The Parton Model

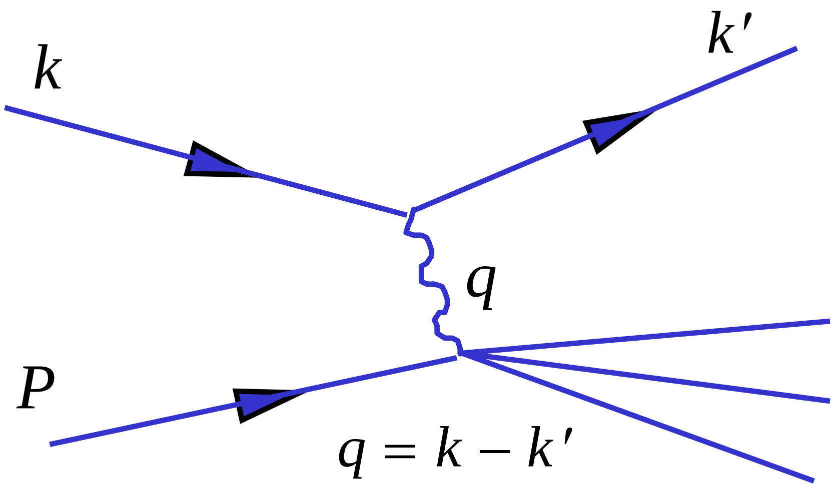
- **Naive Parton Model**
- **QCD Improved Parton Model**

Michel Lefebvre
University of Victoria
Physics and Astronomy

Naive Parton Model

- Deep Inelastic Scattering
- Electron-Nucleon Scattering
- Neutrino-Nucleon Scattering
- Scale Invariance and Partons
- The Parton Model
- Structure Functions
- Sum Rules
- Parton Density Functions
- $e^+ + e^- \to$ Hadrons
- The Drell-Yan Process

Deep Inelastic Scattering



Only 2 independent variables

k incoming lepton

k' outgoing lepton

P Nucleon with mass M

q exchanged particle (probe)

$\gamma, W^{\pm}, Z^{\circ}$

The following invariant quantities are often used:

$$\nu = \frac{q \cdot P}{M} \qquad Q^2 = -q^2$$

$$y = \frac{q \cdot P}{k \cdot P} \qquad x = \frac{Q^2}{2M\nu}$$

x = 1 is the elastic case

$$W^2 = (q + P)^2 = M^2 + 2M\nu - Q^2$$

mass squared of the system recoiling against the lepton

$$s = (k + P)^2 = \frac{Q^2}{xy} + M^2$$

center of mass energy squared

Deep Inelastic Scattering

In the lab frame, where $P = (M, 0, 0, 0)$

we have

$$\nu = E - E'$$ lepton energy loss

$$y = \frac{\nu}{E}$$ fraction of energy lost by lepton

$$Q^2 = 4EE' \sin^2 \frac{\theta}{2}$$ θ is the lepton scattering angle

where it is assumed that

$$EE' \sin^2 \frac{\theta}{2} >> m^2,\ m'^2$$

Of interest,

$$\frac{d^2\sigma}{dx\, dy} = \nu\left(s - M^2\right)\frac{d^2\sigma}{d\nu\, dQ^2} = \frac{2\pi M \nu}{E'} \frac{d^2\sigma}{d\Omega\, dE'} = x\left(s - M^2\right)\frac{d^2\sigma}{dx\, dQ^2}$$

$$dE' d\Omega = \frac{\pi}{EE'} dQ^2 d\nu = \frac{2ME}{E'} \pi y\ dx\ dy$$

Electron-Nucleon Scattering

This is the neutral-current process eN→eX

The differential cross section can be expressed as

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}E'} = \frac{4\alpha^2 E'^2}{Q^4}\left(2W_1 \sin^2\frac{\theta}{2} + W_2 \cos^2\frac{\theta}{2}\right) \qquad W_i = W_i\left(\nu, Q^2\right)$$

or in a Lorentz invariant way as

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\,\mathrm{d}y} = \frac{4\pi\alpha^2}{Q^4}\left(s - M^2\right)\left[\left(1 - y - \frac{M^2 xy}{s - M^2}\right)F_2 + xy^2 F_1\right] \qquad F_i = F_i\left(x, Q^2\right)$$

dimensionless structure functions

$$F_1 = MW_1$$

$$F_2 = \nu W_2$$

2 functions for 2 polarization states of the photon

In the elastic case eN→eN, for high Q^2 (essentially probing the magnetic form factor), one can write

$$F_1 = \tfrac{1}{2} x \mu_\mathrm{N}^2 G^2\left(Q^2\right)\delta\left(1 - x\right)$$

$$F_2 = \mu_\mathrm{N}^2 G^2\left(Q^2\right)\delta\left(1 - x\right)$$

where G is the nucleon form factor, where we have used the following experimental scaling laws:

Electron-Nucleon Scattering

$$G_E^p(Q^2) = \frac{G_M^p(Q^2)}{\mu_p} = \frac{G_M^n(Q^2)}{-\mu_n} \equiv G(Q^2)$$

see Perkins p.194-195

$$G_E^n(Q^2) = 0$$ where the anomalous magnetic moments of the nucleons are $\mu_p = 2.79$ and $\mu_n = -1.91$

For example, the nucleon form factor is well represented by the dipole function

$$G(Q^2) = \left(1 + \frac{Q^2}{0.71\ \text{GeV}^2}\right)^{-2}$$ where $(0.71\ \text{GeV}^2)^{-1/2} \cong 0.23$ fm

is characteristic of the nucleon size. 0.71GeV^2 yields an rms charge radius of the proton of 0.81 fm. Slightly different values are also found in the literature. As Q^2 increases,

- G depresses the chance of elastic scattering;
- the inelastic scattering becomes more likely and the nucleon is more likely to break up;
- the resolution of the probe (here a photon) increases so that we can begin to "see" the nucleon constituents.

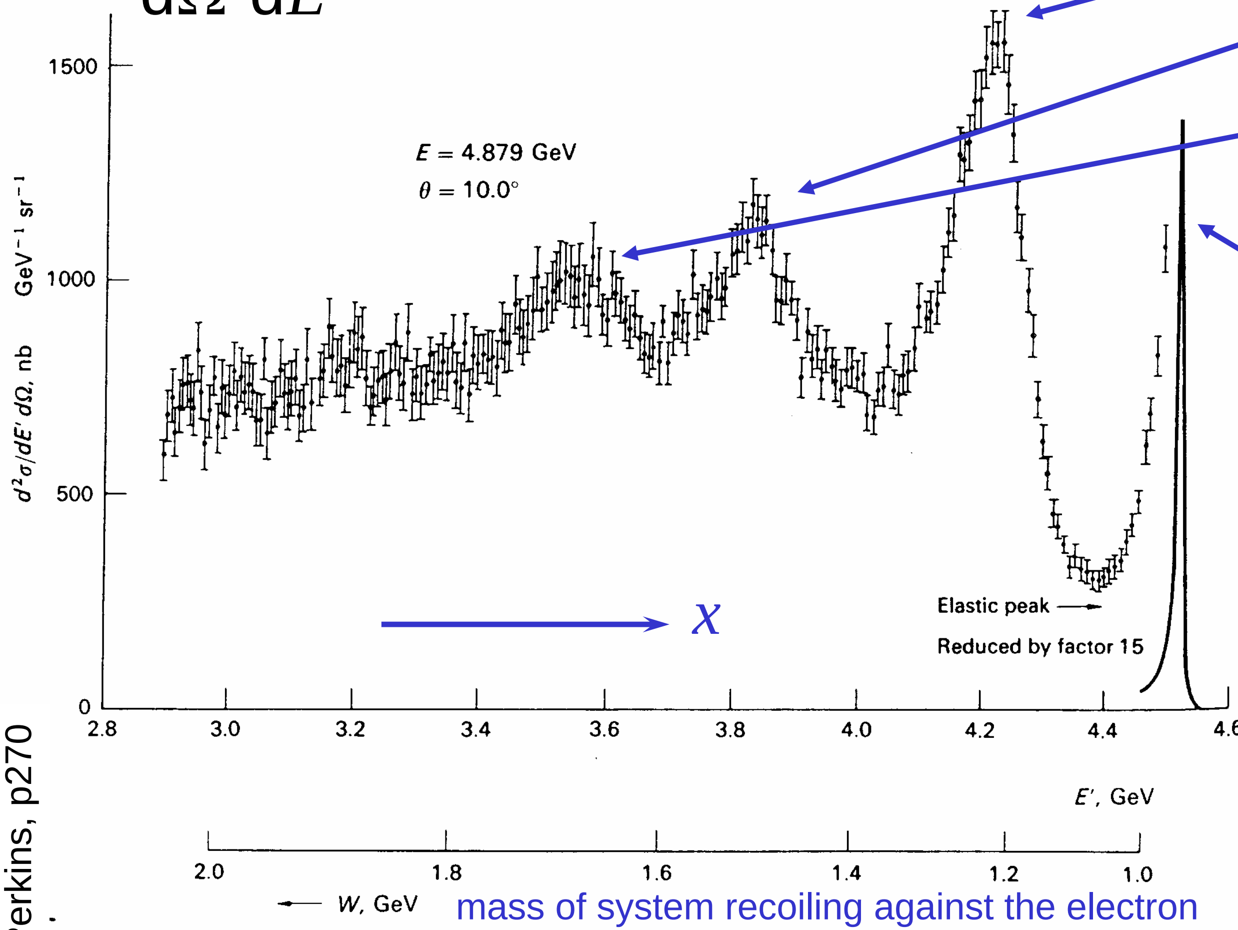
Electron-Nucleon Scattering

$$\frac{d^2\sigma}{d\Omega\, dE'} \text{ vs } E' \quad \text{for ep scattering}$$

$ep \to e\Delta^+(1232)$

$ep \to eN^+(1520)$

$ep \to e\Delta^+(1700)$



elastic ep→ep corresponds to x=1

mass of system recoiling against the electron

Perkins, p270

Figure 8.6 Excitation curve of inelastic *ep* scattering, obtained at the DESY electron accelerator (Bartel *et al.* 1968). *E* and *E′* are the energies of the incident and the scattered electron, and *W* is the mass of the recoiling hadronic state. The peaks due to the pion-nucleon resonances of masses 1.24, 1.51, and 1.69 GeV are clearly visible.

Neutrino-Nucleon Scattering

These are the charged-current processes

$$e^-N \to \nu X$$
$$\nu N \to e^- X$$
$$\bar{\nu} N \to e^+ X$$

The differential cross section can be expressed as

$$\frac{d^2\sigma}{d\Omega\, dE'} = \frac{G_F^2}{2\pi^2} E'^2 \frac{M_W^4}{\left(Q^2 + M_W^2\right)^2} \left[\left(2W_1 \sin^2\frac{\theta}{2} + W_2 \cos^2\frac{\theta}{2} \mp W_3 \left(\frac{E+E'}{M}\right) \sin^2\frac{\theta}{2} \right) \right]$$

where $W = W\left(\nu, Q^2\right)$, or in a Lorentz invariant way

$$\frac{d^2\sigma}{dx\, dy} = \frac{G_F^2\left(s - M^2\right)}{2\pi} \frac{M_W^4}{\left(Q^2 + M_W^2\right)^2} \left[\left(1 - y - \frac{M^2 xy}{s - M^2}\right) F_2^{\nu N} + xy^2 F_1^{\nu N} \pm y\left(1 - \frac{y}{2}\right) x F_3^{\nu N} \right]$$

where the top sign is for neutrino and electron processes, and the bottom one for antineutrino processes.

$F_i = F_i\left(x, Q^2\right)$ dimensionless structure functions

Neutrino-Nucleon Scattering

$$F_1^{\nu N} = MW_1$$
$$F_2^{\nu N} = \nu W_2$$
$$F_3^{\nu N} = \nu W_3$$

3 functions for 3 polarization states of the $W^{\pm}$

The third structure function is sensitive to the parity violating part of the cross section.

Note that the neutral-current cross section is recovered by the replacements

$$\frac{G_F^2}{2\pi}\left(s - M^2\right)\frac{M_W^4}{\left(Q^2 + M_W^2\right)^2} \to \frac{4\pi\alpha^2}{Q^4}\left(s - M^2\right)$$

$$\frac{G_F^2}{2\pi^2}E'^2\frac{M_W^4}{\left(Q^2 + M_W^2\right)^2} \to \frac{4\alpha^2 E'^2}{Q^4}$$

$$F_1^{\nu N} \to F_1$$
$$F_2^{\nu N} \to F_2$$
$$F_3^{\nu N} \to 0$$

Scale Invariance and Partons

Bjorken postulated the scaling law

$$\lim_{\substack{Q^2 \to \infty \\ \nu \to \infty}} MW_1\left(\nu, Q^2\right) \to F_1\left(x\right)$$

$$\lim_{\substack{Q^2 \to \infty \\ \nu \to \infty}} \nu W_2\left(\nu, Q^2\right) \to F_2\left(x\right)$$

$$\lim_{\substack{Q^2 \to \infty \\ \nu \to \infty}} \nu W_3\left(\nu, Q^2\right) \to F_3\left(x\right)$$

Since *x* is dimensionless, there is no scale of mass or length: scale invariance.

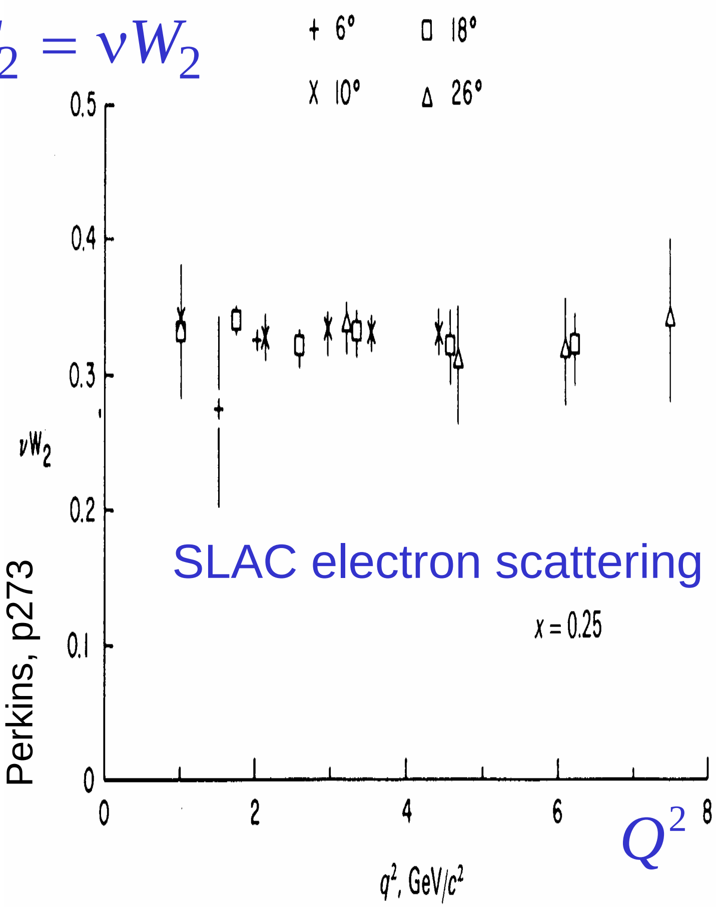
This implies that the virtual photon scatters off pointlike constituents since otherwise the dimensionless structure functions *F(x)* would depend on the ratio

$$Q/Q_\circ$$

where $Q_\circ^{-1}$ is some length scale characteristic of the size of the constituents.

Scale Invariance and Partons

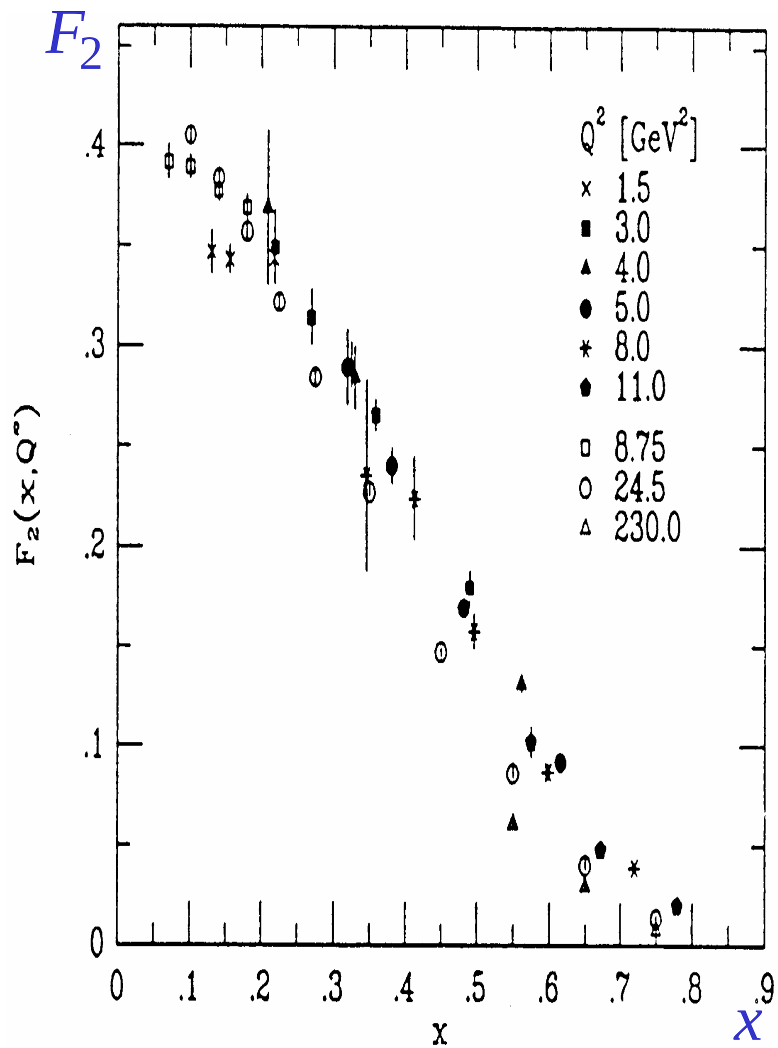
$F_2 = \nu W_2$



SLAC electron scattering

Perkins, p273

Figure 8.8 νW_2 (or F_2) as a function of q^2 at $x = 0.25$. For this choice of x, there is practically no q^2-dependence, that is, exact "scaling." (After Friedman and Kendall (1972).)



FROM STIRLING, FERMILAB-CONF-90/164-T
DATA: SLAC-MIT, PHYS REV D5 (1972) 528
BCDMS, PHYS LETT 223B (1989) 485

Scale Invariance and Partons

In the case of exact scaling

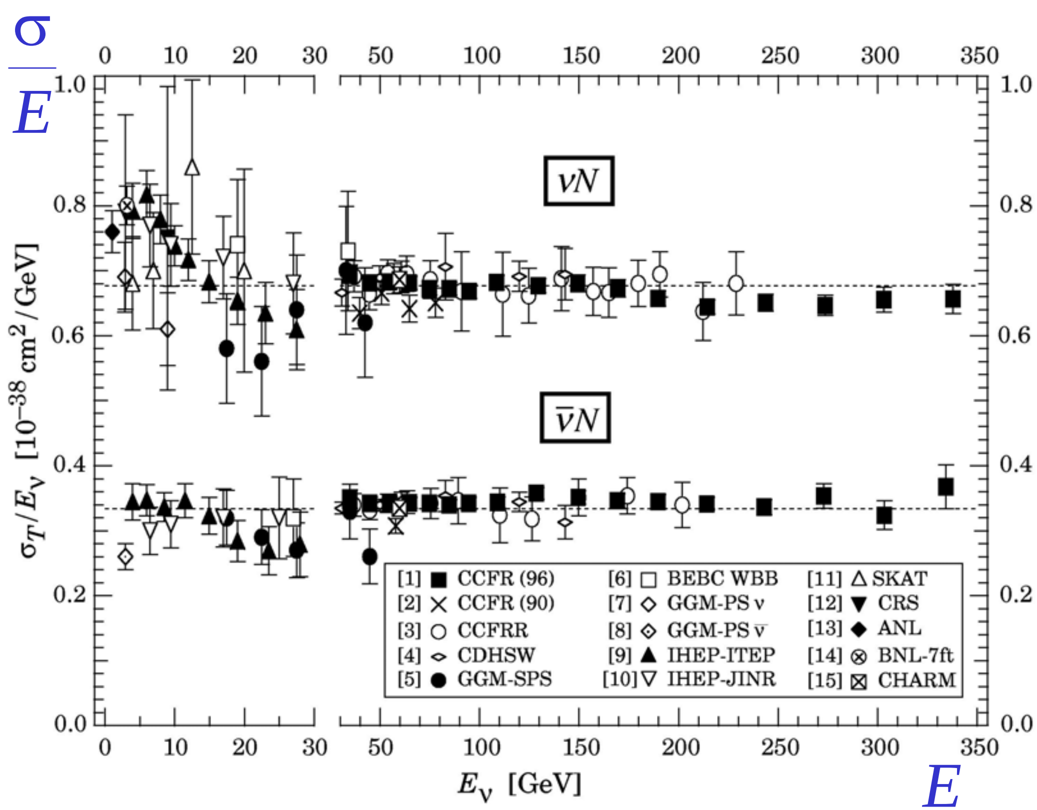
$$F_i^{\nu N} = F_i^{\nu N}(x) \quad i = 1, 2, 3$$

Then

$$\sigma = \int_0^1 \int_0^1 \mathrm{d}x\, \mathrm{d}y \frac{\mathrm{d}^2\sigma}{\mathrm{d}x\, \mathrm{d}y} = aE$$

where *a* is a constant independent of *E*

"The ratio of cross section per nucleon to incident energy is practically constant over two orders of magnitude and is a direct demonstration of pointlike constituents (partons) inside the nucleon" (Perkins p.264)



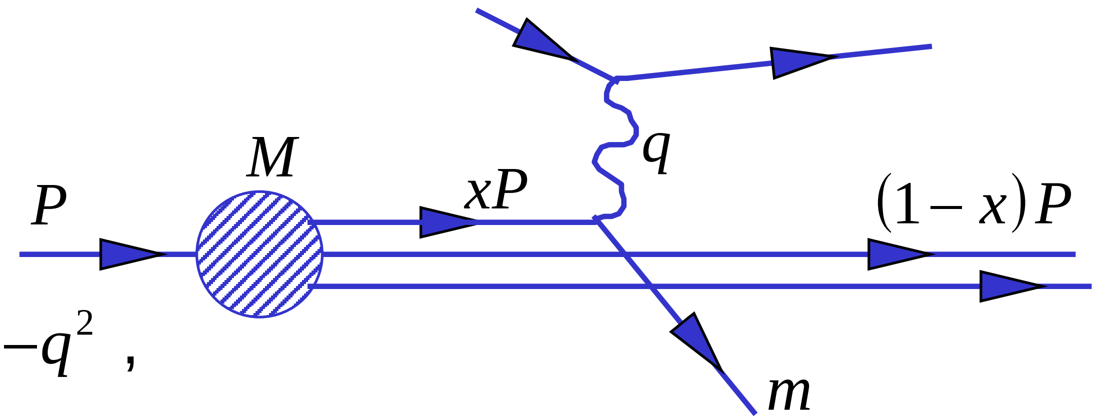
P.D.G. 2002 Fig 39.10

Scale Invariance and Partons

Feynman gave a physical interpretation of scale invariance in the parton model. Go to the “infinite momentum frame”, where the target nucleon has a very large 3-momentum

$$P = P\left(\left|\vec{P}\right|, \left|\vec{P}\right|, 0, 0\right).$$

Since the 3-momentum is large, we can neglect masses and parton transverse momenta



$$(xP+q)^2 = m^2 \approx 0$$

$$x^2P^2 + q^2 + 2xP\cdot q \approx 0$$

if $x^2P^2 = x^2M^2 << Q^2 = -q^2$,

then $x = \dfrac{Q^2}{2P\cdot q} = \dfrac{Q^2}{2M\nu}$

Therefore *x* is the fractional 3-momentum of the parton in the infinite momentum frame.

The Parton Model

- Assume the nucleon is made of pointlike constituents
- In the infinite momentum frame, consider the probe to scatter off a pointlike parton q of mass *m* and charge e_q which carries a fraction ξ of the nucleon's momentum. In the Lab frame, this means

$$m = \xi M$$

Recall the $e\mu \rightarrow e\mu$ cross section (Lab frame, neglecting electron mass)

$$\frac{d\sigma}{d\Omega} = \frac{4\alpha^2 E'^2}{Q^4} \frac{E'}{E} \left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2m^2} \sin^2\frac{\theta}{2} \right)$$

where $m = m_\mu$. Since E' is fixed for a given E and θ, we obtain (Halzen+Martin p 132)

$$\frac{d^2\sigma}{d\Omega\, dE'} = \frac{4\alpha^2 E'^2}{Q^4} \left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2m^2} \sin^2\frac{\theta}{2} \right) \delta\left(\nu - \frac{Q^2}{2m} \right)$$

The Parton Model

Using $m = \xi M$ and assuming a parton of charge e_q instead of a muon, this can be written as

$$\frac{d^2\sigma}{d\Omega\, dE'} = \frac{4\alpha^2 E'^2}{Q^4}\left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2m^2}\sin^2\frac{\theta}{2}\right) e_q^2 \delta\left(\nu - \frac{Q^2}{2m}\right)$$

Comparing this model with the deep inelastic neutral-current result eN→eX

$$\frac{d^2\sigma}{d\Omega\, dE'} = \frac{4\alpha^2 E'^2}{Q^4}\left(2W_1 \sin^2\frac{\theta}{2} + W_2 \cos^2\frac{\theta}{2}\right)$$

and using $\delta\left(\nu - \frac{Q^2}{2m}\right) = \frac{x}{\nu}\delta(\xi - x)$

we obtain for this model the following structure functions

$$\hat{F}_2(x) = x e_q^2 \delta(\xi - x) = 2x\hat{F}_1$$

This suggests that *F(x)* probes a parton constituent with momentum fraction *x*.

The Parton Model

Let $\mathrm{q}(\xi)\ \mathrm{d}\xi$ probability that a parton carries a momentum fraction between ξ and ξ+dξ .

$\mathrm{q}(\xi)$ parton's probability density function, or p.d.f.

then

$$F_2(x)=\sum_{\mathrm{q}}\int_0^1 \mathrm{d}\xi\ \mathrm{q}(\xi)\hat{F}_2(x)=\sum_{\mathrm{q}}\int_0^1 \mathrm{d}\xi\ \mathrm{q}(\xi)xe_{\mathrm{q}}^2\delta(x-\xi)=\sum_{\mathrm{q}} e_{\mathrm{q}}^2 x\mathrm{q}(x)$$

The sum runs overall charged partons q since all the elementary neutral-current processes $e^- q \to e^- q$ are allowed. The result

$$F_2(x)=2xF_1(x)$$ Callan-Gross relation

is verified experimentally. Note that

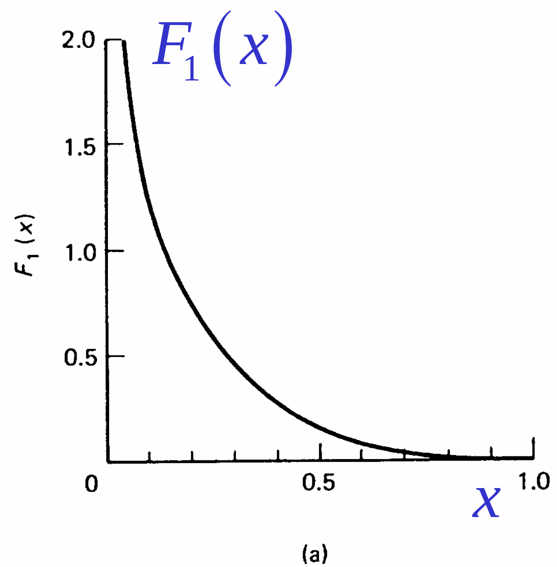
$$\frac{2xF_1(x)}{F_2(x)}=1 \Leftrightarrow \text{ spin } \tfrac{1}{2} \text{ partons}$$

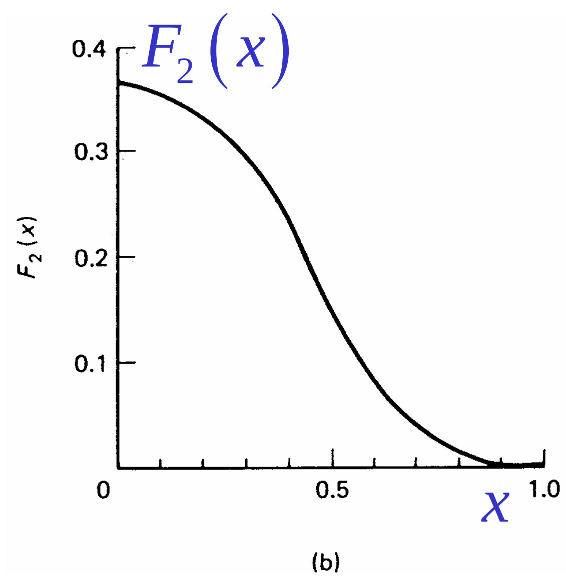
$$\frac{2xF_1(x)}{F_2(x)}=0 \Leftrightarrow \text{ spin } 0 \text{ partons}$$

Therefore the charged partons inside the nucleon are spin 1/2 !

We associate them to quarks.

■ The Parton Model





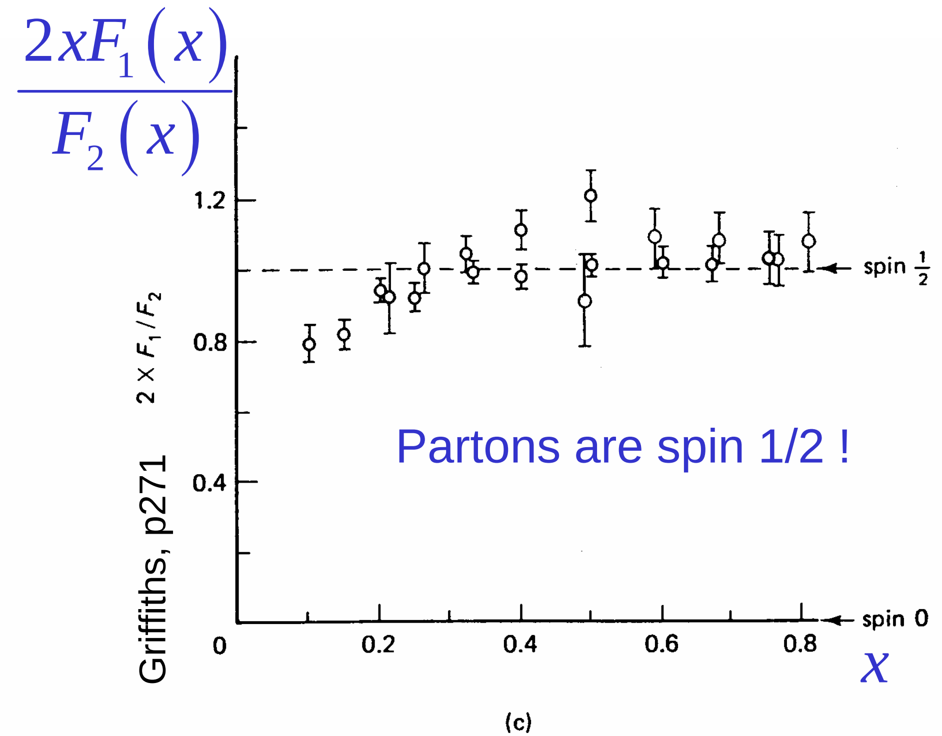


Figure 8.7 Scaling functions and the Callan–Gross relation. In (a) and (b) I plot the experimental measurements of $F_1(x)$ and $F_2(x)$. In (c) the ratio $2xF_1/F_2$ is plotted against x, as a test of the Callan–Gross relation, which evidently holds well for $x \gtrsim 0.2$. [Data from A. Bodek et al., *Phys. Rev.* **D20,** 1471 (1979).]

The Parton Model

A similar analysis can be carried out for the charged-current reactions. Comparing $\nu e^- \to \nu e^-$ $\quad \bar{\nu} e^- \to \bar{\nu} e^-$

with $\nu N \to e^- N$ $\quad \bar{\nu} N \to e^+ N$

one obtains (Perkins p278)

$$F_2^{\nu N}(x) = \sum_{q,\bar{q}'} 2x\left[q(x) + \bar{q}'(x)\right]$$

$$xF_3^{\nu N}(x) = \sum_{q,\bar{q}'} 2x\left[q(x) - \bar{q}'(x)\right]$$

where $q(x)$ and $\bar{q}'(x)$ correspond to the quark and antiquark p.d.f. that are probed in the process.

Note that (Perkins p275)

$$F_1^{\nu n} = F_1^{\bar{\nu} p} \qquad F_2^{\nu n} = F_2^{\bar{\nu} p} \qquad F_3^{\nu n} = -F_3^{\bar{\nu} p}$$

$$F_1^{\nu p} = F_1^{\bar{\nu} n} \qquad F_2^{\nu p} = F_2^{\bar{\nu} n} \qquad F_3^{\nu p} = -F_3^{\bar{\nu} n}$$

In particular, for isoscalar targets we have

$$F_i^{\nu N} = \tfrac{1}{2}\left(F_i^{\nu p} + F_i^{\nu n}\right) = \begin{cases} \tfrac{1}{2}\left(F_1^{\bar{\nu} p} + F_1^{\bar{\nu} n}\right) = F_1^{\bar{\nu} N} \\ \tfrac{1}{2}\left(F_2^{\bar{\nu} p} + F_2^{\bar{\nu} n}\right) = F_2^{\bar{\nu} N} \\ -\tfrac{1}{2}\left(F_3^{\bar{\nu} p} + F_3^{\bar{\nu} n}\right) = -F_3^{\bar{\nu} N} \end{cases}$$

Structure Functions

Let us consider the four lightest quarks $\begin{pmatrix} \mathrm{u} \\ \mathrm{d} \end{pmatrix} \quad \begin{pmatrix} \mathrm{c} \\ \mathrm{s} \end{pmatrix} \quad \begin{matrix} +\frac{2}{3} \\ -\frac{1}{3} \end{matrix}$

Let q(x) denoted by

$$\mathrm{u}(x),\ \ \mathrm{d}(x),\ \ \mathrm{c}(x),\ \ \mathrm{s}(x)$$

to represent the parton density function of the u, d, c and s quarks in the proton.

Isospin invariance relates the proton and neutron p.d.f. for the u and d quarks:

$$\mathrm{u}^{\mathrm{n}}(x) = \mathrm{d}(x)$$
$$\mathrm{d}^{\mathrm{n}}(x) = \mathrm{u}(x)$$
$$\mathrm{c}^{\mathrm{n}}(x) = \mathrm{c}(x)$$
$$\mathrm{s}^{\mathrm{n}}(x) = \mathrm{s}(x)$$

Therefore any $F^{n}(x)$ can be obtained from $F^{p}(x)$ using u$\leftrightarrow$d

Structure Functions

While all $e^- q \to e^- q$ processes are allowed, only the following elementary charged-current processes are permitted:

$$\nu \mathrm{d} \to \mathrm{e}^- \mathrm{u} \quad \text{(or c)}$$
$$\nu \bar{\mathrm{u}} \to \mathrm{e}^- \bar{\mathrm{d}} \quad \text{(or } \bar{\mathrm{s}}\text{)}$$
$$\bar{\nu} \mathrm{u} \to \mathrm{e}^+ \mathrm{d} \quad \text{(or s)}$$
$$\bar{\nu} \bar{\mathrm{d}} \to \mathrm{e}^+ \bar{\mathrm{u}} \quad \text{(or } \bar{\mathrm{c}}\text{)}$$

Cabibbo suppressed

… likewise for (u,d)↔(c,s)

Therefore we obtain

$$F_2^{\mathrm{ep}} = x\left[\tfrac{4}{9}\left(\mathrm{u}+\bar{\mathrm{u}}+\mathrm{c}+\bar{\mathrm{c}}\right)+\tfrac{1}{9}\left(\mathrm{d}+\bar{\mathrm{d}}+\mathrm{s}+\bar{\mathrm{s}}\right)\right]$$
$$F_2^{\nu\mathrm{p}} = 2x\left(\mathrm{d}+\mathrm{s}+\bar{\mathrm{u}}+\bar{\mathrm{c}}\right)$$
$$F_2^{\bar{\nu}\mathrm{p}} = 2x\left(\mathrm{u}+\mathrm{c}+\bar{\mathrm{d}}+\bar{\mathrm{s}}\right)$$
$$xF_3^{\nu\mathrm{p}} = 2x\left(\mathrm{d}+\mathrm{s}-\bar{\mathrm{u}}-\bar{\mathrm{c}}\right)$$
$$-xF_3^{\bar{\nu}\mathrm{p}} = 2x\left(\mathrm{u}+\mathrm{c}-\bar{\mathrm{d}}-\bar{\mathrm{s}}\right)$$

Structure Functions

When the target has an even mixture of p and n we use

$$F_2^{\mathrm{eN}} \equiv \tfrac{1}{2}\left(F_2^{\mathrm{ep}} + F_2^{\mathrm{en}}\right) = x\left[\tfrac{5}{18}\left(\mathrm{u}+\bar{\mathrm{u}}+\mathrm{d}+\bar{\mathrm{d}}\right)+\tfrac{4}{9}\left(\mathrm{c}+\bar{\mathrm{c}}\right)+\tfrac{1}{9}\left(\mathrm{s}+\bar{\mathrm{s}}\right)\right]$$

$$F_2^{\nu\mathrm{N}} \equiv \tfrac{1}{2}\left(F_2^{\nu\mathrm{p}} + F_2^{\nu\mathrm{n}}\right) = x\left[\mathrm{u}+\mathrm{d}+\bar{\mathrm{u}}+\bar{\mathrm{d}}+2\left(\mathrm{s}+\bar{\mathrm{c}}\right)\right]$$

$$F_2^{\bar{\nu}\mathrm{N}} \equiv \tfrac{1}{2}\left(F_2^{\bar{\nu}\mathrm{p}} + F_2^{\bar{\nu}\mathrm{n}}\right) = x\left[\mathrm{u}+\mathrm{d}+\bar{\mathrm{u}}+\bar{\mathrm{d}}+2\left(\mathrm{c}+\bar{\mathrm{s}}\right)\right]$$

$$xF_3^{\nu\mathrm{N}} \equiv \tfrac{1}{2}\left(xF_3^{\nu\mathrm{p}} + xF_3^{\nu\mathrm{n}}\right) = x\left[\mathrm{u}+\mathrm{d}-\bar{\mathrm{u}}-\bar{\mathrm{d}}+2\left(\mathrm{s}-\bar{\mathrm{c}}\right)\right]$$

$$-xF_3^{\bar{\nu}\mathrm{N}} \equiv -\tfrac{1}{2}\left(xF_3^{\bar{\nu}\mathrm{p}} + xF_3^{\bar{\nu}\mathrm{n}}\right) = x\left[\mathrm{u}+\mathrm{d}-\bar{\mathrm{u}}-\bar{\mathrm{d}}+2\left(\mathrm{c}-\bar{\mathrm{s}}\right)\right]$$

With sufficient measurements of the structure functions, these relations can be inverted to obtain the p.d.f.'s.

Structure Functions

As we will see, the following picture emerges:

The proton consists of 3 valence quarks

$$\mathrm{P} = \left(\mathrm{u_v}, \mathrm{u_v}, \mathrm{d_v}\right)$$

which carry the electric charge and the baryon number, and an infinite sea of light quark-antiquark pairs

$$\mathrm{u_s},\ \overline{\mathrm{u}}_\mathrm{s},\ \mathrm{d_s},\ \overline{\mathrm{d}}_\mathrm{s},\ \mathrm{c_s},\ \overline{\mathrm{c}}_\mathrm{s},\ \mathrm{s_s},\ \overline{\mathrm{s}}_\mathrm{s}$$

We set

$$S \equiv \mathrm{u_s} = \overline{\mathrm{u}}_\mathrm{s} = \mathrm{d_s} = \overline{\mathrm{d}}_\mathrm{s}$$

and we have

$$\mathrm{u} = \mathrm{u_v} + S$$
$$\mathrm{d} = \mathrm{d_v} + S$$
$$\overline{\mathrm{u}} = \overline{\mathrm{d}} = S$$
$$\mathrm{c} = \overline{\mathrm{c}} = \mathrm{c_s} = \overline{\mathrm{c}}_\mathrm{s}$$
$$\mathrm{s} = \overline{\mathrm{s}} = \mathrm{s_s} = \overline{\mathrm{s}}_\mathrm{s}$$

Structure Functions

We can get an estimate of the antiquark content of the proton by measuring *R* on isoscalar targets

$$R = \frac{\sigma^{\bar{\nu}N\to\mu^+X}}{\sigma^{\nu N\to\mu^-X}}$$

Let δ_Q and $\delta_{\bar{Q}}$ be the nucleon momentum fraction carried by the quarks and antiquarks respectively

$$\delta_Q \equiv \int_0^1 dx\, xQ = \int_0^1 dx\, x\left(u+d+c+s\right)$$

$$\delta_{\bar{Q}} \equiv \int_0^1 dx\, x\bar{Q} = \int_0^1 dx\, x\left(\bar{u}+\bar{d}+\bar{c}+\bar{s}\right)$$

From angular distribution considerations we have the following cross section weights (Halzen-Martin pages 274-276)

$$\nu q \to X : 1 \qquad \bar{\nu} q \to X : \frac{1}{3} \qquad \nu \bar{q} \to X : \frac{1}{3}\frac{\delta_{\bar{Q}}}{\delta_Q} \qquad \bar{\nu}\bar{q} \to X : \frac{\delta_{\bar{Q}}}{\delta_Q}$$

Therefore if there were no antiquarks in the proton, we would have *R*=1/3. Otherwise we would have

$$R = \frac{\frac{1}{3} + \frac{\delta_{\bar{Q}}}{\delta_Q}}{1 + \frac{1}{3}\frac{\delta_{\bar{Q}}}{\delta_Q}} \Rightarrow \frac{\delta_{\bar{Q}}}{\delta_Q} = \frac{3R-1}{3-R}$$

Experimentally we obtain

$$R \approx 0.45 \Rightarrow \frac{\delta_{\bar{Q}}}{\delta_Q} \approx 0.14$$

Structure Functions

Also of interest, we have

$$\frac{18}{5x}F_2^{eN} = u + \bar{u} + d + \bar{d} + \tfrac{8}{5}(c + \bar{c}) + \tfrac{2}{5}(s + \bar{s})$$

$$\frac{1}{x}F_2^{\nu N} = u + \bar{u} + d + \bar{d} + 2c + 2\bar{s}$$

If we assume c(x) ≤ s(x), we obtain

$$\frac{18}{5}F_2^{eN}(x) \le F_2^{\nu N}(x)$$

equality holds if $c = \bar{c} = s = \bar{s}$.

This is confirmed experimentally. Since

$$\tfrac{1}{2}\left(e_u^2 + e_d^2\right) = \tfrac{1}{2}\left(\tfrac{4}{9} + \tfrac{1}{9}\right) = \tfrac{5}{18}$$

this result is a confirmation of the fractional charge assignments for the quarks.

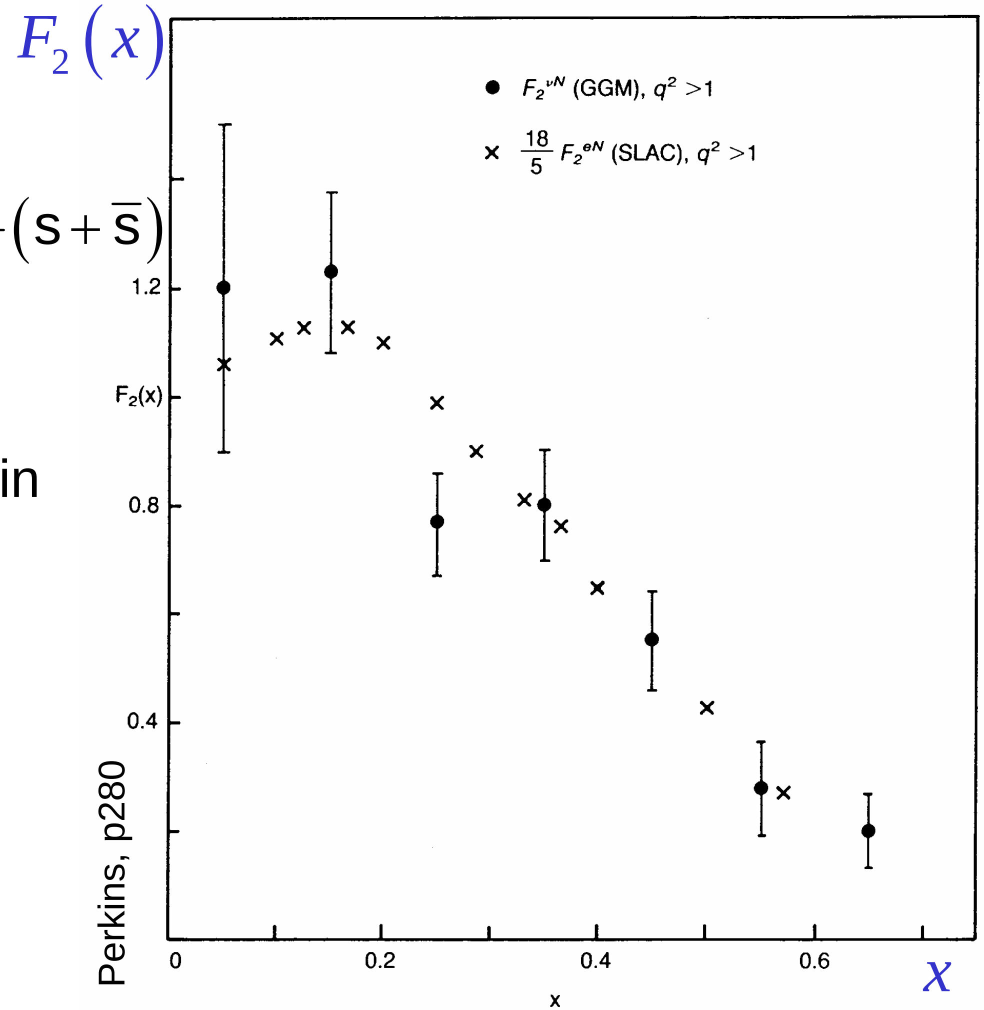


Figure 8.12 (a) First comparison of $F_2^{\nu N}$ measured in neutrino-nucleon scattering in the Gargamelle heavy-liquid bubble chamber in a PS neutrino beam at CERN, with SLAC data on F_2^{eN} from electron-nucleon scattering, in the same region of q^2. The two sets of data agree when the electron points are multiplied by the factor $\frac{18}{5}$, which is the reciprocal of the mean squared charge of u- and d-quarks in the nucleon. This is a confirmation of the fractional charge assignments for the quarks. Note that the total area under the curve, measuring the total momentum fraction in the nucleon carried by quarks, is about 0.5. The remaining mass is ascribed to gluon constituents, which are the postulated carriers of the interquark color field.

Structure Functions

We can also consider
$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} = \frac{\frac{4}{9}\left(d+\bar{d}+c+\bar{c}\right)+\frac{1}{9}\left(u+\bar{u}+s+\bar{s}\right)}{\frac{4}{9}\left(u+\bar{u}+c+\bar{c}\right)+\frac{1}{9}\left(d+\bar{d}+s+\bar{s}\right)}$$

Since $\lim_{x\to 1} S(x)=0$ we expect
$$\lim_{x\to 1}\frac{F_2^{en}(x)}{F_2^{ep}(x)} = \lim_{x\to 1}\frac{u_v+4d_v}{4u_v+d_v} = \left\{\frac{\frac{1}{4}\ \text{if } u_v \text{ dominates}}{4\ \text{if } d_v \text{ dominates}}\right.$$

Since $\lim_{x\to 0} S(x) > \lim_{x\to 0} q_v(x)$ then $\lim_{x\to 0}\frac{F_2^{en}(x)}{F_2^{ep}(x)}=1$

Also $\left.\frac{F_2^{en}(x)}{F_2^{ep}(x)}\right|_{u=d}=1$

Structure Functions

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)}$$

We see that u_V dominates at large *x*, while the sea dominates at low *x*.

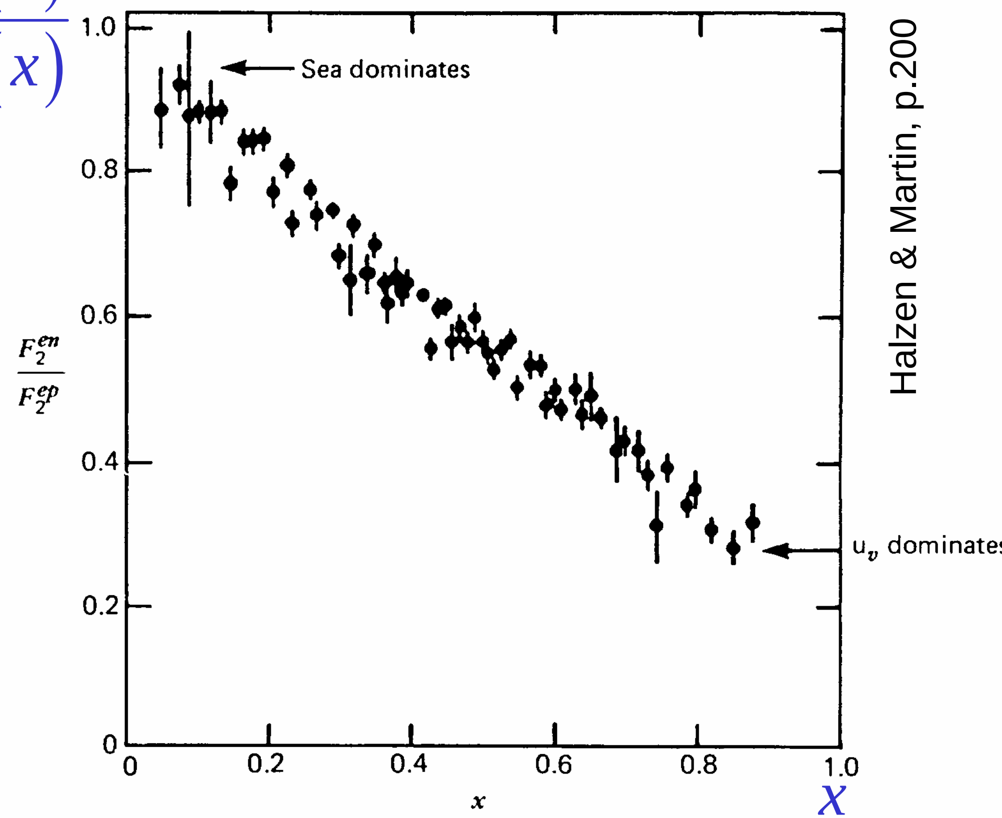


Fig. 9.6 The ratio F_2^{en}/F_2^{ep} as a function of x, measured in deep inelastic scattering. Data are from the Stanford Linear Accelerator.

Sum Rules

If the parton model is correct, one expects

$$\int_0^1 \mathrm{d}x\ \left(\mathrm{u}-\bar{\mathrm{u}}\right) = \int_0^1 \mathrm{d}x\ \mathrm{u_v} = N_\mathrm{u} = 2$$

$$\int_0^1 \mathrm{d}x\ \left(\mathrm{d}-\bar{\mathrm{d}}\right) = \int_0^1 \mathrm{d}x\ \mathrm{d_v} = N_\mathrm{d} = 1$$

$$\int_0^1 \mathrm{d}x\ \left(\mathrm{c}-\bar{\mathrm{c}}\right) = \int_0^1 \mathrm{d}x\ \left(\mathrm{s}-\bar{\mathrm{s}}\right) = 0$$

We therefore have the Gross-Llewellyn-Smith sum rule

$$\int_0^1 \frac{xF_3^{\nu\mathrm{N}}}{x}\ \mathrm{d}x = \int_0^1 \left(\mathrm{u_v}+\mathrm{d_v}\right)\ \mathrm{d}x = N_\mathrm{u} + N_\mathrm{d} = 3$$

Experimentally we obtain 2.79 ± 0.16 (CCFR 1988) and the difference from 3 is understood to come from QCD corrections (see Mishra and Sciulli)

This indicates that the valence quarks do carry baryon number 1/3.

Sum Rules

From neutrino data using hydrogen and deuterium targets, one can extract the Adler sum rule

$$\frac{1}{2}\int_0^1 \frac{\mathrm{d}x}{x}\left(F_2^{\nu \mathrm{n}} - F_2^{\nu \mathrm{p}}\right) = \int_0^1 \mathrm{d}x\ \left(\mathrm{u_v} - \mathrm{d_v}\right) = N_\mathrm{u} - N_\mathrm{d} = 1$$

This result is free of QCD corrections.

Experimentally we obtain 1.01 $\pm$ 0.20 (WA25 1985, Neutrino and antineutrino interactions in deuterium, CERN SPS).

We also have the Gottfried sum rule

$$\int_0^1 \frac{\mathrm{d}x}{x}\left(F_2^{\mathrm{ep}} - F_2^{\mathrm{en}}\right) = \int_0^1 \mathrm{d}x\ \left(\mathrm{e_u^2} - \mathrm{e_d^2}\right)\left(\mathrm{u_v} - \mathrm{d_v}\right)$$

$$= \left(N_\mathrm{u} - N_\mathrm{d}\right)\left(\mathrm{e_u^2} - \mathrm{e_d^2}\right) = \left(2-1\right)\left(\tfrac{4}{9} - \tfrac{1}{9}\right) = \tfrac{1}{3}$$

Experimentally we obtain 0.24 $\pm$ 0.11 (EMC 1987, CERN SPS), using muons

Sum Rules

Finally we have the momentum sum rule. Neglecting the sea contribution (which are further suppressed here by the weight x), we have

$$\frac{18}{5}\int_0^1 \mathrm{d}x\ F_2^{\mathrm{eN}} \approx \int_0^1 \mathrm{d}x\ F_2^{\nu\mathrm{N}} \approx \int_0^1 \mathrm{d}x\ x\left(\mathrm{u}+\bar{\mathrm{u}}+\mathrm{d}+\bar{\mathrm{d}}\right) = \delta_{\mathrm{Q}\bar{\mathrm{Q}}}$$

where $\delta_{\mathrm{Q}\bar{\mathrm{Q}}}$ is the "expectation value" of the fraction of the nucleon momentum carried by the charged partons.

Experimentally , at $Q^2 \approx 1$ to 10 GeV^2,

$$\int_0^1 \mathrm{d}x\ F_2^{\mathrm{ep}} = \tfrac{4}{9}\delta_{\mathrm{u}\bar{\mathrm{u}}} + \tfrac{1}{9}\delta_{\mathrm{d}\bar{\mathrm{d}}} = 0.18$$

$$\int_0^1 \mathrm{d}x\ F_2^{\mathrm{en}} = \tfrac{1}{9}\delta_{\mathrm{u}\bar{\mathrm{u}}} + \tfrac{4}{9}\delta_{\mathrm{d}\bar{\mathrm{d}}} = 0.12$$

where $\delta_{\mathrm{u}\bar{\mathrm{u}}} \equiv \int_0^1 \mathrm{d}x\ x\left(\mathrm{u}+\bar{\mathrm{u}}\right)$

This yields $\delta_{\mathrm{u}\bar{\mathrm{u}}} = 0.36$ $\delta_{\mathrm{d}\bar{\mathrm{d}}} = 0.18$

Which means that a fraction

$$\delta_{\mathrm{g}} = 1 - \delta_{\mathrm{Q}\bar{\mathrm{Q}}} = 1 - \delta_{\mathrm{u}\bar{\mathrm{u}}} - \delta_{\mathrm{d}\bar{\mathrm{d}}} = 0.46$$

of the momentum is carried by neutral partons, which are not seen by the weak or electromagnetic probes. They are associated with the gluons.

Parton Density Functions

The following approximate form of the p.d.f. valid in the limit $x \to 0$ and $x \to 1$ is obtained (Collin and Martin, p34)

$$\mathrm{q}(x) = C_{\mathrm{q}} x^{-\alpha} (1-x)^{2n_s - 1}$$

where n_s is the minimum number of the other partons whose momentum have to vanish in the limit $x \to 1$:

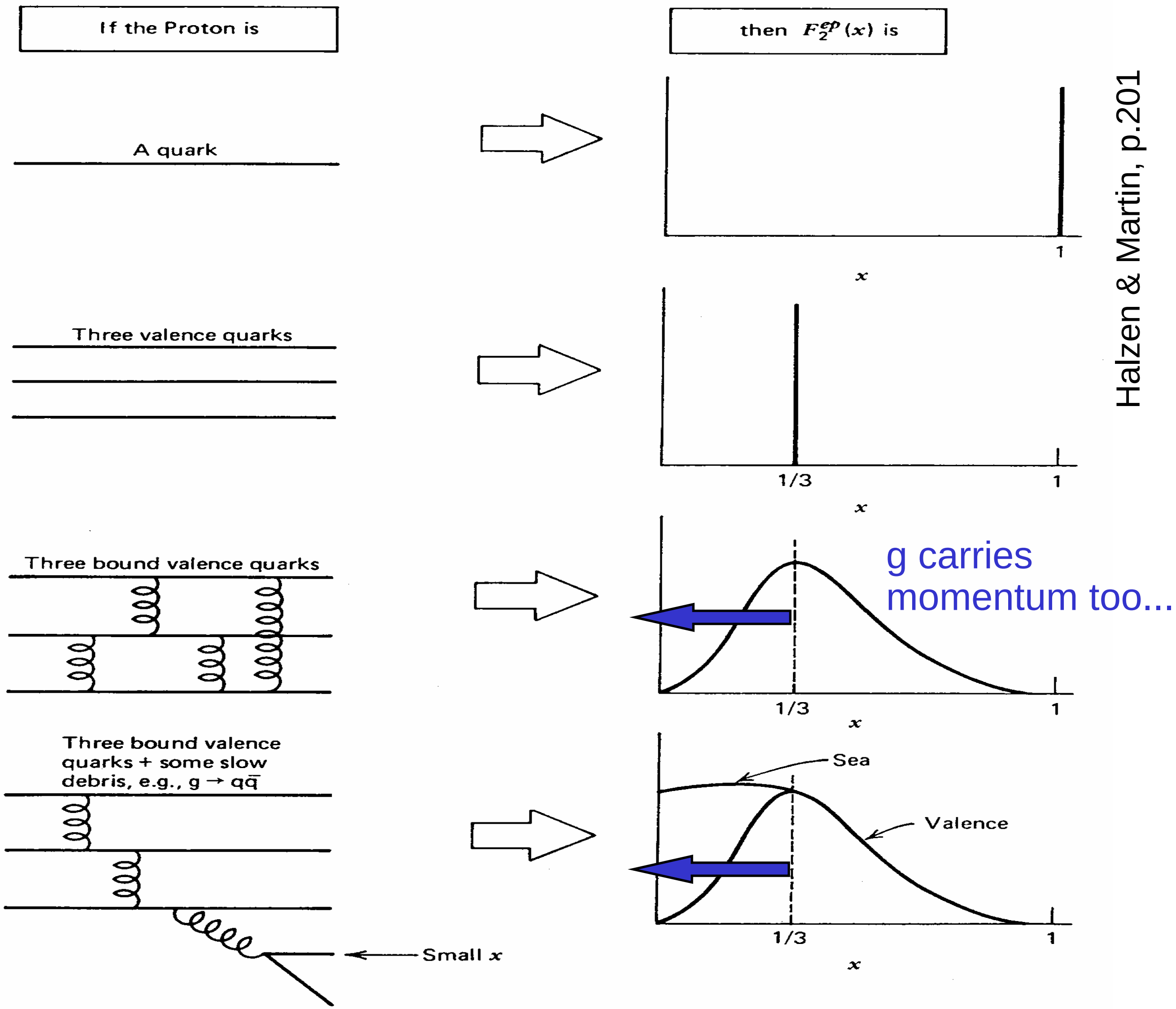
$$n_s = 2 \quad \text{for} \quad \mathrm{q_v}$$
$$n_s = 4 \quad \text{for} \quad \mathrm{q_s}$$

and where

$$\alpha = \tfrac{1}{2} \quad \text{for} \quad \mathrm{q_v}$$
$$\alpha = 1 \quad \text{for} \quad \mathrm{q_s}$$

The constant C_{q} is constrained by the sum rules.

Parton Density Functions

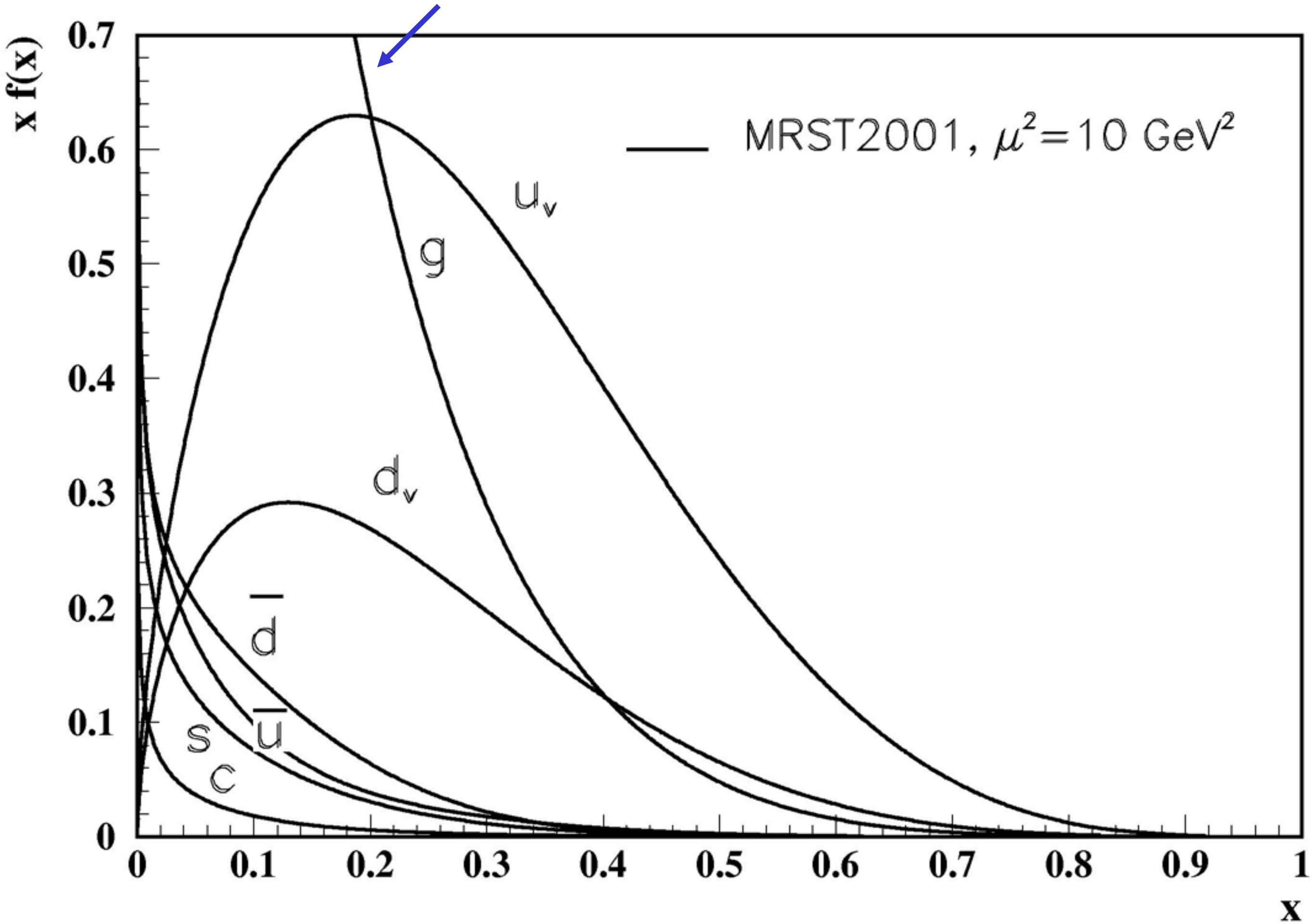


Halzen & Martin, p.201

Parton Density Functions

Quarks and gluons distribution functions at $Q^2 = 10$ GeV^2

Note that g dominates at low *x*.



P.D.G. 2002 Fig 14.4

$e^+ e^- \to$ Hadrons

Consider the ratio
$$R = \frac{\sigma\left(e^+e^- \to \text{hadrons}\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)}$$

If we consider the hadron production to proceed in two stages

$$e^+e^- \to q\bar{q} \qquad \text{then} \qquad q\bar{q} \to \text{hadrons}$$

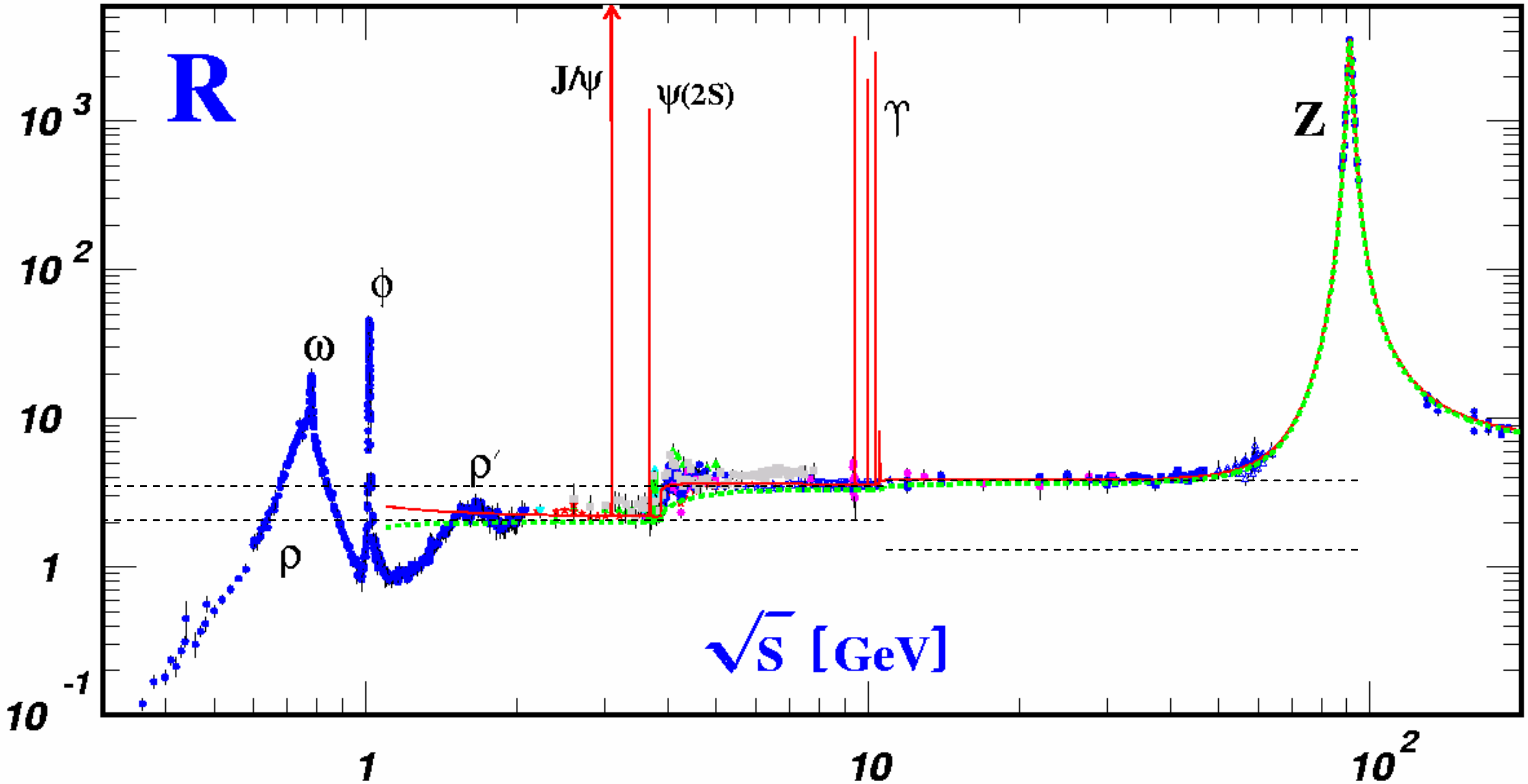
Then the parton model yields

$$R = 3\sum_q e_q^2$$

where the sum runs over all quarks kinematically available. The factor 3 counts the colours, and is confirmed by experiment. This is a compelling evidence for the colour hypothesis.

Note that as the center of mass energy becomes close to M_Z, R is expected to grow up to
$$\frac{B\left(Z \to \text{hadrons}\right)}{B\left(Z \to \mu^+\mu^-\right)} = 20.7$$

e$^+$ e$^-$ → Hadrons



P.D.G. 2004, fig 40.6b

The solid curve is 3-loop QCD predictions, and the dashed curve is the naïve quark parton model.

The dashed lines represent

- u+d+s+c+b no colour 11/9
- u+d+s 2
- u+d+s+c 10/3
- u+d+s+c+b 11/3

■ $e^+ e^- \rightarrow$ Hadrons

Since the e+ e- $\rightarrow \mu^+ \mu^-$ center of mass cross section is given by

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left(1 + \cos^2\theta\right)$$

We expect the angular distribution of the quarks-antiquark to follow the same angular distribution (as long as the effect of the Z is negligible).

At high energy, the quarks-antiquark pair gives rise to two jets.

Their experimental angular distribution confirms the spin 1/2 nature of the quarks.

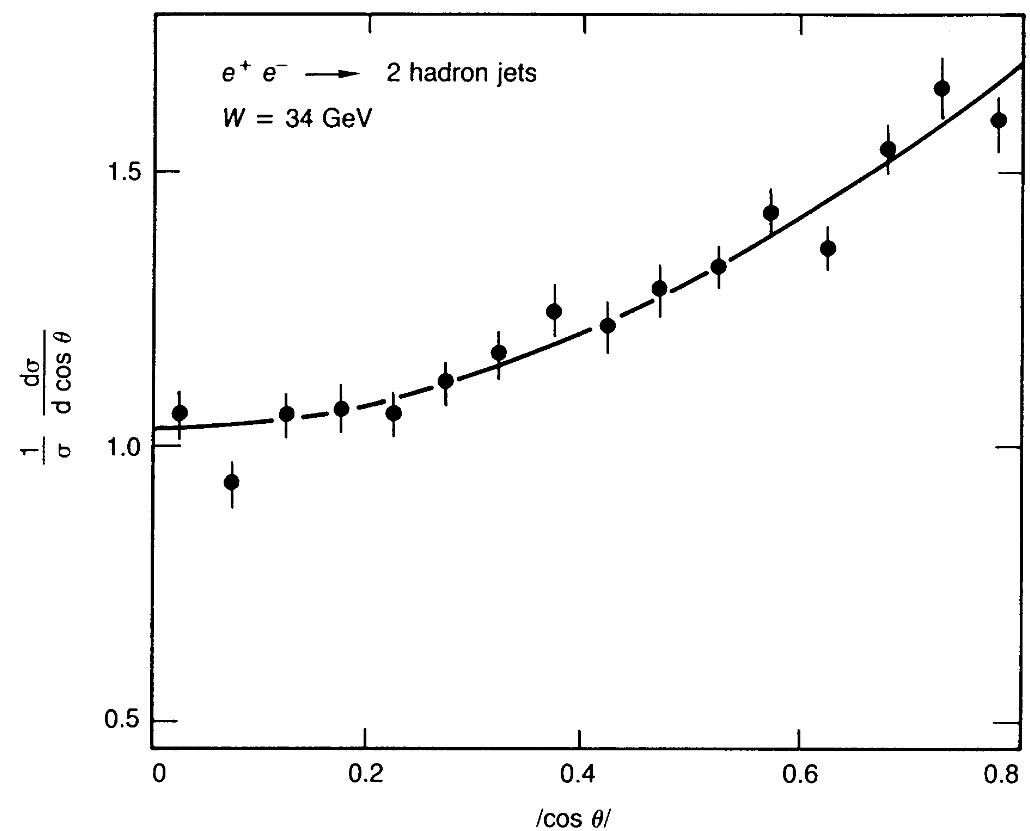
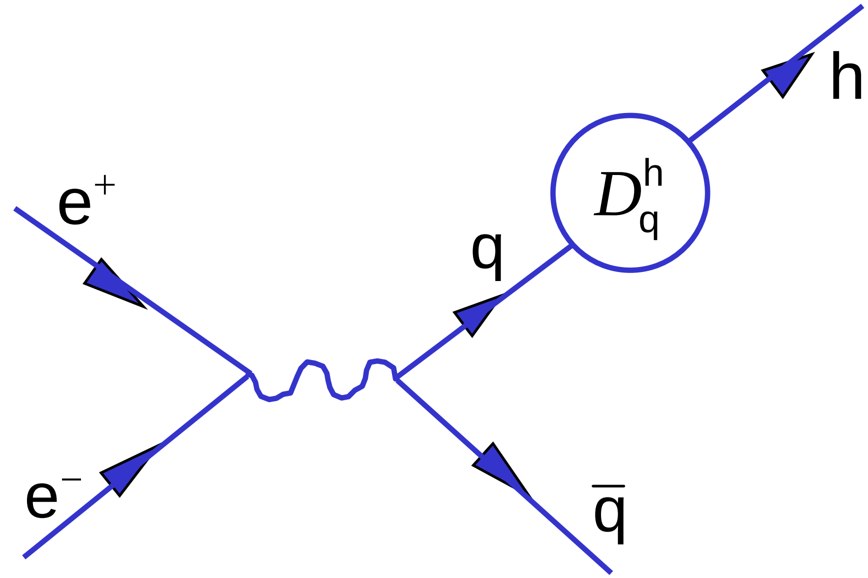


Figure 8.14 Center-of-mass angular distribution of the two hadron jets (as in Fig. 2.23) relative to the beam axis in e^+e^- annihilation at high energy. It is consistent with a $(1 + \cos^2\theta)$ distribution, as expected if the fundamental process is $e^+e^- \rightarrow Q\bar{Q}$.

Perkins, p285

$e^+ e^- \to$ Hadrons



Consider the process $e^+ e^- \to hX$

The production of the hadron should be independent of how the quark was produced. Therefore in the parton model we have

$$\frac{d\sigma}{dz}\left(e^+e^- \to hX\right) = \left(\frac{4\pi\alpha^2}{3s}\right) 3\sum_q e_q^2 \left[D_q^h(z) + D_{\bar{q}}^h(z)\right]$$

where

$D_q^h(z)dz$ probability that the hadron h is found in the debris of q carrying a fraction *z* of its energy

$D_q^h(z)$ fragmentation function

note that

$$z = \frac{E_h}{E_q} = \frac{E_h}{E_{beam}}$$

$e^+ e^- \to$ Hadrons

We have the following sum rules

$$\sum_{\mathrm{h}} \int_0^1 z D_{\mathrm{q}}^{\mathrm{h}} \, \mathrm{d}z = 1 \qquad \text{conservation of energy}$$

$$\sum_{\mathrm{h}} e_{\mathrm{h}} \int_0^1 D_{\mathrm{q}}^{\mathrm{h}} \, \mathrm{d}z = e_{\mathrm{q}} \qquad \text{conservation of charge}$$

The average multiplicity of hadrons of type h is given by

$$\langle n_{\mathrm{h}} \rangle = \sum_{\mathrm{q}} \int_{z_{\min}}^1 \left(D_{\mathrm{q}}^{\mathrm{h}} + D_{\bar{\mathrm{q}}}^{\mathrm{h}} \right) \mathrm{d}z \ , \qquad z_{\min} = \frac{m_{\mathrm{h}}}{E_{\mathrm{beam}}}$$

The following approximate form of the fragmentation function valid in the limit $z \to 0$ and $z \to 1$ is obtained (Collin and Martin),

$$D_{\mathrm{q}}^{\mathrm{h}}(z) = C_{\mathrm{q}}^{\mathrm{h}} z^{-1} (1-z)^{2n_s - 1}$$

where n_{s} is defined as for the p.d.f.

$e^+ e^- \to$ Hadrons

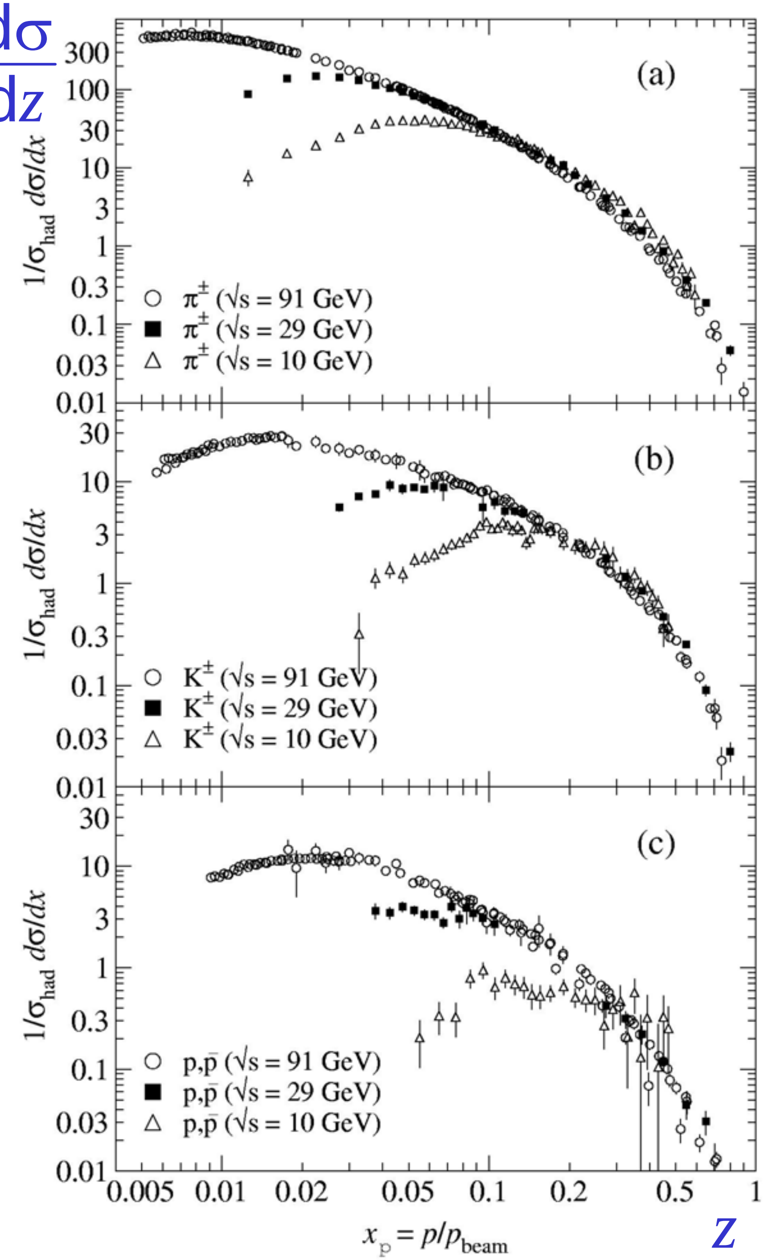
The parton model therefore predicts

$$\frac{1}{\sigma_{\text{had}}}\frac{d\sigma}{dz}\left(e^+e^- \to hX\right) = \frac{\sum_q e_q^2\left[D_q^h(z)+D_{\bar{q}}^h(z)\right]}{\sum_q e_q^2} = F_h(z)$$

This is expected to be scale invariant in the parton model. This agrees well with experiment.

The large scaling violations for z smaller than about 0.2 are associated to c and b quark production (large multiplicity and low z) (Halzen-Martin p.234)

$$\frac{1}{\sigma_{\text{had}}}\frac{d\sigma}{dz}$$



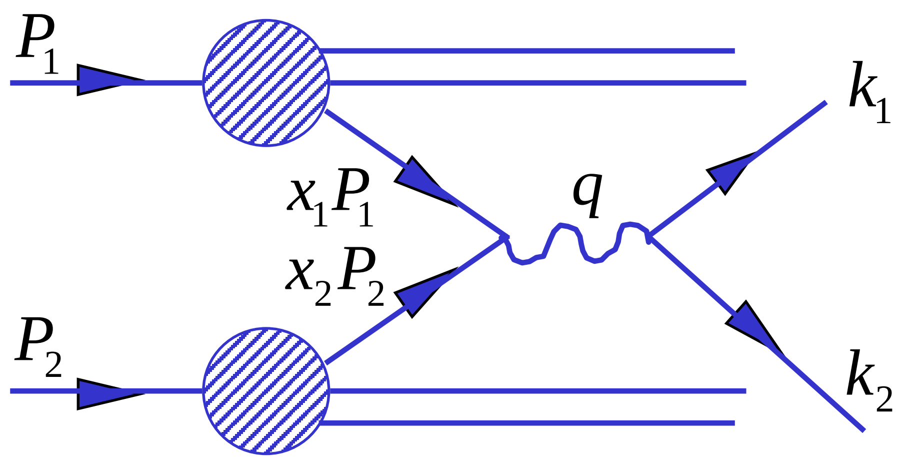
z

P.D.G. 2004, fig 17.4

The Drell-Yan Process

Consider the lepton pair production in hadron-hadron collisions

$$AB \rightarrow \mu^+\mu^- X$$



via an annihilation process.
We have

$$s = \left(P_1 + P_2\right)^2 = E_{\text{CM}}^2$$

$$\hat{s} = \left(x_1 P_1 + x_2 P_2\right)^2 = q^2 = Q^2 = \left(k_1 + k_2\right)^2$$

$$P^2 = M^2$$

$$k^2 = m_k^2$$

In the limit $s >> M^2$ and $s >> m_k{}^2$,we obtain $\hat{s} = x_1 x_2 s$

Let $\hat{\sigma} = \sigma\left(\text{q}\bar{\text{q}} \rightarrow \mu^+\mu^-\right)$ then $\dfrac{\text{d}\hat{\sigma}}{\text{d}\hat{s}} = \hat{\sigma}\delta\left(\hat{s} - x_1 x_2 s\right)$

In the case of pure QED, we have (no sum over colours)

$$\hat{\sigma} = \sigma\left(\text{q}\bar{\text{q}} \rightarrow \mu^+\mu^-\right) = \frac{4\pi\alpha^2}{3\hat{s}} e_{\text{q}}^2$$

The Drell-Yan Process

The parton model yields

$$\frac{d\sigma}{d\hat{s}}\left(AB \to \mu^+\mu^-\right)$$

$$= \frac{1}{3}\sum_q \int dx_1 \int dx_2 \left[q^A(x_1)\bar{q}^B(x_2) + \bar{q}^A(x_1)q^B(x_2)\right]\frac{d\hat{\sigma}}{d\hat{s}}$$

$$= \frac{1}{3\hat{s}}\sum_q \int dx_1 \int dx_2 \left[x_1 q^A(x_1)x_2\bar{q}^B(x_2) + x_1\bar{q}^A(x_1)x_2 q^B(x_2)\right]\hat{\sigma}\delta\left(x_1x_2 - \tau\right)$$

where we have used

$$\delta\left(\hat{s} - x_1x_2s\right) = \frac{x_1x_2}{\hat{s}}\delta\left(x_1x_2 - \tau\right) \qquad \tau \equiv \frac{\hat{s}}{s}$$

In the case of pure QED we see that

$$\hat{s}^2\frac{d\sigma}{d\hat{s}}\left(AB \to \mu^+\mu^-\right) = F(\tau)$$

is expected to be scale invariant.

The Drell-Yan Process

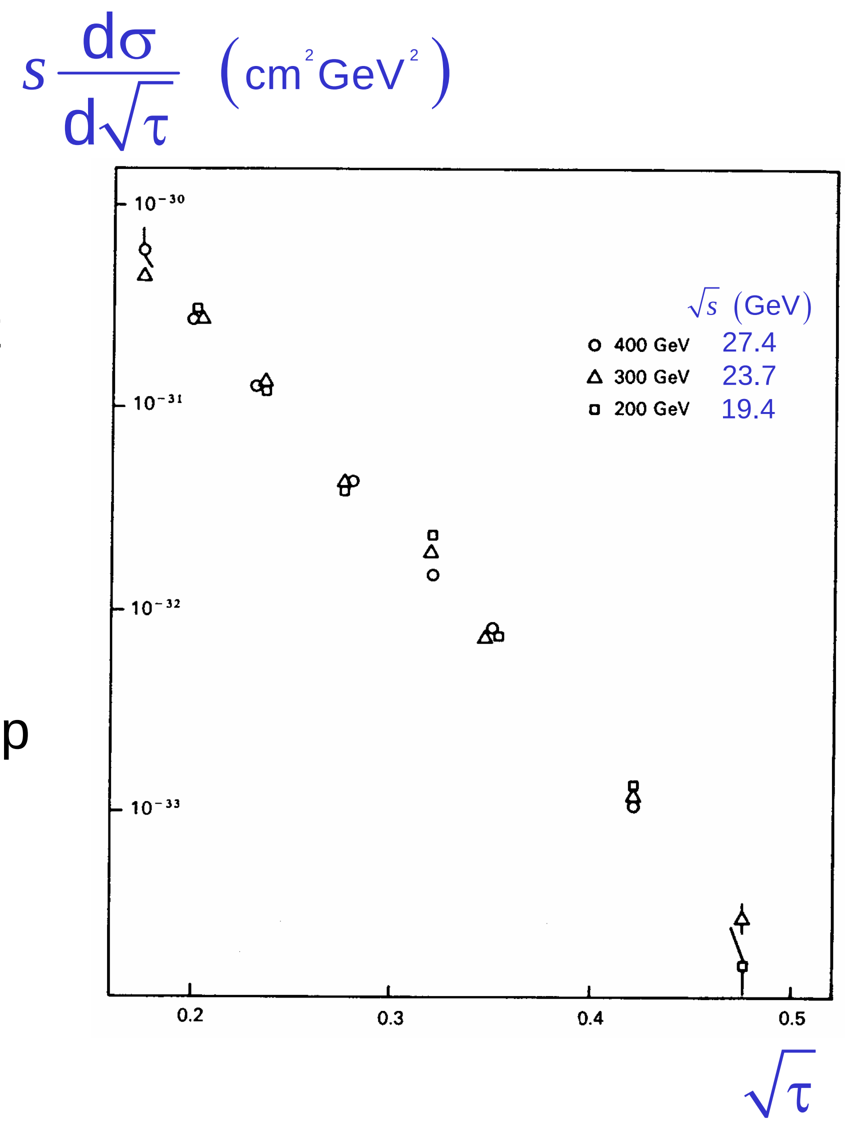
Note that, with $\hat{s} = m^2$

$$\hat{s}^2 \frac{d\sigma}{d\hat{s}} = \frac{m^3}{2}\frac{d\sigma}{dm} = \frac{s}{2\tau\sqrt{\tau}}\frac{d\sigma}{d\sqrt{\tau}}$$

Muon pair production in fixed target proton-proton collision (BFS Collaboration, Fermilab, 1978).

$$\sqrt{s} = \sqrt{2M\left(E+M\right)}$$

Note that a measurement of F(τ) along with the quark p.d.f. from deep inelastic scattering, allows the extraction of the antiquark p.d.f., which is in agreement with the antiquark p.d.f. obtained from neutrino and antineutrino cross section differences.



Perkins, p289

The Drell-Yan Process

Since the center of mass cross section of

$$q\bar{q} \to \mu^+\mu^-$$

is given by

$$\frac{d\hat{\sigma}}{d\Omega} = \frac{\alpha^2}{4\hat{s}} e_q^2 \left(1 + \cos^2\theta\right)$$

we expect the same angular distribution to hold in the Drell-Yan process (in the dimuon rest frame).