Calculation of cosBY

1. $B \rightarrow Dl + \nu$

In the decay of $B \rightarrow Dl + \nu$, 4-momentum of each particles are conserved

$$p_B = p_{Dl} + p_\nu$$

As the mass of neutrino is equal to zero

$$0 = p_\nu^2 = (p_B - p_{Dl})^2 = p_B^2 + p_{Dl}^2 - 2p_B \cdot p_{Dl} = m_B^2 + m_{Dl}^2 - 2p_B \cdot p_{Dl}$$

where

$$p_B \cdot p_{Dl} = E_B E_{Dl} - \vec{p}_B \cdot \vec{p}_{Dl} = E_B E_{Dl} - |\vec{p}_B||\vec{p}_{Dl}| \cos \Theta_{B-Dl}$$

Hence

$$\begin{aligned} 0 &= m_B^2 + m_{Dl}^2 - 2(E_B E_{Dl} - |\vec{p}_B||\vec{p}_{Dl}| \cos \Theta_{B-Dl}) \\ &\Rightarrow m_B^2 + m_{Dl}^2 = 2E_B E_{Dl} - 2|\vec{p}_B||\vec{p}_{Dl}| \cos \Theta_{B-Dl} \end{aligned}$$

Therefore

$$\cos \Theta_{B-Dl} = \frac{2E_B E_{Dl} - m_B^2 - m_{Dl}^2}{2|\vec{p}_B||\vec{p}_{Dl}|}$$

In practice, E_B is given by the half of the beam energy in the CM frame. $|\vec{p}_B|$ is given by

$$|\vec{p}_B| = \sqrt{E_B^2 - m_B^2}$$

2. With an additional pion

If ther is an additional particl $\cos \Theta_{B-Dl} < -1$ is possible.
Let us consider an additional pion, for example, from D^*

$$p_B = p_{Dl} + p_\pi + p_\nu$$

Then,

$$\begin{aligned} &(p_\nu + p_\pi)^2 = (p_B - p_{Dl})^2 \\ &\Rightarrow p_\nu^2 + p_\pi^2 + 2p_\nu \cdot p_\pi = p_B^2 + p_{Dl}^2 - 2p_B \cdot p_{Dl} \\ &\Rightarrow 0 + m_\pi^2 + 2p_\nu \cdot p_\pi = m_B^2 + m_{Dl}^2 - 2p_B \cdot p_{Dl} \end{aligned}$$

Since $m_\pi \ll m_B$,

$$0 + 0 + 2p_\nu \cdot p_\pi = m_B^2 + m_{Dl}^2 - 2p_B \cdot p_{Dl}$$

and also

$$\begin{aligned} p_\nu \cdot p_\pi &= E_\nu E_\pi - \vec{p}_\nu \cdot \vec{p}_\pi = E_\nu E_\pi - |\vec{p}_\pi||\vec{p}_B| \cos \Theta_{\pi-B} \\ &= E_\nu E_\pi - E_\nu E_\pi \cos \Theta_{\pi-B} = E_\nu E_\pi \left(1 - \cos \Theta_{\pi-B}\right) \geq 0 \end{aligned}$$

The Θ_{B-Dl} changes to Θ'_{B-Dl}

$$p_B \cdot p_{Dl} = E_B E_{Dl} - \vec{p}_B \cdot \vec{p}_{Dl} = E_B E_{Dl} - |\vec{p}_B||\vec{p}_{Dl}| \cos \Theta'_{B-Dl}$$

Hence,

$$\begin{array}{rcl}\cos\Theta'_{B-Dl} & = & \frac{2E_BE_{Dl}-m_B^2-m_{Dl}^2}{2|\vec{p}_B||\vec{p}_{Dl}|}+\frac{2p_\nu\cdot p_\pi}{2|\vec{p}_B||\vec{p}_{Dl}|} \\ & = & \cos\Theta_{B-Dl}+\alpha\end{array}$$

where

$$\alpha=\frac{2p_\nu\cdot p_\pi}{2|\vec{p}_B||\vec{p}_{Dl}|}\geq 0$$

Considering $-1\leq\cos\Theta'_{B-Dl}\leq 1$,

$$\cos\Theta_{B-Dl}=\cos\Theta'_{B-Dl}-\alpha$$

is

$$-1-\alpha\leq\cos\Theta_{B-Dl}\leq 1-\alpha\leq 1$$

Since $\alpha\geq 0$, the lower end of $\cos\Theta_{B-Dl}$ becomes smaller than -1, where as the upper end remains to 1.

3. If D and l came from different B.

In the case that D and l came from different B, $\cos\Theta_{B-Dl}>1$ is possible.
The p_{Dl} is

$$p_{Dl}=p_D+p_l=(E_D+E_l,-\vec{p}_D-\vec{p}_l)=(E_{Dl},-\vec{p}_{Dl})$$

Hence

$$\begin{array}{l}E_{Dl}=E_D+E_l=\sqrt{m_D^2+|\vec{p}_D|^2}+\sqrt{m_l^2+|\vec{p}_l|^2}=\sqrt{m_D^2+|\vec{p}_D|^2}+|\vec{p}_l| \\ |\vec{p}_{Dl}|^2=|\vec{p}_D-\vec{p}_l|^2=|\vec{p}_D|^2+|\vec{p}_l|^2-2|\vec{p}_D||\vec{p}_l|\cos\theta\end{array}$$

and

$$\begin{array}{rcl}m_{Dl}^2 & = & p_{Dl}^2=E_{Dl}^2-|\vec{p}_{Dl}|^2 \\ & = & m_D^2+|\vec{p}_D|^2+2|\vec{p}_l|\sqrt{m_D^2+|\vec{p}_D|^2}-|\vec{p}_D|^2-|\vec{p}_l|^2+2|\vec{p}_D||\vec{p}_l|\cos\theta \\ & = & m_D^2+2|\vec{p}_l|\sqrt{m_D^2+|\vec{p}_D|^2}-|\vec{p}_l|^2+2|\vec{p}_D||\vec{p}_l|\cos\theta \\ & = & m_D^2+|\vec{p}_l|(2\sqrt{m_D^2+|\vec{p}_D|^2}+2|\vec{p}_D|\cos\theta-|\vec{p}_l|)\end{array}$$

If D and l came from different B, p_D and p_l are not restricted by energy momentum conservation. The $|\vec{p}_D|$ and $|\vec{p}_l|$ can be any value. If $|\vec{p}_D|$ and $|\vec{p}_l|$ both become very large, E_{Dl} becomes very large but m_{Dl}^2 can be as small as m_D^2 if

$$2\sqrt{m_D^2+|\vec{p}_D|^2}+2|\vec{p}_D|\cos\theta\simeq|\vec{p}_l|$$

Hence

$$\cos\Theta_{B-Dl}=\frac{2E_BE_{Dl}-m_B^2-m_{Dl}^2}{2|\vec{p}_B||\vec{p}_{Dl}|}$$

can be larger than 1.