Calculation of w

1. Definition of w

In the decay of $B \to D\ell\nu$, the w is defined by

$$w = \frac{M_B^2 + M_D^2 - q^2}{2M_B M_D} = v_B \cdot v_D = \frac{p_B \cdot p_D}{M_B M_D}$$

where the q^2 is the square of momentum transfer from the B to the D or the invariant mass squared of the vertual W, and v and p are 4-velocity and 4-momentum. This relation can be proved easily

$$\begin{aligned}
&p_B = p_D + q \\
&\Rightarrow q^2 = (p_B + p_D)^2 = p_B^2 + p_D^2 - 2p_B \cdot p_D = M_B^2 + M_D^2 - 2p_B \cdot p_D \\
&\Rightarrow p_B \cdot p_D = \tfrac{M_B^2 + M_D^2 - q^2}{2}
\end{aligned}$$

The w is a Lorentz invariant and linearly related to q^2.
The w correspond to the relativistic boost γ of the D in the B rest frame.

$$\gamma_D = \frac{E_D}{M_D}$$

2. $1 \leq w < 1.5$

The range of w and q^2 are restrictred by kinematics of the decay.
When $q^2 = 0$, w is maximum

$$w_{\text{max}} = \frac{M_B^2 + M_D^2}{2M_B M_D} = 1.504$$

The minimum of the w is

$$w_{\text{min}} = 1$$

which corresponds to

$$q^2_{\text{max}} = (M_B - M_D)^2 = 10.69(\text{GeV}/c^2)^2$$

2. Calculation of w

We can calculate w by this formula

$$w = \frac{p_B \cdot p_D}{M_B M_D} = \frac{E_B E_D - |\vec{P}_B||\vec{P}_D| \cos\theta_{B-D}}{M_B M_D}$$

The E_B is the half of the beam energy.
The $|\vec{P}_B|$ is

$$|\vec{P}_B| = \sqrt{E_B^2 - M_B^2}$$

The $\cos\theta_{B-D}$ can be calcurated from $\cos\theta_{B-Dl}$