## Equation 2.56

Tha equation (2.56) is

$$
\begin{aligned}
(\partial^2 + m^2) D_R(x-y) &= (\partial^2\theta(x^0-y^0))\langle 0|[\phi(x),\phi(y)]|0\rangle && (1)\\
&+2(\partial_\mu\theta(x^0-y^0))(\partial^\mu\langle 0|[\phi(x),\phi(y)]|0\rangle) && (2)\\
&+\theta(x^0-y^0)(\partial^2+m^2)\langle 0|[\phi(x),\phi(y)]|0\rangle && (3)
\end{aligned}
$$

Here

$$
\begin{aligned}
&\int(\partial_\mu\partial^\mu\theta(x^0-y^0))\langle 0|[\phi(x),\phi(y)]|0\rangle\\
&=\int\partial_\mu[(\partial^\mu\theta(x^0-y^0))\langle 0|[\phi(x),\phi(y)]|0\rangle]-\int(\partial_\mu\theta(x^0-y^0))(\partial^\mu\langle 0|[\phi(x),\phi(y)]|0\rangle)\\
&=-\int(\partial_\mu\theta(x^0-y^0))(\partial^\mu\langle 0|[\phi(x),\phi(y)]|0\rangle)
\end{aligned}
\qquad (4)
$$

and

$$
\begin{aligned}
&\partial_0\theta(x^0-y^0)=\delta(x^0-y^0)\\
&\partial_i\theta(x^0-y^0)=0
\end{aligned}
\qquad (5)
$$

Thus

$$
\begin{aligned}
(\partial^2\theta(x^0-y^0))\langle 0|[\phi(x),\phi(y)]|0\rangle &= -(\partial_\mu\theta(x^0-y^0))(\partial^\mu\langle 0|[\phi(x),\phi(y)]|0\rangle) && (6)\\
&= -(\partial_0\theta(x^0-y^0))(\partial^0\langle 0|[\phi(x),\phi(y)]|0\rangle) && (7)\\
&= -\delta(x^0-y^0)\langle 0|[\partial^0\phi(x),\phi(y)]|0\rangle && (8)\\
&= -\delta(x^0-y^0)\langle 0|[\pi(x),\phi(y)]|0\rangle && (9)
\end{aligned}
$$

Hence

$$
\begin{aligned}
(\partial^2 + m^2) D_R(x-y) &= -\delta(x^0-y^0)\langle 0|[\pi(x),\phi(y)]|0\rangle && (10)\\
&+2\delta(x^0-y^0)\langle 0|[\pi(x),\phi(y)]|0\rangle + 0 && (11)
\end{aligned}
$$

The comutation relation is

$$[\pi(x),\phi(y)] = -i\delta^{(3)}(x-y) \qquad (12)$$

Thus

$$(\partial^2+m^2)D_R(x-y) = -i\delta^{(4)}(x-y) \qquad (13)$$