P317 assignment 5 due June 15, 2007

P317

Problems from Carter 5,5,6,8,11,13

6.2) (a) 1 kg water heated reversibly from 20°C to 80°C

$c_p = 4.18 \cdot 10^3 \frac{J}{kg\,K}$ so $dS = \frac{(mc_p\,dT)}{T}$ ($= \delta Q$) and $\Delta S = mc_p \ln \frac{353.15}{293.15} = 778\,J/K$

(b) melt 1 kg of ice, $l = 3.34 \cdot 10^5\,J/kg$

$$\Delta S = \frac{ml}{T} = \frac{3.34 \cdot 10^5\,J/kg \cdot 1\,kg}{273.15\,K} = 1.22 \cdot 10^3\,J/K$$

(c) 1 kg steam condenses, $l = 2.26 \cdot 10^6\,J/kg$

$$\Delta S = -\frac{ml}{373.15\,K} = -6.06 \cdot 10^3\,J/K$$

6.5) $R = 20\,\Omega$, $I = 2A$, $\Delta t = 1s$, $T_0 = 20°C$, $m = 5g$, $c_p = 850\,J/kg\,K$

(a) Heating (dissipative work) is $(I^2R)\Delta t = 80J = Q$

$Q = mc_p\Delta T$ so $T_f - T_0 = \frac{80J}{mc_p} = 19\,K$

(b) For resister need to find reversible process with same final state; e.g. reversible isobaric heating

$$\Delta S = \int_{T_0}^{T_f} \frac{mc_p\,dT}{T} = mc_p \ln \frac{T_f}{T_0} = 0.27\,J/K \text{ (resistor)}$$

$\Delta S_{universe} = \Delta S_{resistor}$ since the resistor is isolated

6.8) (a) isobaric expansion of ideal gas

$$ds = \frac{\delta q_{rev}}{T} = \frac{c_p\,dT}{T} \Rightarrow \Delta s = c_p \ln \frac{T_f}{T_i} = c_p \ln \frac{P_f V_f}{P_i V_i} = c_p \ln \frac{V_f}{V_i}$$

(b) isothermal $\rightarrow U = U(T) = $ constant, $q = w = \int \frac{nRT}{v}\,dv$

so $\Delta S = \int \frac{nR}{v}\,dv = nR \ln \frac{V_f}{V_i}$

(c) ΔS larger in isobaric: $\frac{\Delta S_P}{\Delta S_T} = \frac{c_P}{R} = \frac{5/2\,R}{R} = \frac{5}{2}$ (monatomic)

6-6) $T_2 = 400K$, $T_1 = 200K$

$Q_2 = 10^8 J$, $Q_1 = -4\cdot10^7 J$, $W_{mech} = 15 kWh = 5.4\cdot10^7 J$

$W_{diss} = (Q_2 - |Q_1|) - W_{mech} = 0.6\cdot10^7 J$ of dissipative work (ok)

so 1st law not violated

$\eta = \frac{W}{Q_2} = 0.54 > 0.5 = 1 - \frac{T_1}{T_2} = \eta_{carnot}$

$\eta > \eta_{carnot}$ violates 2nd law – bad investment

6-11) $\left(P + \frac{a}{v^2}\right)(v-b) = RT$

(a) $\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_v - P = T\left(\frac{R}{v-b}\right) - P = \frac{a}{v^2}$ so

$u(T,v) = f(T) + \frac{-a}{v}$ and $c_v = \left(\frac{\partial u}{\partial T}\right)_v = \left(\frac{\partial f(T)}{\partial T}\right)_T = f'(T)$

(b) given that $c_v = \left(\frac{\partial u}{\partial T}\right)_v$ and $\left(\frac{\partial u}{\partial v}\right)_T = \frac{a}{v^2}$, integrate to find

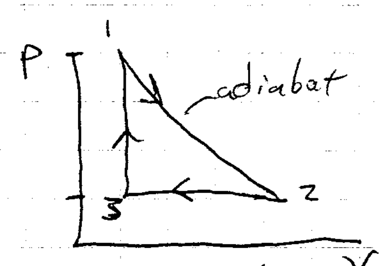
$u = \int^T c_v dT + \int^v \frac{a}{v^2} + u_0 = \int^T c_v dT - \frac{a}{v} + u_0$

(c) $ds = \frac{du + Pdv}{T} = \frac{c_v dT + \frac{a}{v^2}dv + \left(\frac{RT}{v-b} - \frac{a}{v^2}\right)dv}{T}$ so

$\Delta s = \int c_v \frac{dT}{T} + R\ln(v-b) + s_0$

6-13) $P_1 = 10 atm$, $V_1 = 2m^3$, $V_2 = 4m^3$, ideal gas (monatomic)

$n = 1$ kilomole



reversible closed cycle

For adiabatic leg, $\Delta S = S_2 - S_1 = 0$

isobar: $\Delta S = nc_p \ln\frac{T_3}{T_2} = n\left(\frac{5}{2}R\right)\ln\frac{V_1}{V_2} = -1.44\cdot10^4 \frac{J}{K}$

isochore: $\Delta S = nc_v \ln\frac{T_1}{T_3} = n\left(\frac{3}{2}R\right)\ln\frac{P_1}{P_3} = n\left(\frac{3}{2}R\right)\frac{5}{3}\ln\frac{V_2}{V_1} = 1.44\cdot10^4 J/K$

$P_3 = P_2 = P_1\left(\frac{V_1}{V_2}\right)^\gamma$, $\gamma = \frac{5}{3}$

$\Delta S_{cycle} = (0 - 1.44 + 1.44)\cdot10^4 \frac{J}{K} = 0$