P317 assignment 6 due June 26, 2007

P317

Carter problems 7.3, 7, 10, 15, 17

7.3) (a) $m = 0.4kg$, $C_p = 150\,J/K$, $T_o = 100°C$, $T_{lake} = 10°C$

$\Delta S_{block} = C_p \ln \frac{283}{373} = -41.4\,J/K$

$\Delta S_{lake} = Q/T_{lake} = \frac{-C_p(T_{lake} - T_o)}{T_{lake}} = 47.7\,J/K$

$\Delta S_{uni} = 6.3\,J/K$

(b) The block is unchanged $\rightarrow \Delta S_{sys} = 0$

$\Delta S_{lake} = \Delta S_{uni} = \frac{mgh}{T_{lake}} = 1.38\,J/K$

(c) Carter is wrong here. $\Delta S = C_p\left[\ln \frac{T_{ave}}{T_{100}} + \ln \frac{T_{ave}}{T_{10}}\right] = C_p \ln \frac{328^2}{283 \cdot 373}$

$\Delta S_{uni} = 150 \cdot 0.019 = 2.85\,J/K$

(d) $n = 1$ kilomole, $V_o \rightarrow 2V_o$

$\Delta S_{sys} = nR \ln \frac{V_f}{V_o} = 5.76 \cdot 10^3\,J/K$ (system), $\Delta S_{uni} = 0$ (reversible)

(e) reversible + adiabatic $\rightarrow$ isentropic, $\Delta S_{uni} = 0 = \Delta S_{sys}$

7.7) 50kg bag of sand at 25°C falls 10m to pavement.

(a) The change in gravitational potential energy shows up as dissipative work: $W_{diss} = -mg\Delta h = 4.9 \cdot 10^3\,J$ done by the sand

$\rightarrow -4.9 \cdot 10^3\,J$ work done on the sand

(b) $\cancel{Q}^{\,0} = \Delta U + W$ so $\Delta U = 4.9 \cdot 10^3\,J$

(c) T = constant so $\Delta S_{sand} = \frac{Q}{T} = \frac{\Delta U}{T} = 16.4\,J/K$

$\Delta S_{surr} = 0$ so $\Delta S_{uni} = 16.4\,J/K$

7-10) (a) $Tds = du + Pdv$. In a process at constant volume,

$ds_v = \frac{1}{T}[du_v] = \frac{1}{T}\left[\left(\frac{\partial u}{\partial T}\right)_v dT_v + \left(\frac{\partial u}{\partial v}\right)_T \cancel{dv}^{\,0}\right] = \frac{c_v}{T} dT_v$

Differentiating by P (dividing by dP_v) $\rightarrow \left(\frac{\partial s}{\partial P}\right)_v = \frac{c_v}{T}\left(\frac{\partial T}{\partial P}\right)_v$

Similarly, at constant pressure $ds_p = \frac{1}{T}\left[dh_p - v\cancel{dP}^{\,0}\right] = \frac{c_p}{T} dT_p + \left(\frac{\partial h}{\partial P}\right)_T \cancel{dP}^{\,0}$

so $(\partial s/\partial v)_p = (c_p/T)(\partial T/\partial v)_p$

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7-10)(b) $ds = \left(\frac{\partial s}{\partial P}\right)_v dP + \left(\frac{\partial s}{\partial v}\right)_P dv$

$= \frac{c_v}{T}\left(\frac{\partial T}{\partial P}\right)_v dP + \frac{c_P}{T}\left(\frac{\partial T}{\partial v}\right)_P dv$

Since $\left(\frac{\partial T}{\partial P}\right)_v = \kappa/\beta$ and $\left(\frac{\partial T}{\partial v}\right)_P = \frac{1}{\beta v}$

$\hookrightarrow T ds = c_v \frac{\kappa}{\beta} dP + \frac{c_P}{\beta v} dv$

7-11) $\left(P + \frac{a}{v^2}\right)(v-b) = RT$. $\left(\frac{\partial P}{\partial T}\right)_v = \frac{R}{v-b}$

$T ds = c_v dT + T\left(\frac{\partial P}{\partial T}\right)_v dv \rightarrow ds = c_v \frac{dT}{T} + \frac{R}{v-b} dv$

$s(T, v) = s_0 + c_v \ln \frac{T}{T_0} + R \ln \frac{v-b}{v_0-b}$

7-15)(a) $\eta \equiv \left(\frac{\partial T}{\partial v}\right)_u = -\left(\frac{\partial T}{\partial u}\right)_v \left(\frac{\partial u}{\partial v}\right)_T = -\frac{1}{c_v}\left[T\left(\frac{\partial P}{\partial T}\right)_v - P\right] = \frac{1}{c_v}\left[P - T\frac{\beta}{\kappa}\right]$

(b) $\mu \equiv \left(\frac{\partial T}{\partial P}\right)_h = -\left(\frac{\partial T}{\partial h}\right)_P \left(\frac{\partial h}{\partial P}\right)_T = \frac{-1}{c_P}\left[v - T\left(\frac{\partial v}{\partial T}\right)_P\right] = \frac{v}{c_P}(T\beta - 1)$

(c) van der Waals: $\left(\frac{\partial P}{\partial T}\right)_v = \frac{R}{v-b} = \frac{P + a/v^2}{T}$, $\left(\frac{\partial v}{\partial T}\right)_P = \frac{1}{\left(\frac{\partial T}{\partial v}\right)_P} = \left[\frac{T}{v-b} + \frac{2a(v-b)}{Rv^3}\right]^{-1}$

$\eta = \frac{-a}{c_v v^2}$, $\mu = -\frac{1}{c_P} \frac{RTb v^3 - 2av(v-b)^2}{RTv^3 - 2a(v-b)^2}$

In the limit $a \to 0$, $b \to 0$ the van der Waals eq^n is $Pv = RT$ (ideal gas) so we have

$\eta_{ideal} = 0$, $\mu_{ideal} = 0$

7-17) $v \simeq v_0 \Rightarrow \beta, \kappa$ are small. $v - v_0 \simeq 0 = (\beta \Delta T - \kappa \Delta P) v_0$

(a) $ds = c_P \frac{dT}{T} - \left(\frac{\partial v}{\partial T}\right)_P dP = c_P \frac{dT}{T} - \left(\frac{\partial v}{\partial T}\right)_P \left[\left(\frac{\partial P}{\partial T}\right)_v dT + \underbrace{\left(\frac{\partial P}{\partial v}\right)_T dv}_{\simeq 0}\right] = c_P \frac{dT}{T} - \frac{\beta^2 v_0}{\kappa} dT$

$\Delta s = c_P \ln\left(\frac{T}{T_0}\right) - \frac{\beta^2 v_0}{\kappa}(T - T_0)$

(b) Put in numbers: 1^{st} term is 12.79 J/K/kg

2^{nd} term is −0.32 J/K/kg

$\Delta s = 12.5$ J/K/kg