Equation 3.38

Pauli matrices are

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{1}$$

and

$$\{\sigma^i, \sigma^j\} = 2g^{ij} \tag{2}$$

For $i \neq 2$

$$\sigma^2(\sigma^i)^* = \sigma^2\sigma^i = -\sigma^i\sigma^2 \tag{3}$$

For $i = 2$

$$\sigma^2(\sigma^2)^* = \sigma^2(-\sigma^2) = -\sigma^2\sigma^2 \tag{4}$$

Hence

$$\sigma^2\sigma^* = -\sigma\sigma^2 \tag{5}$$

The transformation of $\sigma^2\psi_L^*$ is

$$\begin{aligned} &\sigma^2\psi_L^* \to \sigma^2(1 + i\theta \cdot \tfrac{\sigma^*}{2} - \beta \cdot \tfrac{\sigma^*}{2})\psi_L^* \\ &= (\sigma^2 - i\theta \cdot \tfrac{\sigma}{2}\sigma^2 + \beta \cdot \tfrac{\sigma}{2}\sigma^2)\psi_L^* \\ &= (1 - i\theta \cdot \tfrac{\sigma}{2} + \beta \cdot \tfrac{\sigma}{2})\sigma^2\psi_L^* \end{aligned} \tag{6}$$