Questions on LLSW paper 2

Summary of formulae in the paper.

First we summarize what is written in the paper based on our understanding. In the paper two constants ε_b and ε_c are defined as follows

$$\varepsilon_b \equiv \frac{1}{2m_b}, \quad \varepsilon_c \equiv \frac{1}{2m_c} \tag{1}$$

with numerical values

$$\begin{aligned} m_b &= 4.8\text{GeV} \\ m_c &= 1.4\text{GeV} \end{aligned} \tag{2}$$

The difference between a hadrom mass and a heavy quark mass in the first order :

$$\begin{aligned} &\bar{\Lambda} \text{ for ground state doublet } (D \text{ and } D^*) \\ &\bar{\Lambda}' \text{ for exited } \tfrac{3}{2}^+ \text{ doublet } (D_1 \text{ and } D_2^*) \\ &\bar{\Lambda}^* \text{ for exited } \tfrac{1}{2}^+ \text{ doublet } (D_0^* \text{ and } D_1') \end{aligned} \tag{3}$$

with numerical values

$$\begin{aligned} \bar{\Lambda} &= 0.4\text{GeV} \\ \bar{\Lambda}' - \bar{\Lambda} &= 0.39\text{GeV} \\ \bar{\Lambda}^* - \bar{\Lambda} &= 0.35\text{GeV} \end{aligned} \tag{4}$$

$B \to D_1 \ell \nu$ **case**

The matrix elements are

$$\frac{< D_1(v', \varepsilon)|V^\mu|B(v) >}{\sqrt{m_{D_1} m_B}} = f_{V_1} \varepsilon^{*\mu} + (f_{V_2} v^\mu + f_{V_3} v'^\mu)(\varepsilon^* \cdot v) \tag{5}$$

$$\frac{< D_1(v', \varepsilon)|A^\mu|B(v) >}{\sqrt{m_{D_1} m_B}} = i f_A \epsilon^{\mu\alpha\beta\gamma} \varepsilon^*_\alpha v_\beta v'_\gamma \tag{6}$$

The differential decay rate is

$$\begin{aligned} \frac{d^2\Gamma_{D_1}}{dw\, d\cos\theta} = 3\Gamma_0 r_1^3 \sqrt{w^2-1} \{ &\sin^2\theta [(w - r_1) f_{V_1} + (w^2 - 1)(f_{V_3} + r_1 f_{V_2})]^2 \\ &+ (1 - 2r_1 w + r_1^2)[(1 + \cos^2\theta)(f_{V_1}^2 + (w^2 - 1) f_A^2) \\ &- 4\cos\theta \sqrt{w^2 - 1} f_{V_1} f_A] \} \end{aligned} \tag{7}$$

where

$$\Gamma_0 = \frac{G_F^2 |V_{cb}|^2 m_B^5}{192\pi^3} \tag{8}$$

and

$$r_1 = \frac{m_{D_1}}{m_B} \tag{9}$$

In approximation B, the form factors are given by

$$\begin{aligned}
\sqrt{6}f_A &= -(w+1)\tau \\
&\quad -\varepsilon_b\{(w-1)[(\bar{\Lambda}'+\bar{\Lambda})\tau-(2w+1)\tau_1-\tau_2]+(w+1)\eta_b\} \\
&\quad -\varepsilon_c[4(w\bar{\Lambda}'-\bar{\Lambda})\tau-3(w-1)(\tau_1-\tau_2)+(w+1)(\eta_{\rm ke}-2\eta_1-3\eta_3)] \\
\sqrt{6}f_{V_1} &= (1-w^2)\tau \\
&\quad -\varepsilon_b(w^2-1)[(\bar{\Lambda}'+\bar{\Lambda})\tau-(2w+1)\tau_1-\tau_2+\eta_b] \\
&\quad -\varepsilon_c[4(w+1)(w\bar{\Lambda}'-\bar{\Lambda})\tau-(w^2-1)(3\tau_1-3\tau_2-\eta_{\rm ke}+2\eta_1+3\eta_3)] \\
\sqrt{6}f_{V_2} &= -3\tau \\
&\quad -3\varepsilon_b[(\bar{\Lambda}'+\bar{\Lambda})\tau-(2w+1)\tau_1-\tau_2+\eta_b] \\
&\quad -\varepsilon_c[(4w-1)\tau_1+5\tau_2+3\eta_{\rm ke}+10\eta_1+4(w-1)\eta_2-5\eta_3)] \\
\sqrt{6}f_{V_3} &= (w-2)\tau \\
&\quad +\varepsilon_b\{(2+w)[(\bar{\Lambda}'+\bar{\Lambda})\tau-(2w+1)\tau_1-\tau_2]-(2-w)\eta_b\} \\
&\quad +\varepsilon_c[4(w\bar{\Lambda}'-\bar{\Lambda})\tau+(2+w)\tau_1+(2+3w)\tau_2 \\
&\quad +(w-2)\eta_{\rm ke}-2(6+w)\eta_1-4(w-1)\eta_2-(3w-2)\eta_3]
\end{aligned} \tag{10}$$

where τ is the leading $m_Q \to \infty$ Isgur-Wise function, which is assumed to be a linear form:

$$\tau(w) = \tau(1)[1+\hat{\tau}'(w-1)] \tag{11}$$

where

$$\tau' = \frac{d\tau}{dw}, \quad \hat{\tau}' = \frac{\tau'}{\tau(1)} \tag{12}$$

Numerical values used in the paper are

$$\begin{aligned} \tau(1) &= 0.71 \\ \hat{\tau}' &= -1.5 \end{aligned} \tag{13}$$

The uncertainty in the $\Lambda_{\rm QCD}/m_Q$ corrections is parametrised by τ_1 and τ_2. In Approximation B_1,

$$\begin{aligned} \tau_1 &= 0 \\ \tau_2 &= 0 \end{aligned} \tag{14}$$

In Approximation B_2,

$$\begin{aligned} \tau_1 &= \bar{\Lambda}\tau \\ \tau_2 &= -\bar{\Lambda}'\tau \end{aligned} \tag{15}$$

These are the two end values of τ_1 and τ_2. True τ_1 and τ_2 reside somewher in between.

η_i comes from the correction to the HQET Lagrangian. In Approximation B,

$$\eta_1 = \eta_2 = \eta_3 = 0 \tag{16}$$

Also, in this Approximation, $\eta_1^{(b)} = \eta_2^{(b)} = \eta_3^{(b)} = 0$. Thus

$$\eta_b \equiv \eta_{\rm ke}^{(b)} + 6\eta_1^{(b)} - 2(w-1)\eta_2^{(b)} + \eta_3^{(b)} = \eta_{\rm ke}^{(b)} \tag{17}$$

Hence, the form factors are

$$\begin{aligned}
\sqrt{6} f_A &= -(w+1)(\tau + \varepsilon_c\eta_{\rm ke} + \varepsilon_b\eta_{\rm ke}^{(b)}) \\
&\quad -\varepsilon_b(w-1)[(\bar\Lambda' + \bar\Lambda)\tau - (2w+1)\tau_1 - \tau_2] \\
&\quad -\varepsilon_c[4(w\bar\Lambda' - \bar\Lambda)\tau - 3(w-1)(\tau_1 - \tau_2)] \\
\sqrt{6} f_{V_1} &= (1-w^2)(\tau + \varepsilon_c\eta_{\rm ke} + \varepsilon_b\eta_{\rm ke}^{(b)}) \\
&\quad -\varepsilon_b(w^2-1)[(\bar\Lambda' + \bar\Lambda)\tau - (2w+1)\tau_1 - \tau_2] \\
&\quad -\varepsilon_c[4(w+1)(w\bar\Lambda' - \bar\Lambda)\tau - (w^2-1)(3\tau_1 - 3\tau_2] \\
\sqrt{6} f_{V_2} &= -3(\tau + \varepsilon_c\eta_{\rm ke} + \varepsilon_b\eta_{\rm ke}^{(b)}) \\
&\quad -3\varepsilon_b[(\bar\Lambda' + \bar\Lambda)\tau - (2w+1)\tau_1 - \tau_2] \\
&\quad -\varepsilon_c[(4w-1)\tau_1 + 5\tau_2] \\
\sqrt{6} f_{V_3} &= (w-2)(\tau + \varepsilon_c\eta_{\rm ke} + \varepsilon_b\eta_{\rm ke}^{(b)}) \\
&\quad +\varepsilon_b(2+w)[(\bar\Lambda' + \bar\Lambda)\tau - (2w+1)\tau_1 - \tau_2] \\
&\quad +\varepsilon_c[4(w\bar\Lambda' - \bar\Lambda)\tau + (2+w)\tau_1 + (2+3w)\tau_2]
\end{aligned} \tag{18}$$

$\eta_{\rm ke}$ and $\eta_{\rm ke}^{(b)}$ can be absorbed into τ by the replacement of τ by

$$\tilde\tau = \tau + \varepsilon_c\eta_{\rm ke} + \varepsilon_b\eta_{\rm ke}^{(b)} \tag{19}$$

Thus, the form factors are

$$\begin{aligned}
\sqrt{6} f_A &= -(w+1)\tilde\tau - \varepsilon_b(w-1)[(\bar\Lambda' + \bar\Lambda)\tau - (2w+1)\tau_1 - \tau_2] \\
&\quad -\varepsilon_c[4(w\bar\Lambda' - \bar\Lambda)\tau - 3(w-1)(\tau_1 - \tau_2)] \\
\sqrt{6} f_{V_1} &= (1-w^2)\tilde\tau - \varepsilon_b(w^2-1)[(\bar\Lambda' + \bar\Lambda)\tau - (2w+1)\tau_1 - \tau_2] \\
&\quad -\varepsilon_c[4(w+1)(w\bar\Lambda' - \bar\Lambda)\tau - (w^2-1)(3\tau_1 - 3\tau_2] \\
\sqrt{6} f_{V_2} &= -3\tilde\tau - 3\varepsilon_b[(\bar\Lambda' + \bar\Lambda)\tau - (2w+1)\tau_1 - \tau_2] \\
&\quad -\varepsilon_c[(4w-1)\tau_1 + 5\tau_2] \\
\sqrt{6} f_{V_3} &= (w-2)\tilde\tau + \varepsilon_b(2+w)[(\bar\Lambda' + \bar\Lambda)\tau - (2w+1)\tau_1 - \tau_2] \\
&\quad +\varepsilon_c[4(w\bar\Lambda' - \bar\Lambda)\tau + (2+w)\tau_1 + (2+3w)\tau_2]
\end{aligned} \tag{20}$$

Questions.

1. Can we also replace other τ in the above formulae by $\tilde{\tau}$? I suppose that this replacement makes only the order $(\Lambda_{\mathrm{QCD}}/m_Q)^2$ change.

2. In the code used by DELPHI collaboration, they did not do this replacement. Instead they treat η_{ke} as an free parameter and used an assumption

$$\eta_{\mathrm{ke}}^{(b)} = \eta_{\mathrm{ke}}$$

Does this auumption correct within the the order $\Lambda_{\mathrm{QCD}}/m_Q^2$?