PHYS506B Asignment #3

given 15/03/2005
due 05/04/2005

1 (15pts) Consider the Dirac spinor field ψ. Let

$$\psi_L = P_L \psi \qquad \bar\psi_L \equiv \overline{(\psi_L)} \neq (\bar\psi)_L$$

$$\psi_R = P_R \psi \qquad \bar\psi_R \equiv \overline{(\psi_R)} \neq (\bar\psi)_R$$

where

$$P_L \equiv \tfrac{1}{2}\left(1-\gamma^5\right)$$

$$P_R \equiv \tfrac{1}{2}\left(1+\gamma^5\right)$$

Show that

a) $\bar\psi\psi = \bar\psi_L\psi_R + \bar\psi_R\psi_L$

b) $\left(\bar\psi_L\psi_R\right)^\dagger = \bar\psi_R\psi_L$

c) $\bar\psi\gamma^\mu\psi = \bar\psi_L\gamma^\mu\psi_L + \bar\psi_R\gamma^\mu\psi_R$

2 (30pts) From the lecture notes (Running Coupling Constant), obtain the equations

a) $\left[-\dfrac{\partial}{\partial t} + \beta(\alpha)\dfrac{\partial}{\partial\alpha}\right] R\left(\tfrac{Q^2}{\mu^2}, \alpha\right) = 0$

b) $\dfrac{\partial\alpha(Q)}{\partial t} = \beta\left(\alpha(Q)\right)$ and $\dfrac{\partial\alpha(Q)}{\partial\alpha} = \dfrac{\beta\left(\alpha(Q)\right)}{\beta(\alpha)}$

c) $\left[-\dfrac{\partial}{\partial t} + \beta(\alpha)\dfrac{\partial}{\partial\alpha}\right] R\left(1, \alpha(Q)\right) = 0$

3 (25pts) Estimate the energy scale *Q* at which QED and QCD coupling constants meet. (Compare your result with the Planck mass scale given by $G^{-1/2}$).

Hint: use the renormalization point $\mu = M_Z$ with

$$\alpha_s(M_Z) = 0.118 \qquad \alpha(M_Z) = 128^{-1} \qquad M_Z = 91.187 \text{ GeV}$$

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4 30pts

Consider the definition of $\alpha(Q)$: $\ln\frac{Q^2}{\mu^2} = \int_{\alpha(\mu)}^{\alpha(Q)} \frac{dx}{\beta(x)}$

a) Show that to leading order in $\beta(x)$, that is with $\beta(x) = -bx^2$, we obtain the following result for $\alpha(Q)$:

$$\alpha(Q)^{-1} - \alpha(\mu)^{-1} = b\ln\frac{Q^2}{\mu^2}$$

b) Using the next to leading order in $\beta(x)$, that is with $\beta(x) = -bx^2(1+b'x)$, obtain an expression for $\alpha(Q)$.

5 40pts

Consider the experimental results $\alpha_s(M_Z) = 0.118$ and $M_Z = 91.187$ GeV. Evolve this result for α_s down to the scales $Q = 35$ GeV and $Q = 2$ GeV

a) using the leading order running of α_s;

b) using the next to leading order for the running of α_s.

Hint: assume only one sharp mass threshold at the bottom quark mass, that is for each one of questions a) and b)

1) starting from the given $\alpha_s(M_Z)$, using $\mu = M_Z$, find α_s at $Q = 35$ GeV and at $Q = m_b = 4.3$ GeV;

2) starting from $\alpha_s(m_b)$ found in 1), using $\mu = m_b$, find α_s at $Q = 2$ GeV.

Photocopy the figure on page 200 and plot your points on it.

N.B.: This treatment neglects a very small α_s shift at m_b when going from $n_f=5$ to $n_f=4$.