*+}1}}{2} \right) \right]$$

4.3.6 $j = 6$: $B^+ \to D^{**}\ell\nu$ $(D^{**} = D_1)$

Same as $j = 4$ case

$$C_6^{D^0} = B_{B32} f_{+0} \left[\frac{B_{D^{**}21}}{2} + B_{D^{**}22} \left(B_{D^{*+}1} + \frac{1}{2} \right) \right]$$

4.3.7 $j = 7$: $B^0 \to D^{**}\ell\nu$ $(D^{**} = D_1)$

Same as $j = 5$ case

$$C_7^{D^0} = B_{B32} \left[B_{D^{**}21} + B_{D^{**}22} \left(1 + \frac{B_{D^{*+}1}}{2} \right) \right]$$

4.3.8 $j = 8:\ B^+ \to D^{**}\ell\nu\ (D^{**} = D_1')$

Same as $j = 4$ case

$$C_8^{D^0} = B_{B33} f_{+0} \left[\frac{B_{D^{**}31}}{2} + B_{D^{**}32} \left(B_{D^{*+}1} + \frac{1}{2} \right) \right]$$

4.3.9 $j = 9:\ B^0 \to D^{**}\ell\nu\ (D^{**} = D_1')$

Same as $j = 5$ case

$$C_9^{D^0} = B_{B33} \left[B_{D^{**}31} + B_{D^{**}32} \left(1 + \frac{B_{D^{*+}1}}{2} \right) \right]$$

4.3.10 $j = 10:\ B^+ \to D^{**}\ell\nu\ (D^{**} = D_2^*)$

Same as $j = 4$ case

$$C_{10}^{D^0} = B_{B34} f_{+0} \left[\frac{B_{D^{**}41}}{2} + B_{D^{**}42} \left(B_{D^{*+}1} + \frac{1}{2} \right) \right]$$

4.3.11 $j = 11:\ B^0 \to D^{**}\ell\nu\ (D^{**} = D_2^*)$

Same as $j = 5$ case

$$C_{11}^{D^0} = B_{B34} \left[B_{D^{**}41} + B_{D^{**}42} \left(1 + \frac{B_{D^{*+}1}}{2} \right) \right]$$

4.3.12 $j = 12:\ B^+ \to D\pi\ell\nu$

The decay contribute to this mode is
$B^+ \to \bar{D}^0 \pi^0 \ell^+ \nu$
Number of events is

$$N_{12}^{D^0} = N_{chrg}^{prod} \frac{B_{B41}}{2} B_{D^0 1} = N_{neut}^{prod} f_{+0} B_{D^0 1} \frac{B_{B41}}{2}$$

Since N_{neut}^{prod} and $B_{D^0 1}$ are common to data and MC and are not relevant for relative branching fraction measurements, the coefficient is

$$C_{12}^{D^0} = B_{B41} \frac{f_{+0}}{2}$$

4.3.13 $j = 13:\ B^0 \to D\pi\ell\nu$

The decay contribute to this mode is
$B^0 \to \bar{D}^0 \pi^- \ell^+ \nu$
Number of events is

$$N_{13}^{D^0} = N_{neut}^{prod} \frac{B_{B41}}{t_{+0}} B_{D^0 1}$$

Since N_{neut}^{prod} and B_{D^01} are common to data and MC and are not relevant for relative branching fraction measurements, the coefficient is

$$C_{13}^{D^0} = B_{B41}\frac{1}{t_{+0}}$$

4.3.14 $j = 14$: $B^+ \to D^*\pi\ell\nu$

Two decay modes contribute to this mode
1. $B^+ \to \bar{D}^{*0}\pi^0\ell^+\nu$ and $\bar{D}^{*0} \to \bar{D}^0 + \gamma/\pi^0$
2. $B^+ \to D^{*-}\pi^+\ell^+\nu$ and $D^{*-} \to \bar{D}^0 + \pi^-$
Hence, the number of events is

$$\begin{aligned} N_{14}^{D^0} &= N_{chrg}^{prod}\tfrac{B_{B42}}{2}(B_{D^{*0}1} + B_{D^{*0}2})B_{D^01} + N_{chrg}^{prod}B_{B42}B_{D^{*+}1}B_{D^01} \\ &= N_{neut}^{prod}f_{+0}B_{D^01}\left(\tfrac{B_{B42}}{2} + B_{B42}B_{D^{*+}1}\right) \end{aligned}$$

Since N_{neut}^{prod} and B_{D^01} are common to data and MC and are not relevant for relative branching fraction measurements, the coefficient is

$$C_{14}^{D^0} = B_{B42}f_{+0}\left(B_{D^{*+}1} + \frac{1}{2}\right)$$

4.3.15 $j = 15$: $B^0 \to D^*\pi\ell\nu$

Two decay modes contribute to this mode
1. $B^0 \to \bar{D}^{*0}\pi^-\ell^+\nu$ and $\bar{D}^{*0} \to \bar{D}^0 + \gamma/\pi^0$
2. $B^0 \to D^{*-}\pi^0\ell^+\nu$ and $D^{*-} \to \bar{D}^0 + \pi^-$
Hence, the number of events is

$$\begin{aligned} N_{15}^{D^0} &= N_{neut}^{prod}\tfrac{B_{B42}}{t_{+0}}(B_{D^{*0}1} + B_{D^{*0}2})B_{D^01} + N_{neut}^{prod}\tfrac{B_{B42}}{2t_{+0}}B_{D^{*+}1}B_{D^01} \\ &= N_{neut}^{prod}B_{D^01}\tfrac{B_{B42}}{t_{+0}}\left(1 + \tfrac{1}{2}B_{D^{*+}1}\right) \end{aligned}$$

Since N_{neut}^{prod} and B_{D^01} are common to data and MC and are not relevant for relative branching fraction measurements, the coefficient is

$$C_{15}^{D^0} = B_{B42}\frac{2 + B_{D^{*+}1}}{2t_{+0}}$$

4.4 $D^+ \to K^-\pi^+\pi^+$ mode

4.4.1 $j = 1$: $B^0 \to D\ell\nu$

The decay contribute to this mode is
$\bar{B}^0 \to D^+\ell^-\nu$
Hence, the number of events is

$$N_1^{D^+} = N_{neut}^{prod}\frac{B_{B1}}{t_{+0}}B_{D^+1} = N_{neut}^{prod}B_{D^+1}\frac{B_{B1}}{t_{+0}}$$

Since N_{neut}^{prod} and B_{D^+1} are common to data and MC and are not relevant for relative branching fraction measurements, the coefficient is

$$C_1^{D^+} = B_{B1}\frac{1}{t_{+0}}$$

4.4.2 $j = 2$: $B^0 \to D^*\ell\nu$

The decay contribute to this mode is
$\bar{B}^0 \to D^{*+}\ell^-\nu$ and $D^{*+} \to D^+ + \gamma/\pi^0$
Hence, the number of events is

$$N_2^{D^+} = N_{neut}^{prod}\frac{B_{B2}}{t_{+0}}(B_{D^{*+}2} + B_{D^{*+}3})B_{D^+1} = N_{neut}^{prod}B_{D^+1}B_{B2}\frac{B_{D^{*+}2} + B_{D^{*+}3}}{t_{+0}}$$

Since N_{neut}^{prod} and B_{D^+1} are common to data and MC and are not relevant for relative branching fraction measurements, the coefficient is

$$C_2^{D^+} = B_{B2}\frac{B_{D^{*+}2} + B_{D^{*+}3}}{t_{+0}}$$

4.4.3 $j = 3$: $B^0 \to D^{**}\ell\nu$ $(D^{**} = D_0^*)$

Two decay modes contribute to this mode
1. $\bar{B}^0 \to D^{**+}\ell^-\nu$ and $D^{**+} \to D^+ + \pi^0$
2. $\bar{B}^0 \to D^{**+}\ell^-\nu$ and $D^{**+} \to D^{*+} + \pi^0$ and $D^{*+} \to \bar{D}^0 + \pi^+$

Hence, the number of events is

$$\begin{aligned} N_3^{D^+} &= N_{neut}^{prod}\tfrac{B_{B31}}{t_{+0}}\tfrac{B_{D^{**}11}}{2}B_{D^+1} + N_{neut}^{prod}\tfrac{B_{B31}}{t_{+0}}\tfrac{B_{D^{**}12}}{2}B_{D^{*+}1}B_{D^+1} \\ &= N_{neut}^{prod}B_{D^+1}B_{B31}\left(\tfrac{B_{D^{**}11}}{2t_{+0}} + B_{D^{**}12}\tfrac{B_{D^{*+}1}}{2t_{+0}}\right) \end{aligned}$$

Since N_{neut}^{prod} and B_{D^+1} are common to data and MC and are not relevant for relative branching fraction measurements, the coefficient is

$$C_3^{D^+} = B_{B31}\frac{B_{D^{**}11} + B_{D^{**}12}B_{D^{*+}1}}{2t_{+0}}$$

4.4.4 $j = 4$: $B^+ \to D^{**}\ell\nu$ $(D^{**} = D_0^*)$

Two decay modes contribute to this mode
1. $B^- \to D^{**0}\ell^-\nu$ and $D^{**0} \to D^+\pi^-$
2. $B^- \to D^{**0}\ell^-\nu$ and $D^{**0} \to D^{*+} + \pi^-$ and $D^{*+} \to D^+ + \gamma/\pi^0$

Hence, the number of events is

$$\begin{aligned} N_4^{D^+} &= N_{chrg}^{prod}B_{B31}B_{D^{**}11}B_{D^+1} + N_{chrg}^{prod}B_{B31}B_{D^{**}12}(B_{D^{*+}2} + B_{D^{*+}3})B_{D^+1} \\ &= N_{neut}^{prod}B_{D^+1}B_{B31}f_{+0}\left(B_{D^{**}11} + B_{D^{**}12}(B_{D^{*+}2} + B_{D^{*+}3})\right) \end{aligned}$$

Since N_{neut}^{prod} and B_{D^+1} are common to data and MC and are not relevant for relative branching fraction measurements, the coefficient is

$$C_4^{D^+} = B_{B31}f_{+0}\left[B_{D^{**}11} + B_{D^{**}12}(B_{D^{*+}2} + B_{D^{*+}3})\right]$$

4.4.5 $j = 5$: $B^0 \to D^{**}\ell\nu$ $(D^{**} = D_1)$

Same as $j = 3$

$$C_5^{D^+} = B_{B32}\frac{B_{D^{**}21} + B_{D^{**}22}B_{D^{*+}1}}{2t_{+0}}$$

4.4.6 $j = 6$: $B^+ \to D^{**}\ell\nu$ $(D^{**} = D_1)$

Same as $j = 4$

$$C_6^{D^+} = B_{B32}f_{+0}\left[B_{D^{**}21} + B_{D^{**}22}(B_{D^{*+}2} + B_{D^{*+}3})\right]$$

4.4.7 $j = 7$: $B^0 \to D^{**}\ell\nu$ $(D^{**} = D_1')$

Same as $j = 3$

$$C_7^{D^+} = B_{B33}\frac{B_{D^{**}31} + B_{D^{**}32}B_{D^{*+}1}}{2t_{+0}}$$

4.4.8 $j = 8$: $B^+ \to D^{**}\ell\nu$ $(D^{**} = D_1')$

Same as $j = 4$

$$C_8^{D^+} = B_{B33}f_{+0}\left[B_{D^{**}31} + B_{D^{**}32}(B_{D^{*+}2} + B_{D^{*+}3})\right]$$

4.4.9 $j = 9$: $B^0 \to D^{**}\ell\nu$ $(D^{**} = D_2^*)$

Same as $j = 3$

$$C_9^{D^+} = B_{B34}\frac{B_{D^{**}41} + B_{D^{**}42}B_{D^{*+}1}}{2t_{+0}}$$

4.4.10 $j = 10$: $B^+ \to D^{**}\ell\nu$ $(D^{**} = D_2^*)$

Same as $j = 4$

$$C_{10}^{D^+} = B_{B34}f_{+0}\left[B_{D^{**}41} + B_{D^{**}42}(B_{D^{*+}2} + B_{D^{*+}3})\right]$$

4.4.11 $j = 11$: $B^0 \to D\pi\ell\nu$

The decay mode contribute to this mode
$\bar{B}^0 \to D^+\pi^0\ell^-\nu$
Hence, the number of events is

$$N_{11}^{D^+} = N_{neut}^{prod}\frac{B_{B41}}{t_{+0}}B_{D^+1}$$

Since N_{neut}^{prod} and B_{D^+1} are common to data and MC and are not relevant for relative branching fraction measurements,

$$C_{11}^{D^+} = B_{B41}\frac{1}{t_{+0}}$$

4.4.12 $j = 12$: $B^+ \to D\pi\ell\nu$

The decay mode contribute to this mode
$B^- \to D^+\pi^-\ell^-\nu$
Hence, the number of events is

$$N_{12}^{D^+} = N_{chrg}^{prod} B_{B41} B_{D^+1} = N_{neut}^{prod} B_{D^+1} f_{+0} B_{B41}$$

Since N_{neut}^{prod} and B_{D^+1} are common to data and MC and are not relevant for relative branching fraction measurements,

$$C_{12}^{D^+} = B_{B41} f_{+0}$$

4.4.13 $j = 13$: $B^0 \to D^*\pi\ell\nu$

The decay mode contribute to this mode
$\bar{B}^0 \to D^{*+}\pi^0\ell^-\nu$ and $D^{*+} \to D^+ + \gamma/\pi^0$
Hence, the number of events is

$$N_{13}^{D^+} = N_{neut}^{prod} \frac{B_{B42}}{t_{+0}} (B_{D^{*+}2} + B_{D^{*+}3}) B_{D^+1}$$

Since N_{neut}^{prod} and B_{D^+1} are common to data and MC and are not relevant for relative branching fraction measurements,

$$C_{13}^{D^+} = B_{B42} \frac{B_{D^{*+}2} + B_{D^{*+}3}}{t_{+0}}$$

4.4.14 $j = 14$: $B^+ \to D^*\pi\ell\nu$

The decay mode contribute to this mode
$B^- \to D^{*+}\pi^-\ell^-\nu$ and $D^{*+} \to D^+ + \gamma/\pi^0$
Hence, the number of events is

$$N_{14}^{D^+} = N_{chrg}^{prod} B_{B42} (B_{D^{*+}2} + B_{D^{*+}3}) B_{D^+1} = N_{neut}^{prod} B_{D^+1} f_{+0} B_{B42} (B_{D^{*+}2} + B_{D^{*+}3})$$

Since N_{neut}^{prod} and B_{D^+1} are common to data and MC and are not relevant for relative branching fraction measurements,

$$C_{14}^{D^+} = B_{B42} f_{+0} (B_{D^{*+}2} + B_{D^{*+}3})$$

5 Efficiecncy to detect Dl candidates

Since we are not trying to determine branching fraction itself, we do not need the efficiency to detect Dl candidates. However, we can determine an average efficiecncy ϵ to detect Dl candidates from MC by using MC truth info.
The number of produced Dl candidates is calculated from the number of produced MC events in SP5/SP6

5.1 $B^+ \to \bar{D}^0 \ell^+ \nu$ mode : ϵ for D^0

The branching fraction of this decay mode used in SP5/SP6 is

$$\mathcal{B}(B^+ \to \bar{D}^0 \ell^+ \nu) = 0.0210$$

Using 86492000 events of Run3 B^+B^- MC, the number of produced Dl candidates for $D^0 \to K\pi$ (branching fraction = 0.0380) mode is

$$N_{Dl} = 2 \times 86492000 \times 0.0210 \times 0.0380 = 138041$$

The number of observed Dl candidates from signal $B^+ \to \bar{D}^0 \ell^+ \nu$ mode, after D mass side-band subtraction, is 73854

Hence for D^0 mode,

$$\epsilon = 73854/138041 = 0.53501$$

5.2 $\bar{B}^0 \to D^+ \ell^- \nu$ mode : ϵ for D^+

The branching fraction of this decay mode used in SP5/SP6 is

$$\mathcal{B}(\bar{B}^0 \to D^+ \ell^- \nu) = 0.0210$$

Using 61158000 events of Run3 $B^0\bar{B}^0$ MC, the number of produced Dl candidates for $D^+ \to K\pi\pi$ (branching fraction = 0.092) mode is

$$N_{Dl} = 2 \times 61158000 \times 0.0210 \times 0.092 = 236314$$

The number of observed Dl candidates from signal $\bar{B}^0 \to D^+ \ell^- \nu$ mode, after D mass side-band subtraction, is 91775

Hence for D^+ mode,

$$\epsilon = 91775/236314 = 0.38836$$