Discrete Symmetries

Thanks to Ian Blockland and Randy Sobie for these slides

- Parity
- Parity Violation
- Charge Conjugation
- CP Violation
- Time Reversal
- The CPT Theorem
- Lepton number and Baryon number

Parity

- A parity transformation, P, inverts every spatial coordinate:

$$P(t, \mathbf{x}) = (t, -\mathbf{x})$$

  This corresponds to a reflection plus a 180° rotation.

- Clearly, $P^2 = I$, and therefore the eigenvalues of P are ± 1.

Parity Eigenvalues

- Consider an ordinary vector $\mathbf{v}$. By the definition of P, $P(\mathbf{v}) = -\mathbf{v}$.
- Now let's construct a scalar from $\mathbf{v}$: $s = \mathbf{v} \cdot \mathbf{v}$

$$P(s) = P(\mathbf{v} \cdot \mathbf{v}) = (-\mathbf{v}) \cdot (-\mathbf{v}) = \mathbf{v} \cdot \mathbf{v} = +s$$

- Suppose we take the cross product of two vectors: $\mathbf{a} = \mathbf{v} \times \mathbf{w}$

$$P(\mathbf{a}) = P(\mathbf{v} \times \mathbf{w}) = (-\mathbf{v}) \times (-\mathbf{w}) = \mathbf{v} \times \mathbf{w} = +\mathbf{a}$$

- Finally, we can form a scalar from $\mathbf{a}$ and $\mathbf{v}$: $p = \mathbf{a} \cdot \mathbf{v}$

$$P(p) = P(\mathbf{a} \cdot \mathbf{v}) = (+\mathbf{a}) \cdot (-\mathbf{v}) = -\mathbf{a} \cdot \mathbf{v} = -p$$

Types of Scalars and Vectors

Scalar	$P(s) = +s$
Pseudoscalar	$P(p) = -p$
Vector	$P(\mathbf{v}) = -\mathbf{v}$
Pseudovector	$P(\mathbf{a}) = +\mathbf{a}$

- Note: Pseudovectors are also known as axial vectors.

Parity in Physical Systems

- Two-body systems have parity $p_A p_B (-1)^\ell$, where ℓ is the orbital angular momentum eigenvalue
- Intrinsically, particles and antiparticles have opposite parity, therefore bound states like positronium ($e^+ \, e^-$) and mesons ($q\overline{q}$) have an overall parity of $(-1)^{\ell+1}$.
- Photons have a parity of (-1), and this underlies the $\Delta\ell = \pm 1$ selection rule in atomic transitions.
- Note that parity is a *multiplicative* quantum number. This is true for all discrete symmetries. Continuous symmetries have *additive* quantum numbers.

Example: $u\overline{u}$ mesons

By convention: u-quarks have spin 1/2 and + parity and $\overline{u}$-quarks have spin 1/2 and - parity

Parity of a $u\overline{u}$ meson is $P = p_u p_{\overline{u}} (-1)^\ell$

The intrinsic spin(**S**) of the $u\overline{u}$ meson is 0 or 1 but may have any orbital angular momentum (**L**) value.

S	L	J^P	particle	transformation properties
0	0	0^-	π^0	pseudoscalar
1	0	1^-	ρ^0	vector
0	1	1^+	$b_1(1235)$	pseudovector

Parity in the Standard Model

- It has long been realized that parity is a respected symmetry of the strong and electromagnetic interactions. This is built right into the equations. For example, while the Lorentz force law

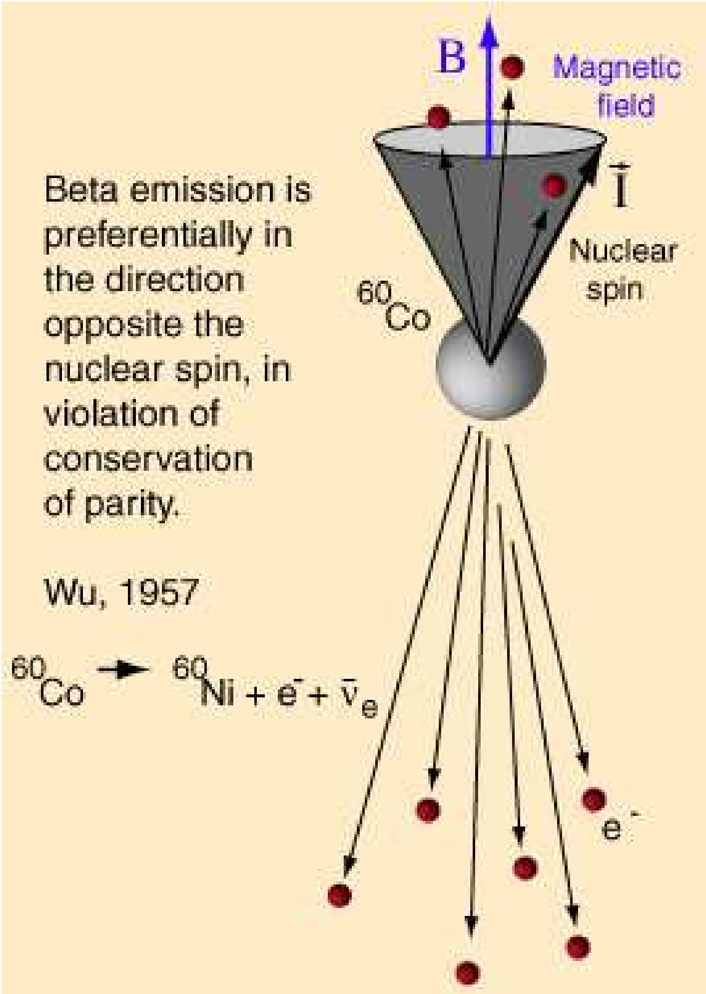
$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$

  contains a pseudovector ($\mathbf{B}$), we are never adding pseudovectors to vectors.

- In 1956, Yang and Lee realized that parity invariance had never been tested experimentally for weak interactions.

Madame Wu's Experiment

The nuclear spin is an axial vector (even under parity) and the electron momentum is a vector (odd under parity), so the relative orientation of the two changes under parity. Any asymmetry in the electron distribution relative to the spin (**B**) direction violates parity.



Parity Violation in π Decay

- Consider the weak decay $\pi^+ \to \mu^+ + \nu_\mu$. Since the π is spin-0 and the μ and ν emerge back-to-back in the CM frame, the spins of the μ and ν must cancel.
- Experiments show that *every* μ^+ is left-handed, and therefore *every* ν_μ is also left-handed. Similarly, in π^- decay, both the μ^- and $\overline{\nu}_\mu$ *always* emerge right-handed.
- If parity were conserved by the weak interaction, we would expect left-handed pairs and right-handed pairs with equal probability (just as we observe with $\pi^0 \to 2\gamma$).

Helicity

- Only the z-component of angular momentum can be specified and it is a good idea to align the z-axis with the direction of motion of a particle.
- Define the *helicity* of a particle by: $h = m_s/s$
- For a spin-1/2 particle the helicity can be either +1 or -1. Similarly, the spin-1 photon can only have $h = \pm 1$ since the $m = 0$ mode is absent (longitudinal polarization).
- Helicity is not Lorentz invariant unless the particle is massless (such that $v = c$).

Chirality in the Standard Model

- Assuming that neutrinos are massless,

  ALL neutrinos are left-handed

  ALL antineutrinos are right-handed

Charge Conjugation

- The charge conjugation operator, C, converts a particle to its antiparticle. $C\left|p\right\rangle = \left|\overline{p}\right\rangle$
- In particular, C reverses *every* internal quantum number (charge, baryon/lepton number, strangeness, etc.).
- $C^2 = I$ implies that the only allowed eigenvalues of C are ± 1.
- Unlike parity, very few particles are C eigenstates. Only those particles that are their own antiparticles (π^0, η, γ, etc.) are C eigenstates.

  For example, $C\left|\pi^+\right\rangle = \left|\pi^-\right\rangle$ and $C\left|\gamma\right\rangle = -\left|\gamma\right\rangle$

Using Charge Conjugation

- The photon has $C = -1$
- $f\overline{f}$ bound states have $C = (-1)^{\ell+s}$.
- Charge conjugation is respected by both the strong and electromagnetic interactions.
- As a result, the π^0 ($\ell = s = 0 \Rightarrow C = +1$) can decay into 2γ but not 3γ.
- $C\,|n\gamma\rangle = (-1)^n\,|\gamma\rangle$ and $C\,\left|\pi^0\right\rangle = \left|\pi^0\right\rangle$
  $\pi^0 \rightarrow 2\gamma$ is allowed (and observed)
  $\pi^0 \rightarrow 3\gamma$ is not allowed (and not observed $< 3.1 \times 10^{-8}$)

G-Parity

- Most particles are not C eigenstates, hence C-symmetry is of limited use.
- The C operator converts π^+ to π^-, and since these two particles have isospin assignments $|\,1\,1\,\rangle$ and $|\,1\;-1\,\rangle$, the charged pions are eigenstates under the G-parity operator, which combines C with a 180° isospin rotation:

$$G = Ce^{i\pi I_2}$$

- G-parity is mainly used to examine decays to pions (which have $G = -1$). $\qquad G\,|n\pi\rangle = (-1)^n\,|n\pi\rangle$

G-Parity of a few light mesons

Particle	J^P	I	G	Decay	width (MeV)
$\rho(770)$	1^-	1	+1	2π	150
$\omega(783)$	1^-	0	-1	3π	8.5
$\phi(1020)$	1^-	0	-1	3π	4.3
$f(1270)$	2^+	0	+1	2π	185

Remember - G-Parity involves isospin, so it only tells us about *strong* decay selection rules.

Reading particle data tables

Citation: S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004) (URL: http://pdg.lbl.gov)

$f_0(600)$ [i] or σ

$$I^G(J^{PC}) = 0^+(0^{++})$$

Mass $m = (400\text{–}1200)$ MeV
Full width $\Gamma = (600\text{–}1000)$ MeV

$f_0(600)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$\pi\pi$	dominant	–
$\gamma\gamma$	seen	–

$\rho(770)$ [j]

$$I^G(J^{PC}) = 1^+(1^{--})$$

Mass $m = 775.8 \pm 0.5$ MeV
Full width $\Gamma = 150.3 \pm 1.6$ MeV
$\Gamma_{ee} = 7.02 \pm 0.11$ keV

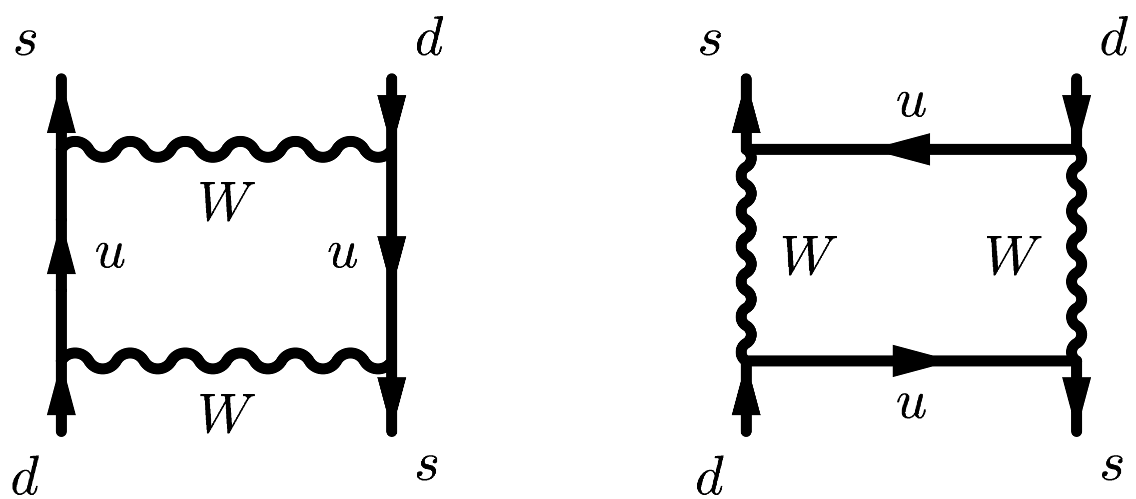
$\rho(770)$ DECAY MODES		Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
$\pi\pi$		~ 100 %		364
$\rho(770)^{\pm}$ decays				
$\pi^{\pm}\gamma$		$(4.5 \pm 0.5) \times 10^{-4}$	S=2.2	375
$\pi^{\pm}\eta$		$< 6 \times 10^{-3}$	CL=84%	153
$\pi^{\pm}\pi^{+}\pi^{-}\pi^{0}$		$< 2.0 \times 10^{-3}$	CL=84%	254
$\rho(770)^{0}$ decays				
$\pi^{+}\pi^{-}\gamma$		$(9.9 \pm 1.6) \times 10^{-3}$		362
$\pi^{0}\gamma$		$(6.0 \pm 1.3) \times 10^{-4}$	S=1.1	376
$\eta\gamma$		$(3.0 \pm 0.4) \times 10^{-4}$	S=1.4	195
$\pi^{0}\pi^{0}\gamma$		$(4.5 \pm 0.8) \times 10^{-5}$		364
$\mu^{+}\mu^{-}$	[k]	$(4.55 \pm 0.28) \times 10^{-5}$		373

CP Symmetry

- Reconsidering the pion decay $\pi^+ \rightarrow \mu^+ \nu_\mu$, we recall that the ν_μ is always left-handed.
- Under charge conjugation, this reaction becomes $\pi^- \rightarrow \mu^- + \overline{\nu}_\mu$, but the $\overline{\nu}_\mu$ is still left-handed, a chirality which *does not* occur in nature.
- If we combine C and P, though, we get a right-handed antineutrino. Perhaps it is CP that is the perfect symmetry of nature that our intuition craves.
- Not in this universe...

CP Violation in the Kaon Sector

- Consider the neutral kaons K^0 and $\bar{K}^0$ (with quark assignments $d\bar{s}$ and $s\bar{d}$, respectively). These particles can mix via a second-order weak interaction *because the weak interaction does not conserve quark flavor*:



Kaon Quantum Numbers

- Both K^0 and $\bar{K}^0$ are pseudoscalar mesons, therefore $P = -1$.
- Since K^0 and $\bar{K}^0$ are a particle-antiparticle pair,

$$C\left|K^0\right\rangle = \left|\bar{K}^0\right\rangle \qquad C\left|\bar{K}^0\right\rangle = \left|K^0\right\rangle$$

- As a result, under CP, we have

$$CP\left|K^0\right\rangle = -\left|\bar{K}^0\right\rangle \qquad CP\left|\bar{K}^0\right\rangle = -\left|K^0\right\rangle$$

Kaon CP Eigenstates

- Defining

$$\begin{aligned} |K_1\rangle &= \left(|K^0\rangle - |\bar{K}^0\rangle\right)/\sqrt{2} \\ |K_2\rangle &= \left(|K^0\rangle + |\bar{K}^0\rangle\right)/\sqrt{2} \end{aligned}$$

  we have

$$\begin{aligned} CP\,|K_1\rangle &= +\,|K_1\rangle \\ CP\,|K_2\rangle &= -\,|K_2\rangle \end{aligned}$$

- If CP is conserved, then $|K_1\rangle$ can only decay to 2π ($CP = +1$) and $|K_2\rangle$ can only decay to 3π ($CP = -1$).
- Based on phase-space considerations, $|K_1\rangle$ should have a much shorter lifetime than $|K_2\rangle$.

Kaon Decay Eigenstates

- We observe the K_S and the K_L with lifetimes 0.9×10^{-10} s and 0.5×10^{-7} s
- K^0 and the $\overline{K}^0$ are mass eigenstates and are each others antiparticles
- K_S and the K_L are approximately CP eigenstates (have different masses) and are not antiparticles
- That they are not exactly CP eigenstates was astounding

The Experiment...

- Start out with a beam of K^0. This will be a superposition of K_1 and K_2:
$$\left|K^0\right\rangle = \left(|K_1\rangle + |K_2\rangle\right)/\sqrt{2}$$
- The K_1 component of the beam will decay away over a few centimeters, thereby leaving a nearly pure beam of K_2. As a result, we would expect to see only 3π decays in a detector several meters down the beam pipe.
- Experimentally, we find that about 1 in 440 decays is to 2π! In other words, the long-lived neutral kaon has a small mixture of K_1:
$$|K_L\rangle = \left(|K_2\rangle + \epsilon\,|K_1\rangle\right)/\sqrt{1+|\epsilon|^2}$$

Other Tests of CP Violation

- There are other CP-violating observables that have been measured in the kaon sector. For example, there is an asymmetry between the branching ratios of K_L to $\pi^+ + e^- + \bar{\nu}_e$ versus $\pi^- + e^+ + \nu_e$.
- Within the last few years, the BaBar and Belle experiments have measured CP violation in the B-meson sector.
- CP violation should also be observable in the D-meson (charm) sector, though this will be a small effect that will be very difficult to measure.

Time Reversal Symmetry

- Time reversal symmetry, as you might guess, reverses the time component:
$$T(t, \mathbf{x}) = T(-t, \mathbf{x})$$
- Although we expect the weak interaction to violate T, direct T violation has not been definitively observed yet.

The CPT Theorem

- The combination CPT is *always* conserved in any local quantum field theory.
- CPT violation in essentially synonymous with a violation of Lorentz invariance.
- CPT symmetry mandates that particles and antiparticles must have certain identical properties, such as the same mass, lifetime, charge, and magnetic moment.

Lepton Number

- There are 3 lepton numbers: L_e, L_μ and L_τ
  $L_e = +1$ for e^- and ν_e
  $L_e = -1$ for e^+ and $\overline{\nu}_e$
- Conserved in the EM and Weak interactions $\gamma \rightarrow e^+e^-$ and $\pi^+ \rightarrow \mu^+\overline{\nu}_\mu$ are allowed whereas $\mu^+ \rightarrow e^+\gamma$ is forbidden
- BaBar (UVic group) put a new limit on the $\tau^+ \rightarrow \mu^+\gamma$ branching fraction (10^{-8})

Baryon Number

- We can associate with each baryon (3-quark bound state) a quantum number
  $B = +1$ for baryons (protons, neutrons...)
  $B = -1$ for anti-baryons (anti-protons, anti-neutrons...)
- The net baryon number appears to be conserved in all interactions; if it weren't we (i.e. protons) might decay into lighter stuff.
- The current experimental limit on the proton lifetime is $> 10^N$ years, where N varies from 31 to 33 for different decay modes (since nobody knows how protons decay, if they do, one has to search in a variety of kinematically allowed modes)

Summary

- The 3 discrete symmetries P, C, and T are respected individually by the strong and electromagnetic forces.
- Parity violation is the signature of the weak interaction.
- The weak interaction also violates CP, as a result of a complex phase in the CKM matrix.
- The combined symmetry of CPT is obeyed by all local quantum field theories.