P317 Assignment 4 due June 6, 2007 $\boxed{P317}$

5.7, 10, 11, 13, 14, 15

5-7) $m = 50\,kg$, $l_{12} = 3.33\cdot10^5\,J/kg$, $T_{room} = 20°C = 293.15\,K$

$Q_c = -m l_{12} = -1.67\cdot10^7\,J$ to be extracted from 0°C reservoir

$Q_c + Q_h = W$, $\frac{|Q_c|}{|Q_h|} = \frac{T_c}{T_h}$ for Carnot refrigerator, so

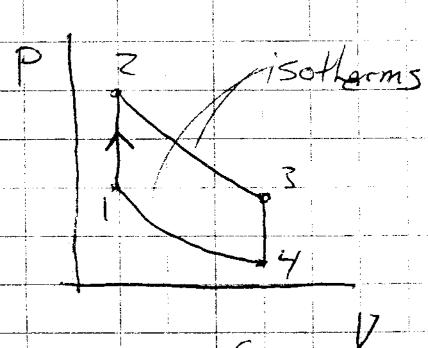
$W = \left(\frac{T_h}{T_c} - 1\right)|Q_c| = 1.22\cdot10^6\,J$, Power $= \frac{W}{3600\,s} = 340$ watts

5-10) a) $Q_{12} = n c_v (T_2 - T_1) = 2\cdot\frac{3}{2}R\cdot100K = 2.49\cdot10^6\,J$

$Q_{34} = 3R(-100K) = -2.49\cdot10^6\,J$

$Q_{23} = W_{23} = nRT_2 \ln\frac{V_3}{V_2} = 10.71\cdot10^6\,J$

$Q_{41} = W_{41} = nRT_1 \ln\frac{V_1}{V_4} = -8.03\cdot10^6\,J$



b) W_{23}, W_{41} given above; $W_{12} = W_{34} = 0$; $W_{net} = 2.68\cdot10^6\,J$

c) $\eta = \frac{W}{Q_{in}} = \frac{W_{23} + W_{41}}{Q_{12} + Q_{23}} = 0.203$

5-11) A reversible heat engine observes the ~~ratio~~ sum $\sum_i \frac{Q_i}{T_i} = 0$ (which for two reservoirs reduces to $\frac{Q_1}{T_1} + \frac{Q_2}{T_2} = 0$ or $\frac{T_1}{T_2} = \frac{|Q_1|}{|Q_2|}$)

In this case $\frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \frac{Q_3}{T_3} = 0 = \frac{1200\,J}{400K} + \frac{Q_2}{200K} + \frac{Q_3}{300K}$

Also $W = Q_1 + Q_2 + Q_3 = 200\,J \longrightarrow Q_2 + Q_3 = -1000\,J$

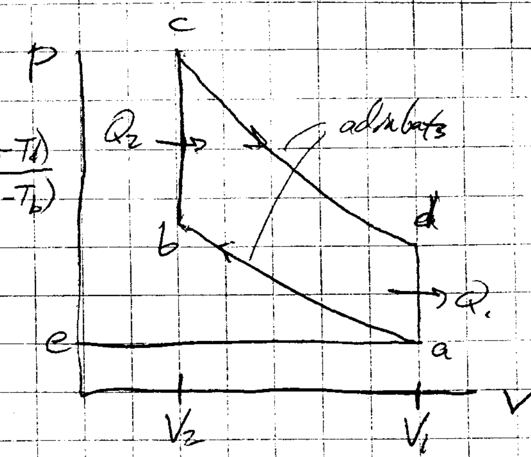
$\therefore\ Q_2 = 200\,J$, $Q_3 = -1200\,J$

5-13) a) $c = \frac{Q_2}{W} = \frac{Q_2}{Q_2 - Q_1} = \frac{T_2}{T_2 - T_1}$

b) $T_1 = 0°C$, $T_2 = 20°C$ so $\frac{T_2}{T_2 - T_1} = \frac{293.15}{20} = 14.6 = c$

You get ~14 times as much heat delivered as you put in in work (real systems give ~~ca~~3) assignment 4 continued

5-14) Otto cycle



(a) $\eta = \frac{W}{Q_2} = \frac{W_{cd} + W_{ab}}{Q_2} = 1 + \frac{Q_1}{Q_2} = 1 + \frac{\cancel{nc_v}(T_a - T_d)}{\cancel{nc_v}(T_c - T_b)}$

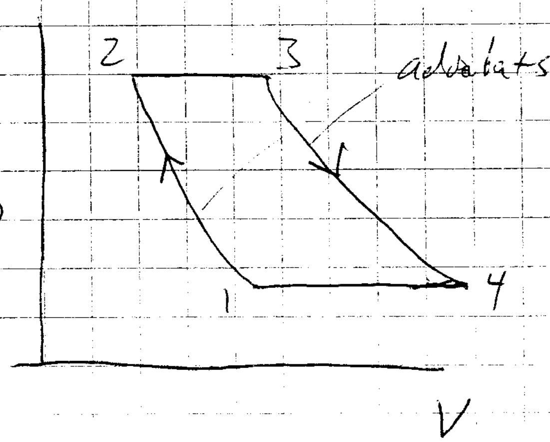
Along adiabats $TV^{\gamma-1} = \text{const} \longrightarrow \frac{T_a}{T_b} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \frac{T_d}{T_c}$

or $(T_a - T_d)V_1^{\gamma-1} = (T_b - T_c)V_2^{\gamma-1}$

so $\eta = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1} = 1 - \left(\frac{1}{r}\right)^{\gamma-1}$

(b) $\eta = 0.55$

5-15) Joule cycle, c_p, c_v constant
ideal gas



$\eta = \frac{W}{Q_{in}} = \frac{Q_{23} + Q_{41}}{Q_{23}} = 1 + \frac{nc_p(T_1 - T_4)}{nc_p(T_3 - T_2)}$

along adiabats $PV^{\gamma} = \text{const}$

$\longrightarrow P^{1-\gamma}T^{\gamma} = \text{const}$

or $TP^{\left(\frac{1-\gamma}{\gamma}\right)} = \text{const}$

so $P_1^{\frac{1-\gamma}{\gamma}}(T_1 - T_4) = P_2^{\frac{1-\gamma}{\gamma}}(T_2 - T_3)$

$\longrightarrow \eta = 1 - \left(\frac{P_2}{P_1}\right)^{\frac{1-\gamma}{\gamma}} = 1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$