CM enrgy of beam

1. Lab frame

Beam energies of e^- and e^+ in the LAB frame are

$$E_- = 9\text{GeV}, \quad E_+ = 3.1\text{GeV}$$

So, the total energy is

$$E_{\text{LAB}} = E_- + E_+ = 12.2\text{GeV}$$

2. CM frame

In the CM frame, 4-momenta of e^- and e^+ may be

$$p_{e^-} = (E, \vec{p}), \quad p_{e^+} = (E, -\vec{p})$$

Hence,

$$(p_{e^-} + p_{e^+})^2 = (2E, \vec{0})^2 = 4E^2$$

In the CM frame, 4-momenta of e^- and e^+ may be

$$p_{e^-} = (E_-, \vec{p}_-), \quad p_{e^+} = (E_+, \vec{p}_+)$$

Hence

$$(p_{e^-}+p_{e^+})^2 = p_{e^-}^2+p_{e^+}^2+2p_{e^-}\cdot p_{e^+} = m^2+m^2+2(E_-E_+-\vec{p}_-\cdot\vec{p}_+) = 2(E_-E_++|\vec{p}_-||\vec{p}_+|) \simeq 4E_-E_+$$

By comparing the two results

$$E = \sqrt{E_-E_+} = \sqrt{(9\text{GeV})(3.1\text{GeV})} = 5.3\text{GeV}$$

Total energy is

$$E_{\text{CM}} = 2E = 10.6\text{GeV}$$

3. Fixed target

If the positron target is fixed

$$p_{e^-} = (m, \vec{0}), \quad p_{e^+} = (E_0, \vec{p}_0)$$

Hence

$$(p_{e^-} + p_{e^+})^2 = m^2 + m^2 + 2[(m, \vec{0}) \cdot (E_0, \vec{p}_0)] \simeq 2E_0 m$$

Therefore

$$2E_0 m = 4E^2 \Rightarrow E_0 = 10^5\text{GeV}$$