P317 Assignment 3 — due May 30, 2007

P317

Carter text problems

4-1) Given $c_v = A\left(\frac{T}{\Theta}\right)^3$, $A = 19.4 \cdot 10^5 \frac{J}{\text{Kmole}\,K}$, $\Theta = 320K$ for NaCl

a) At $T = 10K$, $c_v = 59.2 \frac{J}{\text{Kmole}\,K}$; at $T = 50K$, $c_v = 7.40 \cdot 10^3 \frac{J}{\text{Kmole}\,K}$

b) $Q = \int_{T_1}^{T_2} n c_v \, dT = \frac{nA}{4\Theta^3}(T_2^4 - T_1^4) = 1.85 \cdot 10^5 J$

c) Instead of $\frac{c_v(T=50) + c_v(T=10)}{2}$ I prefer $\langle c_v \rangle = \frac{Q}{\Delta T} = 4.62 \cdot 10^3 \frac{J}{\text{Kmole}\,K}$

4-5) $c_p = 2.6 \cdot 10^4 \frac{J}{\text{Kmole}\,K}$ for Cu. Since $c_p \simeq c_v \simeq$ constant $\Delta U \simeq c_v \Delta T \simeq c_p \Delta T$

so $\Delta U = m c_p \Delta T = \left(8.97 \cdot 10^4 \frac{J}{kg}\right) m$.

$$\frac{\Delta m}{m} = \frac{\Delta U / c^2}{m} = \frac{8.97 \cdot 10^4 \, J/kg}{c^2} = 1.0 \cdot 10^{-12}$$

4-9) Given ideal gas in reversible adiabatic expansion

a) $\delta q = 0 \underset{\text{reversible}}{=} du + P dv \underset{\text{ideal}}{=} c_v dT + \frac{RT}{v} dv$ so $\frac{dT}{T} + \frac{R}{c_v}\frac{dv}{v} = 0$

Integrate $\longrightarrow \ln\left(\frac{T_2}{T_1}\right) + \ln\left(\frac{V_2}{V_1}\right)^{R/c_v} = 0$ or $\ln\left(\frac{T_2}{T_1}\right) = (\gamma - 1)\ln\left(\frac{V_1}{V_2}\right)$

Since $\gamma - 1 = \frac{c_p - c_v}{c_v} = \frac{R}{c_v}$ (ideal)

b) $\frac{T_2}{T_1} = \frac{2}{5}$, $\frac{V_2}{V_1} = 2 \Rightarrow \gamma = 2.32$ from eqn in part a.

However, $\gamma = 1 + \frac{R}{c_v} \rightarrow c_v = \frac{R}{\gamma - 1} = 0.76R < 1.5R = c_v$ for monatomic gas (smallest possible)

So no ideal gas can reach state 2 from state 1 in an adiabatic process

4-10) $P = aT^4/3$, $U = aT^4 V \rightarrow dU_T = aT^4 dV$

a) $T =$ constant, $Q = \int_V^{2V} dU_T + P dV_T = \int_V^{2V} \frac{4}{3} a T^4 dV = \frac{4}{3} a T^4 V$

b) adiabatic $\delta Q = 0 = dU + PdV = 4aT^3 V dT + \frac{4}{3} a T^4 dV$

so $\frac{3\,dT}{T} = -\frac{dV}{V} \Rightarrow \ln T^3 + \ln V =$ constant or $VT^3 =$ const.

P317 assignment 3 continued

4-13) Take ice @ $-20°C$ to steam at $400°C$ at constant P, $m = 1\,kg$

Find $\Delta H = m c_p \Delta T$ or $m l$ (heating / phase change)

$\Delta H_{melting} = 80\,kcal$ $\Delta H_{boiling} = 538\,kcal$

$\Delta H_{ice} = 0.55 \frac{kcal}{K} \cdot 20K = 11.0\ kcal$

$\Delta H_{water} = 1.00 \frac{kcal}{K} \cdot 100K = 100\ kcal$

$\Delta H_{steam} = 0.48 \frac{kcal}{K} \cdot 300K = 144\ kcal$

$\Sigma \Delta H = 873\ kcal \doteq 3.65 \cdot 10^6\ J$

4-15) If we treat the fireball as an ideal gas in a reversible adiabatic expansion then $TV^{\gamma-1} = \text{constant}$. $\gamma(N_2, O_2) \doteq \frac{7}{5}$

$T_1 / T_2 = 3 \cdot 10^5 K / 3000K = 100$, $r_1 = 15m$

$V_2 = V_1 \left(\frac{T_1}{T_2}\right)^{3/2}$ or $r_2 = r_1 \left(\frac{T_1}{T_2}\right)^{5/6} = 696\ m$