

# Tau to (K K pi pi) decay analysis – Reference material

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Version 1 - January 2026

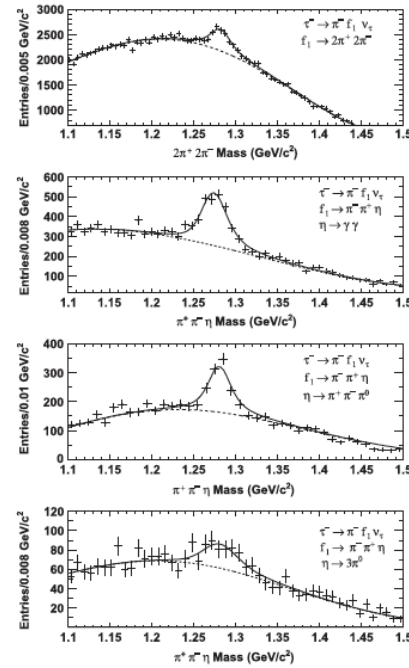
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- 6 Likelihoods for minimizing qqbar and non-signal tau decays
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- 11 Pythia 4-vector fit studies

# 1 Motivation and review of related work

TABLE VIII. Summary of branching fractions excluding contributions from  $K_S^0 \rightarrow \pi^+ \pi^-$ .

Decay mode	Branching fraction
<b>Resonant decays</b>	
$\tau^- \rightarrow 2\pi^- \pi^+ \eta \nu_\tau$ (including $f_1$ )	$(2.25 \pm 0.07 \pm 0.12) \times 10^{-4}$
$\tau^- \rightarrow 2\pi^- \pi^+ \eta \nu_\tau$ (excluding $f_1$ )	$(0.99 \pm 0.09 \pm 0.13) \times 10^{-4}$
$\tau^- \rightarrow \pi^- 2\pi^0 \eta \nu_\tau$ (including $f_1$ )	$(2.01 \pm 0.34 \pm 0.22) \times 10^{-4}$
$\tau^- \rightarrow \pi^- f_1 \nu_\tau$ via $f_1 \rightarrow 2\pi^+ 2\pi^-$	$(5.20 \pm 0.31 \pm 0.37) \times 10^{-5}$
$\tau^- \rightarrow \pi^- f_1 \nu_\tau$ via $f_1 \rightarrow \pi^+ \pi^- \eta$	$(1.26 \pm 0.06 \pm 0.06) \times 10^{-4}$
$\mathcal{B}(f_1 \rightarrow 2\pi^+ 2\pi^-) / \mathcal{B}(f_1 \rightarrow \pi\pi\eta)$	$0.28 \pm 0.02 \pm 0.02$
$\tau^- \rightarrow 2\pi^- \pi^+ \omega \nu_\tau$	$(8.4 \pm 0.4 \pm 0.6) \times 10^{-5}$
$\tau^- \rightarrow \pi^- 2\pi^0 \omega \nu_\tau$	$(7.3 \pm 1.2 \pm 1.2) \times 10^{-5}$
<b>Nonresonant decays</b>	
$\tau^- \rightarrow 3\pi^- 2\pi^+ \nu_\tau$ (excluding $\omega, f_1$ )	$(7.68 \pm 0.04 \pm 0.40) \times 10^{-4}$
$\tau^- \rightarrow 2\pi^- \pi^+ 3\pi^0 \nu_\tau$ (excluding $\eta, \omega, f_1$ )	$(1.0 \pm 0.8 \pm 3.0) \times 10^{-5}$
$\tau^- \rightarrow 2\pi^- \pi^+ 3\pi^0 \nu_\tau$ (excluding $\eta, f_1$ )	$(16.9 \pm 0.8 \pm 4.3) \times 10^{-5}$
$\tau^- \rightarrow 3\pi^- 2\pi^+ \pi^0 \nu_\tau$ (excluding $\eta, \omega, f_1$ )	$(3.6 \pm 0.3 \pm 0.9) \times 10^{-5}$
$\tau^- \rightarrow 3\pi^- 2\pi^+ \pi^0 \nu_\tau$ (excluding $\eta, f_1$ )	$(1.11 \pm 0.04 \pm 0.09) \times 10^{-4}$
<b>Inclusive decays (including <math>\eta, \omega, f_1</math>)</b>	
$\tau^- \rightarrow 2\pi^- \pi^+ 3\pi^0 \nu_\tau$	$(2.07 \pm 0.18 \pm 0.37) \times 10^{-4}$
$\tau^- \rightarrow 3\pi^- 2\pi^+ \nu_\tau$ (excluding $\omega$ )	$(8.33 \pm 0.04 \pm 0.43) \times 10^{-4}$
$\tau^- \rightarrow 3\pi^- 2\pi^+ \pi^0 \nu_\tau$	$(1.65 \pm 0.05 \pm 0.09) \times 10^{-4}$
<b><math>\eta'</math> (958) decays (90% upper level confidence limit)</b>	
$\tau^- \rightarrow \pi^- \pi^0 \eta'(958) \nu_\tau$	$< 1.2 \times 10^{-5}$
$\tau^- \rightarrow K^- \eta'(958) \nu_\tau$	$< 2.4 \times 10^{-6}$
$\tau^- \rightarrow \pi^- \eta'(958) \nu_\tau$	$< 4.0 \times 10^{-6}$
<b>Kaonic decays (90% upper level confidence limit)</b>	
$\tau^- \rightarrow K^- 2\pi^- 2\pi^+ \nu_\tau$	$< 2.4 \times 10^{-6}$
$\tau^- \rightarrow K^+ 3\pi^- \pi^+ \nu_\tau$	$< 5.0 \times 10^{-6}$
$\tau^- \rightarrow K^- K^+ 2\pi^- \pi^+ \nu_\tau$	$< 4.5 \times 10^{-7}$
$\tau^- \rightarrow K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$	$< 1.9 \times 10^{-6}$
$\tau^- \rightarrow K^+ 3\pi^- \pi^+ \pi^0 \nu_\tau$	$< 8 \times 10^{-7}$

STUDY OF HIGH-MULTIPLICITY THREE-PRONG AND ...

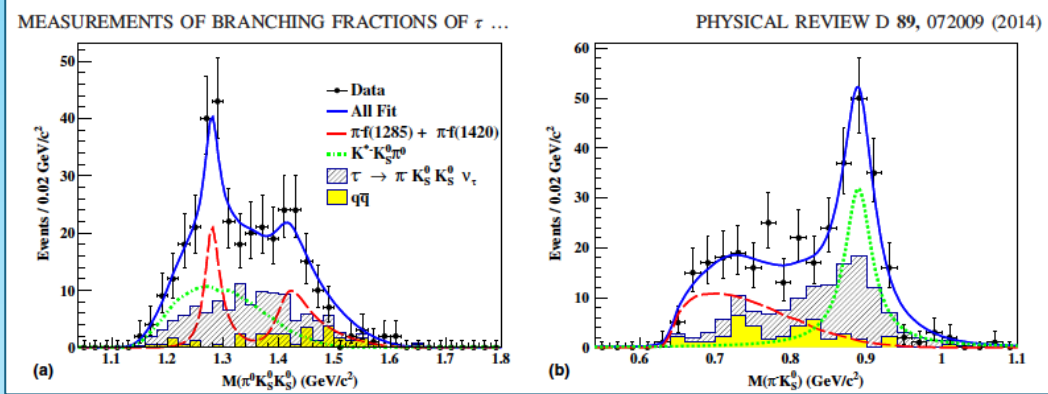
FIG. 2. The  $2\pi^+ 2\pi^-$  and  $\pi^+ \pi^- \eta$  invariant mass distributions for  $\tau^- \rightarrow 2\pi^- \pi^+ \eta \nu_\tau$  decay candidates after all selection criteria are applied. The lower three plots are for the  $\eta \rightarrow \gamma\gamma$ ,  $\eta \rightarrow \pi^+ \pi^- \pi^0$ , and  $\eta \rightarrow 3\pi^0$  decays. The solid lines represent the fit to the  $f_1(1285)$  peak and background. The dashed lines show the extrapolation of the background function under the  $f_1$  peak.

## Study of high-multiplicity three-prong and five-prong tau decays at BABAR

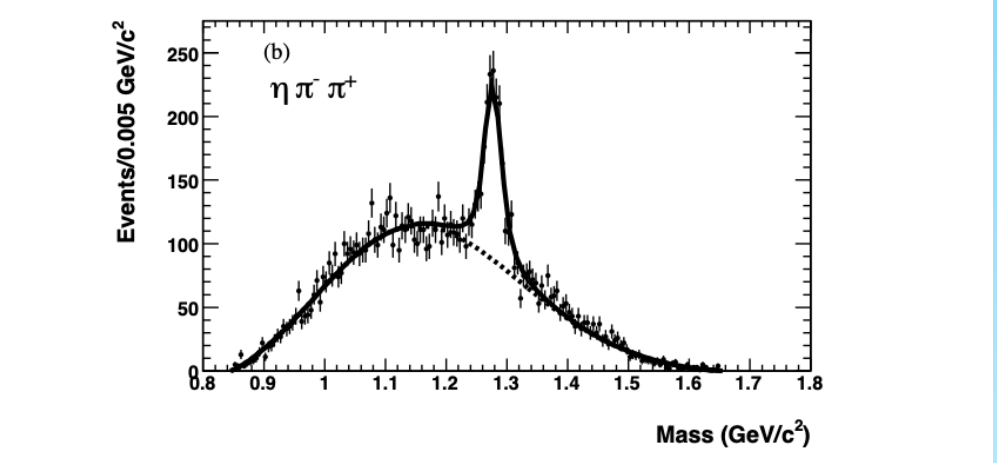
PHYSICAL REVIEW D 86, 092010 (2012)

# Section 1: Belle study of the tau decay to (pi- KS KS pi0)

Belle found a signal for the f1(1285) and f1(1420) in tau decays to (pi- KS KS pi0)



Our BaBar analysis of tau decays showed clear evidence of the f1(1285) but not the f1(1420)



**f<sub>1</sub>(1285)**  $I^G(J^{PC}) = 0^+(1^{++})$

Mass  $m = 1281.8 \pm 0.5$  MeV (S = 1.7)  
Full width  $\Gamma = 23.0 \pm 1.1$  MeV (S = 1.6)

f <sub>1</sub> (1285) DECAY MODES	Fraction ( $\Gamma_i/\Gamma$ )	Scale factor/ Confidence level	$\rho$ (MeV/c)
$4\pi$	(32.7 ± 1.8) %	S=1.2	568
$\pi^0 \pi^0 \pi^+ \pi^-$	(21.8 ± 1.2) %	S=1.2	566
$2\pi^+ 2\pi^-$	(10.9 ± 0.6) %	S=1.2	563
$\rho^0 \pi^+ \pi^-$	(10.9 ± 0.6) %	S=1.2	336
$\rho^0 \rho^0$	seen		†
$4\pi^0$	< 7 × 10 <sup>-4</sup>	CL=90%	568
$\eta \pi^+ \pi^-$	(35 ± 15) %		479
$\eta \pi \pi$	(52.2 ± 1.9) %	S=1.2	482
$a_0(980) \pi$ [ignoring $a_0(980) \rightarrow K \bar{K}$ ]	(38 ± 4) %		238
$\eta \pi \pi$ [excluding $a_0(980) \pi$ ]	(14 ± 4) %		482
$K \bar{K} \pi$	(9.0 ± 0.4) %	S=1.1	308
$K \bar{K}^*(892)$	not seen		†

**f<sub>1</sub>(1420)**  $I^G(J^{PC}) = 0^+(1^{++})$

See the review on "Spectroscopy of Light Meson Resonances."  
Mass  $m = 1428.4^{+1.5}_{-1.3}$  MeV (S = 1.8)  
Full width  $\Gamma = 56.7 \pm 3.3$  MeV (S = 1.3)

f <sub>1</sub> (1420) DECAY MODES	Fraction ( $\Gamma_i/\Gamma$ )	$\rho$ (MeV/c)
$K \bar{K} \pi$	seen	440
$K \bar{K}^*(892) + c.c.$	seen	167
$\eta \pi \pi$	possibly seen	574
$\phi \gamma$	seen	350

**Did we miss the f1(1420)?**

The PDG says the (eta pi pi) decay may be seen

Unfortunately, we do not yet have a large enough sample to study the (pi- KS KS pi0) decay.

**Section 1:**

**Isospin symmetry says  $(\pi^- K^0 K^0 \pi^0) = (\pi^- K^+ K^- \pi^0)$**

CLEO and ALEPH measurements (30% error) and no studies of the decay mechanism

$\Gamma(K^- K^+ \pi^- \pi^0 \nu_\tau) / \Gamma_{\text{total}}$						$\Gamma_{107} / \Gamma$
<u>VALUE (units <math>10^{-4}</math>)</u>	<u>CL%</u>	<u>EVTS</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>COMMENT</u>	
<b>0.61 ± 0.18 OUR FIT</b> (Produced by HFLAV)						
<b>0.60 ± 0.18 OUR AVERAGE</b>						
0.55 ± 0.14 ± 0.12		48	ARMS	05	CLE3	7.6 fb <sup>-1</sup> , $E_{\text{cm}}^{ee} = 10.6$ GeV
7.5 ± 2.9 ± 1.5			BARATE	98	ALEP	1991–1995 LEP runs

B2 should have a sample with a few thousand  $(\pi^- K^- K^+ \pi^0)$  decays

Never studied by BaBar or Belle with tau decays

## Section 1: Rich, complicated decay structures observed in related measurements

ISR, tau, J/Psi and D-meson decays

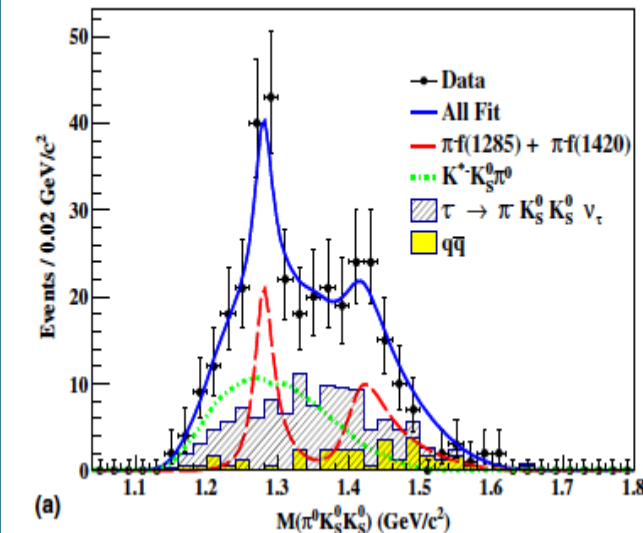
Results are very dependent on the nature of the probe or decay

### Belle tau decays with KS-mesons

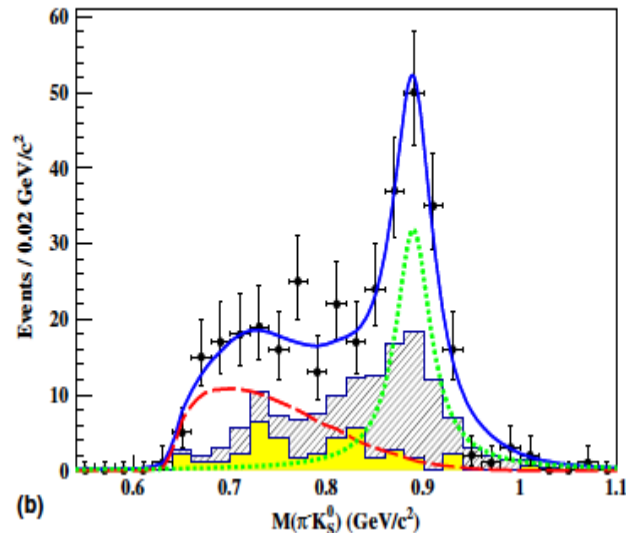
Observed ( $\pi^- f_1(1285)$ ), ( $\pi^- f_1(1420)$ ) and ( $K^{*-} K_S^0 \pi^0$ ) final states

Belle found a signal for the  $f_1(1285)$  and  $f_1(1420)$  in tau decays to ( $\pi^- K_S^0 K_S^0 \pi^0$ )

MEASUREMENTS OF BRANCHING FRACTIONS OF  $\tau \dots$



PHYSICAL REVIEW D 89, 072009 (2014)



$$\mathcal{B}(\tau^- \rightarrow f_1(1285)\pi^- \nu_\tau) \cdot \mathcal{B}(f_1(1285) \rightarrow K_S^0 K_S^0 \pi^0) = (0.68 \pm 0.13 \pm 0.07) \times 10^{-5},$$

$$\mathcal{B}(\tau^- \rightarrow f_1(1420)\pi^- \nu_\tau) \cdot \mathcal{B}(f_1(1420) \rightarrow K_S^0 K_S^0 \pi^0) = (0.24 \pm 0.05 \pm 0.06) \times 10^{-5},$$

$$\mathcal{B}(\tau^- \rightarrow K^{*-} K_S^0 \pi^0 \nu_\tau) \cdot \mathcal{B}(K^{*-} \rightarrow K_S^0 \pi^-) = (1.08 \pm 0.14 \pm 0.15) \times 10^{-5}.$$

$$m_{f_1(1285)} = 1274 \pm 3 \text{ MeV}/c^2,$$

$$\Gamma_{f_1(1285)} = 20 \pm 4 \text{ MeV}/c^2,$$

$$m_{f_1(1420)} = 1425 \pm 2 \text{ MeV}/c^2,$$

$$\Gamma_{f_1(1420)} = 42 \pm 19 \text{ MeV}/c^2.$$

$$m_{K^{*-}} = 890 \pm 3 \text{ MeV}/c^2,$$

$$\Gamma_{K^{*-}} = 48 \pm 2 \text{ MeV}/c^2.$$

# Section1: BES III studied the (KS KS pi0) decay from the J/PSI (Partial Wave Analysis PWA)

[https://doi.org/10.1007/JHEP03\(2023\)121](https://doi.org/10.1007/JHEP03(2023)121)  
200-300K decays

## Simultaneous fit to extract many parameters, of notable interest:

f1285 to the (KS KS pi0) final state via phase space and the (pi- a0(980)) final state  
f1420 via (K\* KS pi0) and (pi- a0) final state (significantly more (x10) f1420s than f1285s)

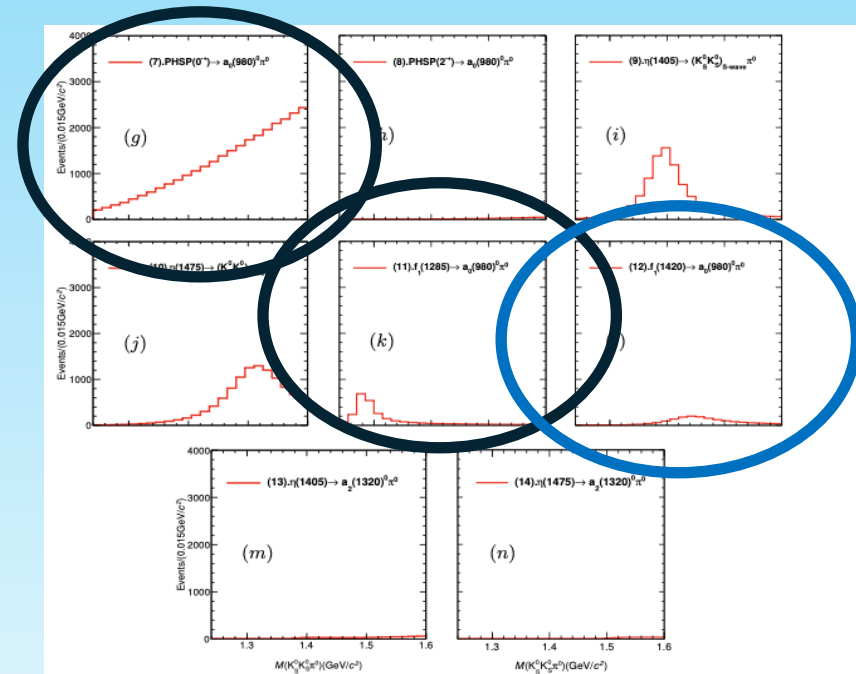
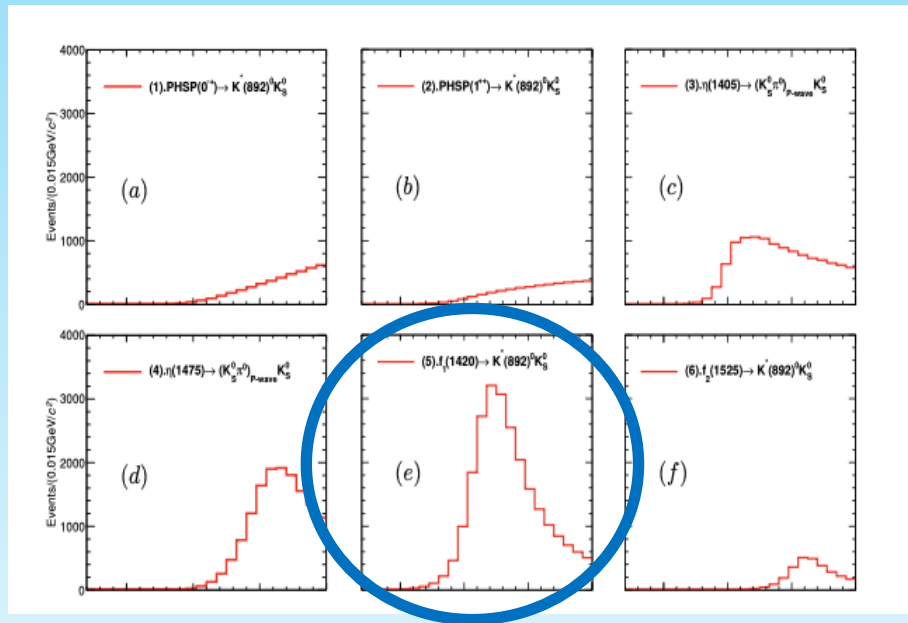


Figure 5. Invariant mass distribution of  $K_S^0 K_S^0 \pi^0$  for each of the 14 individual components corresponding to table 3 included in the MD PWA nominal solution.

# Section 1: BES III sees a much broader width for the f1420

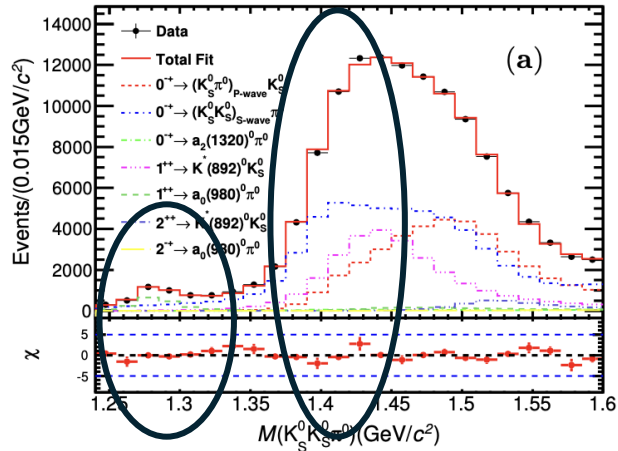
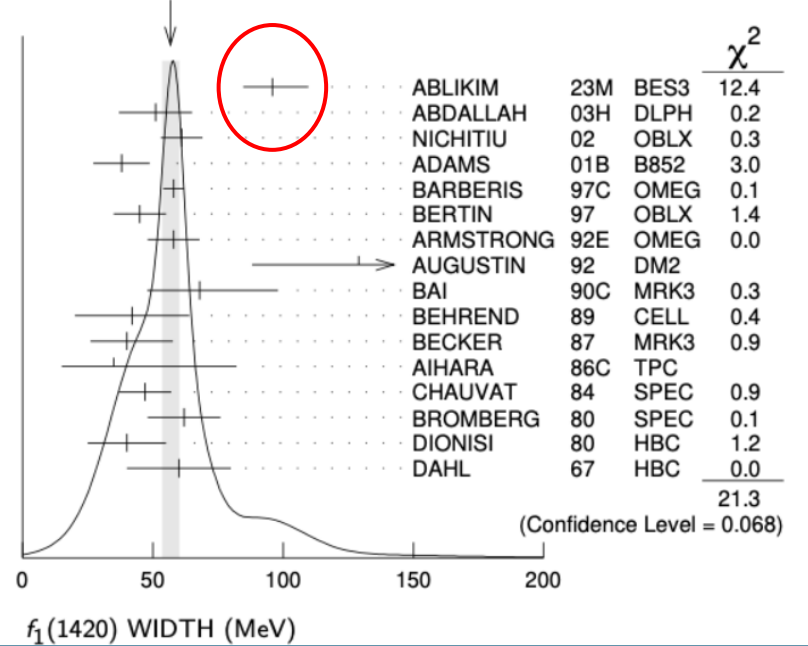
96 MeV vs 57 MeV quoted by the PDG

Belle quote f1420 decay width of (42 +/- 29) MeV

Resonance	$M(\text{MeV}/c^2)$	$\Gamma(\text{MeV})$	Decay Mode	B.F.
$\eta(1405)$	$1391.7 \pm 0.7^{+11.3}_{-0.3}$	$60.8 \pm 1.2^{+5.5}_{-12.0}$	$J/\psi \rightarrow \gamma \eta(1405) \rightarrow \gamma K_S^0(K_S^0 \pi^0)_{P\text{-wave}} \rightarrow \gamma K_S^0 K_S^0 \pi^0$	$(5.84 \pm 0.12^{+2.03}_{-3.36}) \times 10^{-5}$
			$J/\psi \rightarrow \gamma \eta(1405) \rightarrow \gamma (K_S^0 K_S^0)_{S\text{-wave}} \pi^0 \rightarrow \gamma K_S^0 K_S^0 \pi^0$	$(2.88 \pm 0.04^{+1.64}_{-0.38}) \times 10^{-5}$
$\eta(1475)$	$1507.6 \pm 1.6^{+15.5}_{-32.2}$	$115.8 \pm 2.4^{+14.8}_{-10.9}$	$J/\psi \rightarrow \gamma \eta(1475) \rightarrow \gamma K_S^0(K_S^0 \pi^0)_{P\text{-wave}} \rightarrow \gamma K_S^0 K_S^0 \pi^0$	$(6.58 \pm 0.12^{+3.98}_{-2.82}) \times 10^{-5}$
			$J/\psi \rightarrow \gamma \eta(1475) \rightarrow \gamma (K_S^0 K_S^0)_{S\text{-wave}} \pi^0 \rightarrow \gamma K_S^0 K_S^0 \pi^0$	$(3.99 \pm 0.09^{+0.41}_{-0.66}) \times 10^{-5}$
$f_1(1285)$	$1286.2 \pm 0.6^{+1.2}_{-1.5}$	$28.2 \pm 1.1^{+3.5}_{-2.9}$	$J/\psi \rightarrow \gamma f_1(1285) \rightarrow \gamma a_0(980)^0 \pi^0 \rightarrow \gamma K_S^0 K_S^0 \pi^0$	$(8.55 \pm 0.41^{+3.42}_{-1.04}) \times 10^{-6}$
$f_1(1420)$	$1433.5 \pm 1.1^{+27.9}_{-0.7}$	$95.9 \pm 2.3^{+13.6}_{-10.9}$	$J/\psi \rightarrow \gamma f_1(1420) \rightarrow \gamma K^*(892)^0 K_S^0 \rightarrow \gamma K_S^0 K_S^0 \pi^0$	$(7.25 \pm 0.12^{+0.75}_{-1.25}) \times 10^{-5}$
			$J/\psi \rightarrow \gamma f_1(1420) \rightarrow \gamma a_0(980)^0 \pi^0 \rightarrow \gamma K_S^0 K_S^0 \pi^0$	$(4.62 \pm 0.36^{+2.36}_{-1.94}) \times 10^{-6}$
$f_2(1525)$	$1515.4 \pm 2.5^{+3.2}_{-7.0}$	$64.0 \pm 4.3^{+2.0}_{-3.1}$	$J/\psi \rightarrow \gamma f_2(1525) \rightarrow \gamma K^*(892)^0 K_S^0 \rightarrow \gamma K_S^0 K_S^0 \pi^0$	$(9.47 \pm 0.43^{+1.51}_{-0.66}) \times 10^{-6}$

Table 4. Masses ( $M$ ), widths ( $\Gamma$ ), branching fractions (B.F.) and significances (Sig.) of predominant components in the MD PWA solution. The first uncertainties are statistical and the second ones are systematic.

WEIGHTED AVERAGE  
56.7±3.3 (Error scaled by 1.3)



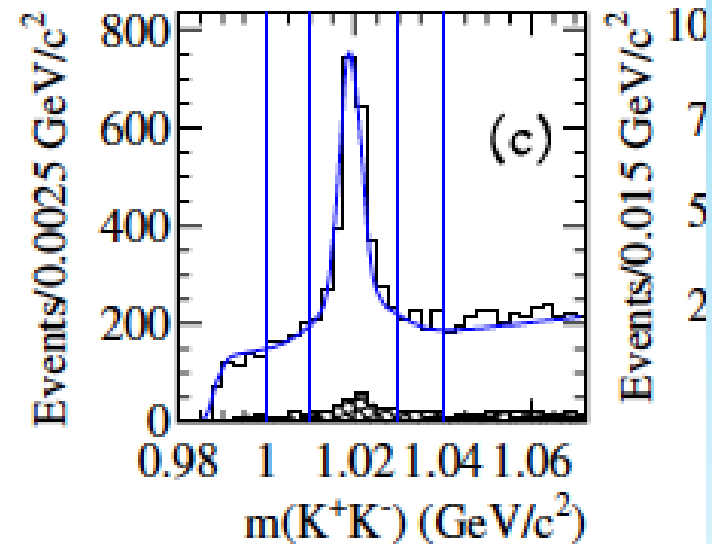
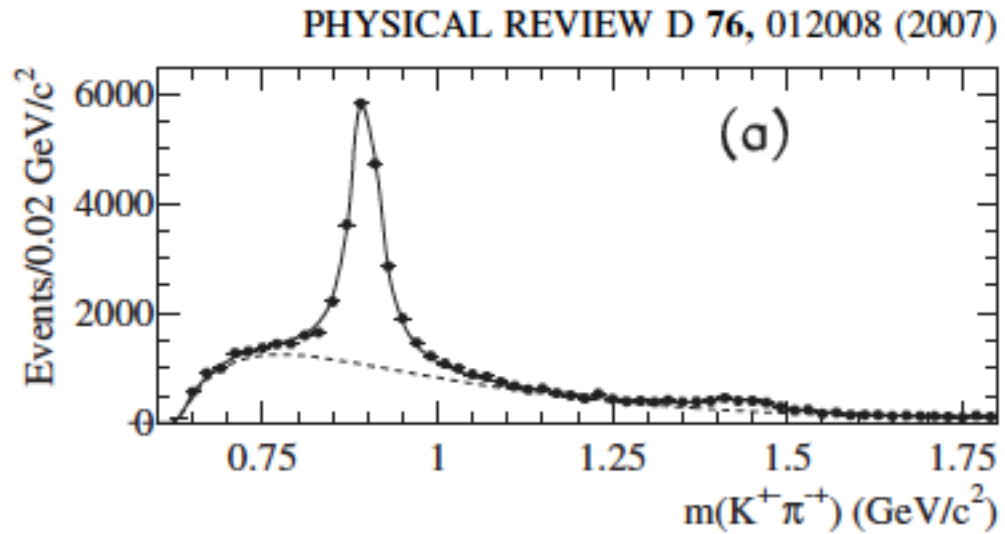
f1285 and f1420

## Section 1: BaBar ISR measurement

Able to study the channel to high-mass but the  $f_1$  mesons are not an allowed final state  
Observe  $K^*$  and  $\phi$ -mesons but their reach is to much higher masses

PHYSICAL REVIEW D 76, 012008 (2007)

$e^+e^- \rightarrow K^+K^-\pi^+\pi^-$ ,  $K^+K^-\pi^0\pi^0$  and  $K^+K^-K^+K^-$  cross sections measured  
with initial-state radiation



## Section1: Possible tau final states and other measurements

<b>Decay modes:</b>		
$\pi^- K^- K^+ \pi^0$	phase space (PS)	
$\pi^- f_1(1285)$	$f_1(1285)$ to $KK\pi^0$ (via PS or $a_0$ )	
$\pi^- f_1(1420)$	$f_1(1420)$ to $KK\pi^0$ (via PS or $a_0$ ) and $K^*-K+\pi^0$	
$\pi^- \phi \pi^0$	PS (or via some resonance)	
$K^- K^* \pi^0$	$K^*$ to $(\pi^- K^+)$	
$\pi^- K^*- K^+$	$K^*$ to $(K^- \pi^0)$	via ( $f_1(1420)$ or PS)
$\pi^- K^- K^{*+}$	$K^*$ to $(K^+ \pi^0)$	via ( $f_1(1420)$ or PS)
<b>f1 properties</b>		
$f_1(1285)$ mass (width already measured precisely)		
$f_1(1420)$ mass and BFs		
Mixing amplitude of f1s	(see discussion on lineshapes)	

## 15 possible measurement plus resonance parameters and phase space contributions

## **2 Lineshapes (low and high mass thresholds, and interference)**

## Section 2: Digression into line shapes of the resonances

Note that there is no visible difference between a Breit-Wigner and Relativistic BW in this work

### Assume tau decay into a final state ( R X nut)

where R is a resonance that decays to a final state of particles R(N) and X are the other particles in the tau decay

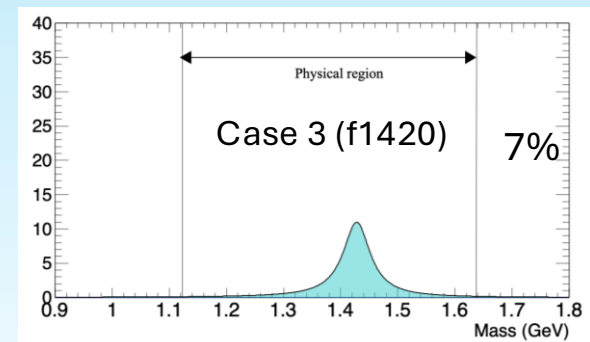
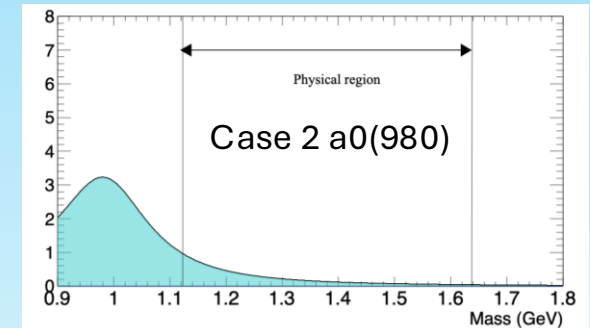
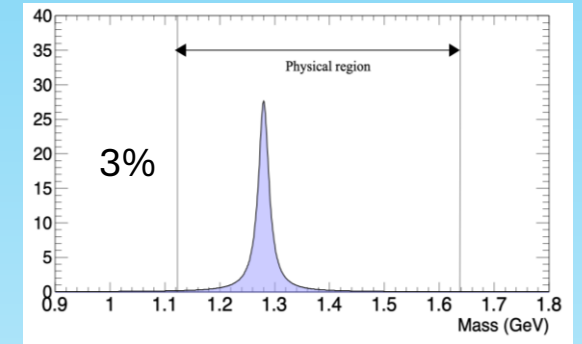
Case 1 – the mass of the R resonance  $M_R$  is above the sum of the masses of its decay particles  $\text{Sum}(M_{Ri})$  but the tails of the lineshape reach into the unphysical region (well known case with analytical forms for 2-body final states)

Case 2 – the mass of the R resonance  $M_R$  is below the sum of the masses of its decay particles  $\text{Sum}(M_{Ri})$  where most of the lineshape lies in the unphysical region

Case 3 – the mass of the R resonance  $M_R$  is below the tau mass less the X mass ( $M_{\text{tau}} - M_X$ ) where some of the lineshape lies in the unphysical region

Case 4 – the interference between the f1285 and f1420

Case 1 (f1285)

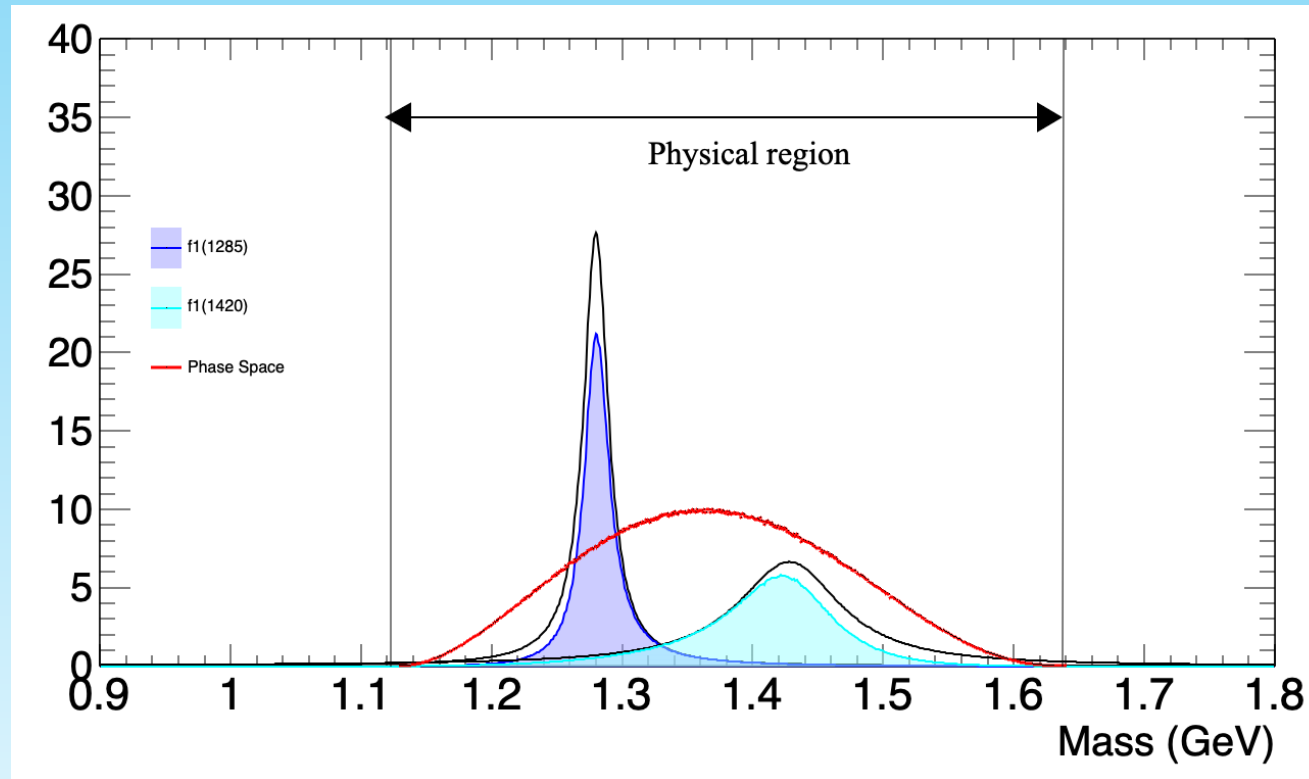


## Section 2: Weight the f1285 and f1420 lineshapes by the phase space distribution

Phase space distribution generated using the ROOT function (TPhaseSpace)

Plot show the f1285 and f1420 lineshapes corrected by phase space value

The uncorrected f1285 and f1420 lineshapes are normalized to unit area



Note that I am using a width of 58 MeV for the f1420 whereas BES-III measured a width of 96 MeV

## Section 2: Compare the f1285 PS-corrected distribution with the analytical form

Screen shot from PHYSICAL REVIEW D 107, 112011 (2023)

### A. Relativistic Breit-Wigner model

The relativistic Breit-Wigner lineshape function is defined as [1]

$$f_{\text{BW}}(M) = \frac{m_{\text{BW}} M \Gamma(M)}{(M^2 - m_{\text{BW}}^2)^2 + m_{\text{BW}}^2 \Gamma(M)^2}, \quad (3)$$

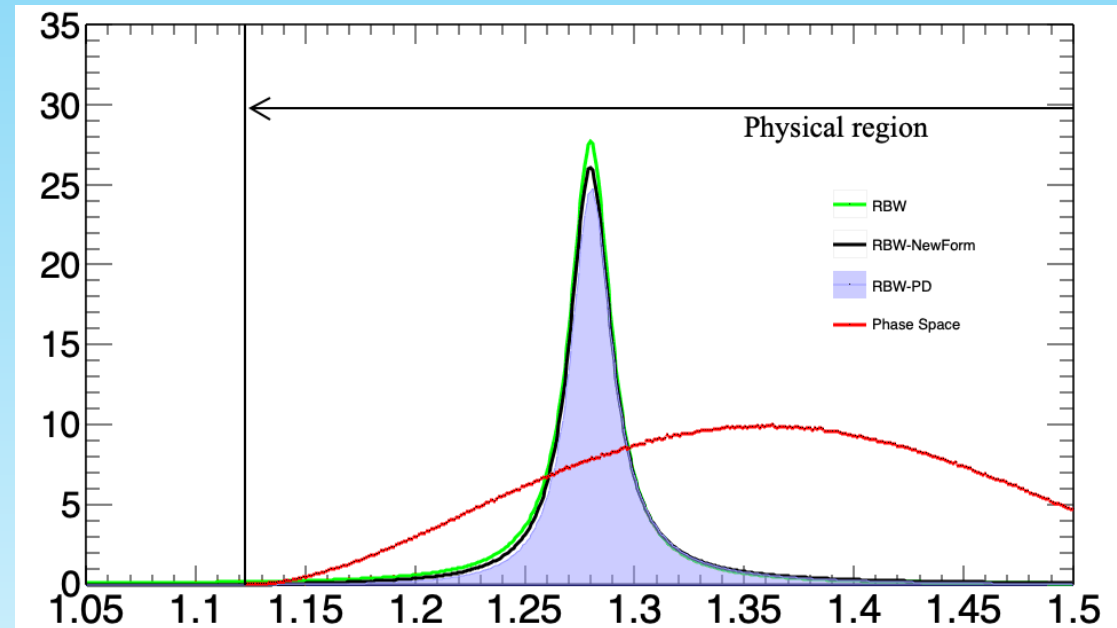
where  $M$  is the observed invariant mass, and  $m_{\text{BW}}$  is the mass of the resonance. The mass-dependent width  $\Gamma(M)$  is defined as

$$\Gamma(M) = \Gamma_{\text{BW}} \frac{m_{\text{BW}}}{M} \left( \frac{p(M)}{p(m_{\text{BW}})} \right)^{2L+1}, \quad (4)$$

where  $\Gamma_{\text{BW}}$  and  $L$  are the width of the resonance and the orbital angular momentum, respectively. Taking account of the closeness to the threshold, the decay is assumed to be pure S-wave ( $L = 0$ ) with no D-wave ( $L = 2$ ) admixture. The momentum of one of the daughters in the rest frame of  $X(3872)$ ,  $p(M)$ , can be calculated as

$$p(M) = \frac{1}{2M} \sqrt{(M^2 - (m_{D^0} + m_{D^{*0}})^2) \times \sqrt{(M^2 - (m_{D^0} - m_{D^{*0}})^2)}}. \quad (5)$$

Using  $(M_k + M_k + M_{\pi})$  and 0 for difference term

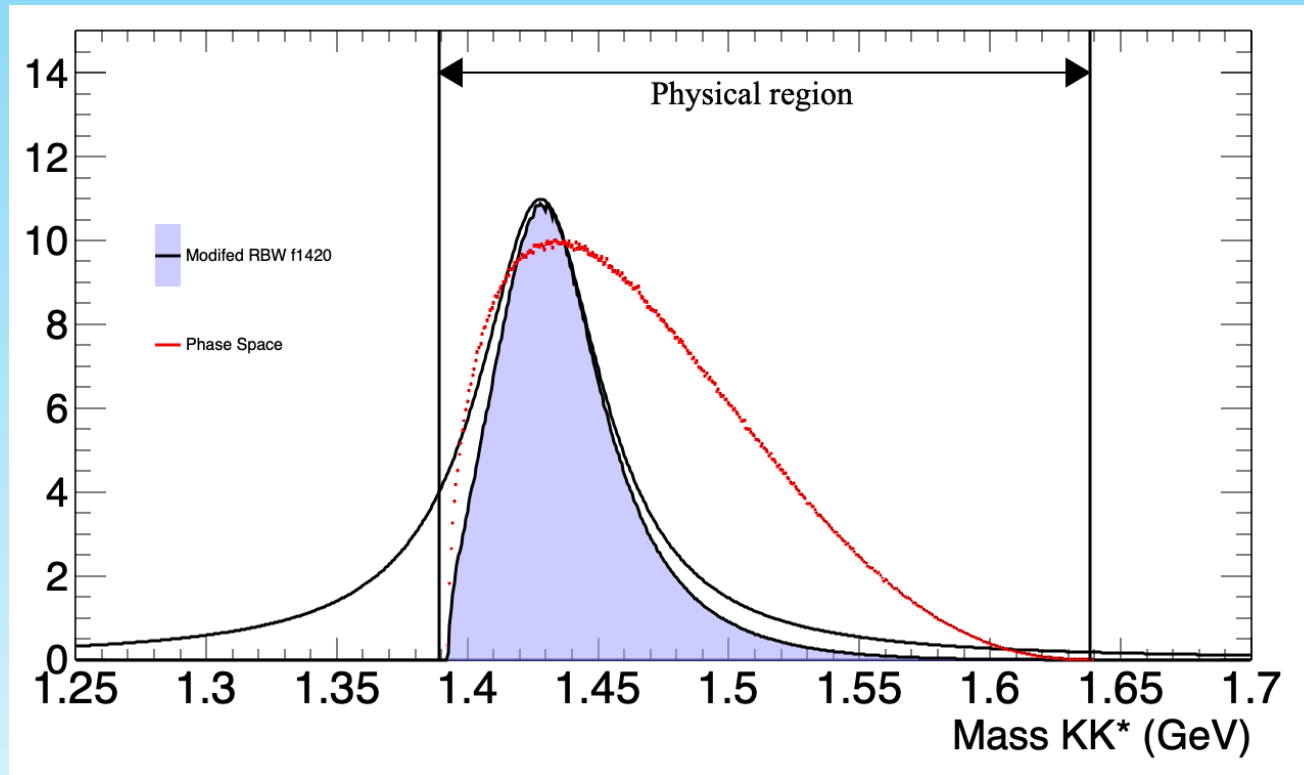


BW-PS and the BW-analytical form are similar

Need to search for analytical lineshape for 3-body decays

## Section 2: Both Belle and BES-III say that the f1420 decays to a K\*K final state

Plot shows RBW (black line) and RBW-PS (shaded histogram), and phase space (red)



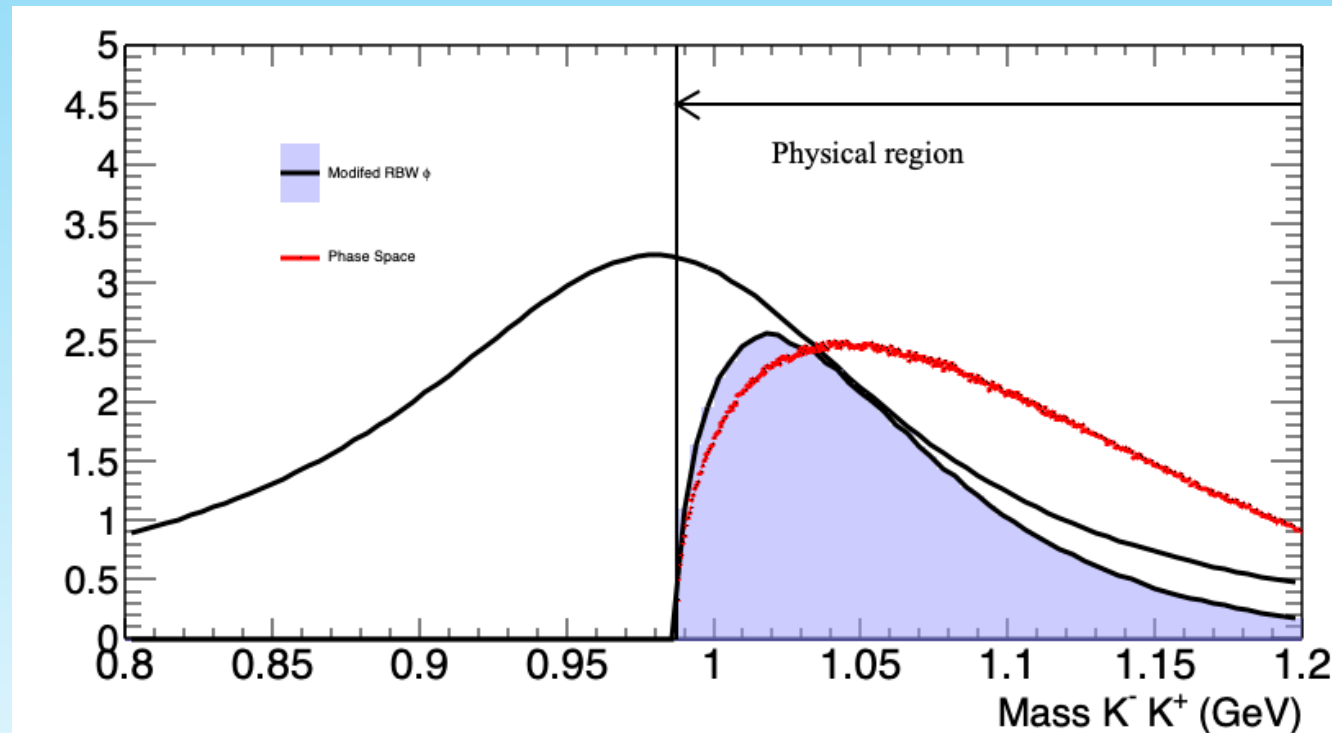
f1420 lineshape is impacted by the low and high mass kinematic limits

## Section 2: Weight the $a_0(980)$ lineshapes by the phase space distribution

PDG:  $a_0 M = (980 \pm 20) \text{ MeV}$  Width = 50-100 MeV

Using  $M=980 \text{ MeV}$  and  $W = 75 \text{ MeV}$

Have not found an analytical form when the peak of the resonance is below the mass threshold



The  $a_0$  BW looks similar to the phase distribution if the width is higher

## Section 2: Interference between the f1285 and f1420 resonances

Similar to what is observed for the rho resonances

On the ambiguity of determination of interfering resonances parameters

<https://arxiv.org/pdf/1505.01509> V.M. Malyshev

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Abstract

The general form of solutions for parameters of interfering Breit-Wigner resonances is found. The number of solutions is determined by the properties of roots of corresponding characteristic equation and does not exceed  $2^{N-1}$ , where  $N$  is the number of resonances. For resonances of more complicated form, provided that their amplitudes satisfy certain conditions, for any  $N \geq 2$  multiple solutions also exist.

### B. Relativistic form of resonance amplitude

Let us consider more accurate dependence of the resonance amplitude on energy:

$$\sigma(E) = \frac{m_1^4 \sqrt{E^2 - 4\mu^2}^3}{E^4 \sqrt{m_1^2 - 4\mu^2}^3} \cdot \left| \frac{2m_1 a}{E^2 - m_1^2 + i\Gamma_1 m_1} + \frac{2m_2 b e^{i\psi}}{E^2 - m_2^2 + i\Gamma_2 m_2} \right|^2$$

Note that the amplitudes that interfere are only for (K- K+ pi0) final state

$$\Gamma_{KK\pi} + \Gamma_{K^*K}$$
$$\Gamma_{KK\pi} = \left| A_{KK\pi}^{1285} + A_{KK\pi}^{1420} \right|^2$$

Adds a phase factor between the two amplitudes for the (KKz) decays

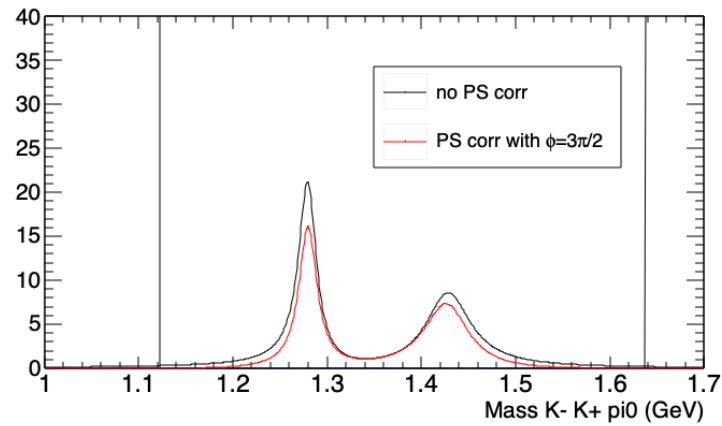
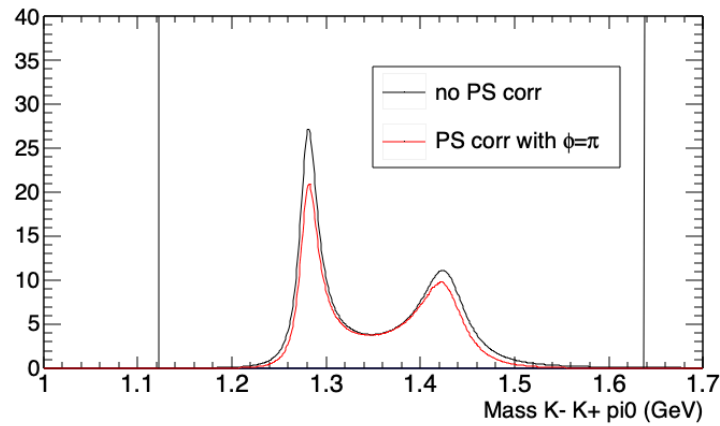
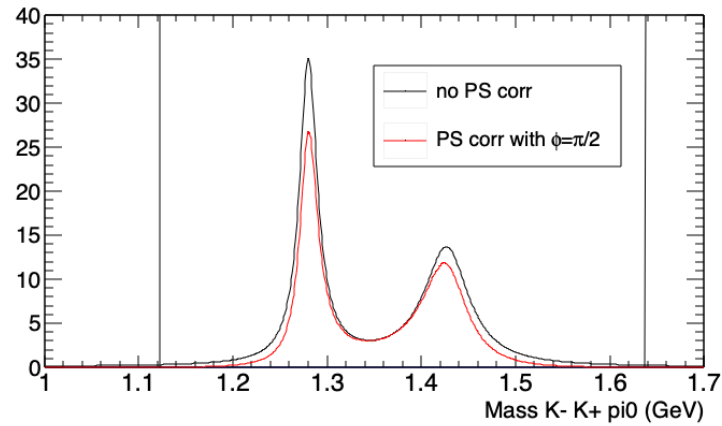
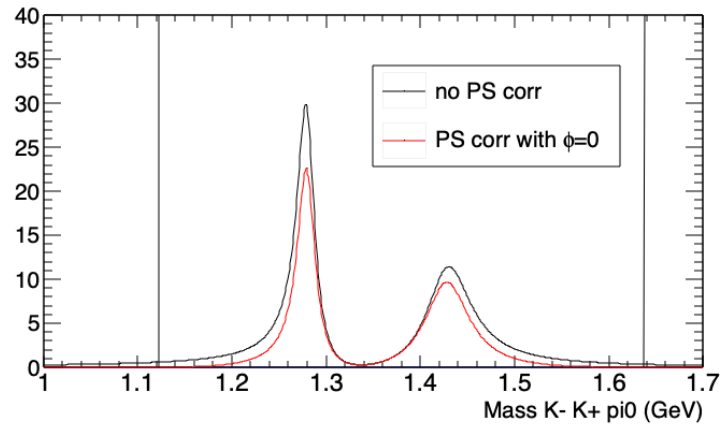
Belle did not include a phase angle and assumed the angle was ( $\pi/2$ ).

## Section 2: Interference between the f1285 and f1420 resonances

RBW (black) and RBW-PS (red) lineshapes using the PDG masses and widths for 4 phase angles

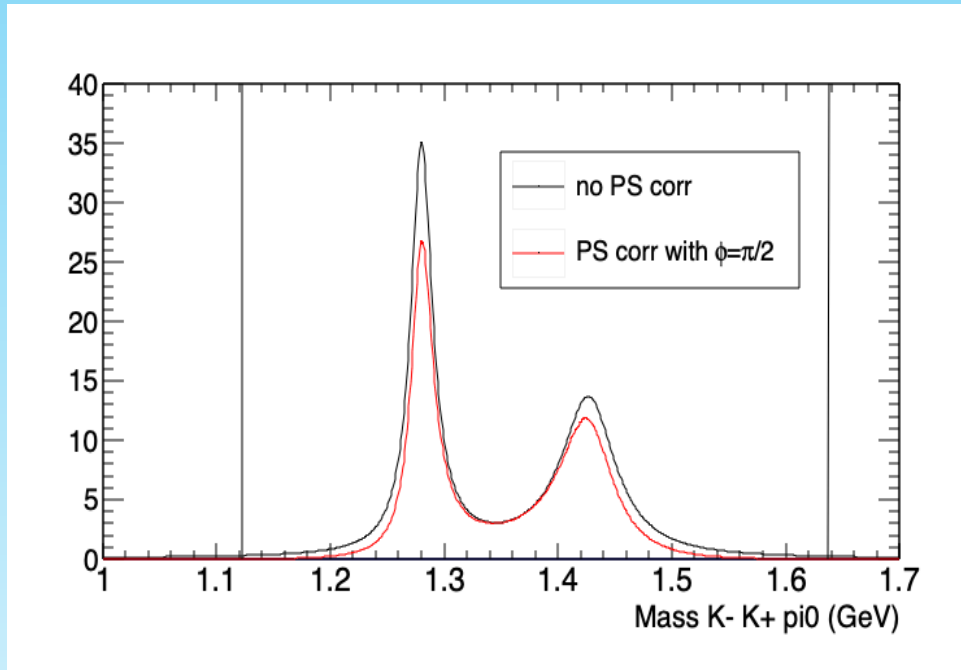
Using f1420 width of 58 MeV (PDG)

BES-III measured f1420 width of 96 MeV

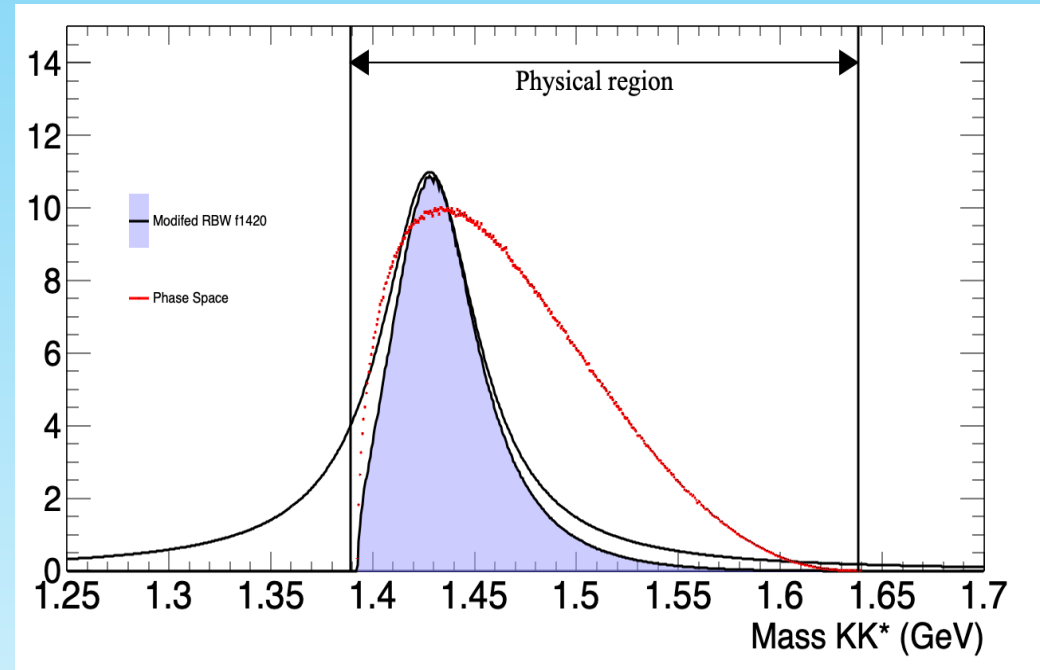


## Section 2: Hence the f1 mesons lineshape is

$$\text{Lineshape} \sim |\text{Amplitude}_{kkz}(f1258) + \text{Amplitude}_{kkz}(f1420)|^2 + \text{BW-PS}_{KK}(f1420)$$



+



BES-III also shows a phase space component for both the (KKpi0) and (K\*K) final state which would add two more terms to the lineshape

(Assume the PS contributions are negligible until necessary)

BES-III only observed the (K-K+pi0) final state via the (a0 pi0) mode

(The lineshape shown above is for the PS mode; not via the a0)

## Unraveling the tau decays to the ( $\pi^- K^- K^+ \pi^0$ ) final state

### Nine mass distributions:

(tau – decay to)

$\pi^- K^- K^+ \pi^0$

$K^- K^+$

$K^- K^+ \pi^0$

$\pi^- K^- K^+$

$\pi^- K^+$

$K^- \pi^0$

$K^+ \pi^0$

$\pi^- \pi^0$

$\pi^- K^+ \pi^0$

### Background from $q\bar{q}$ and other tau decays validated with control samples

(DONE - later in these slides)

### Resonant (peaking) backgrounds validated with control samples

(DONE - later in these slides)

### Calibrate resonance shapes with control samples

(Ongoing finished)

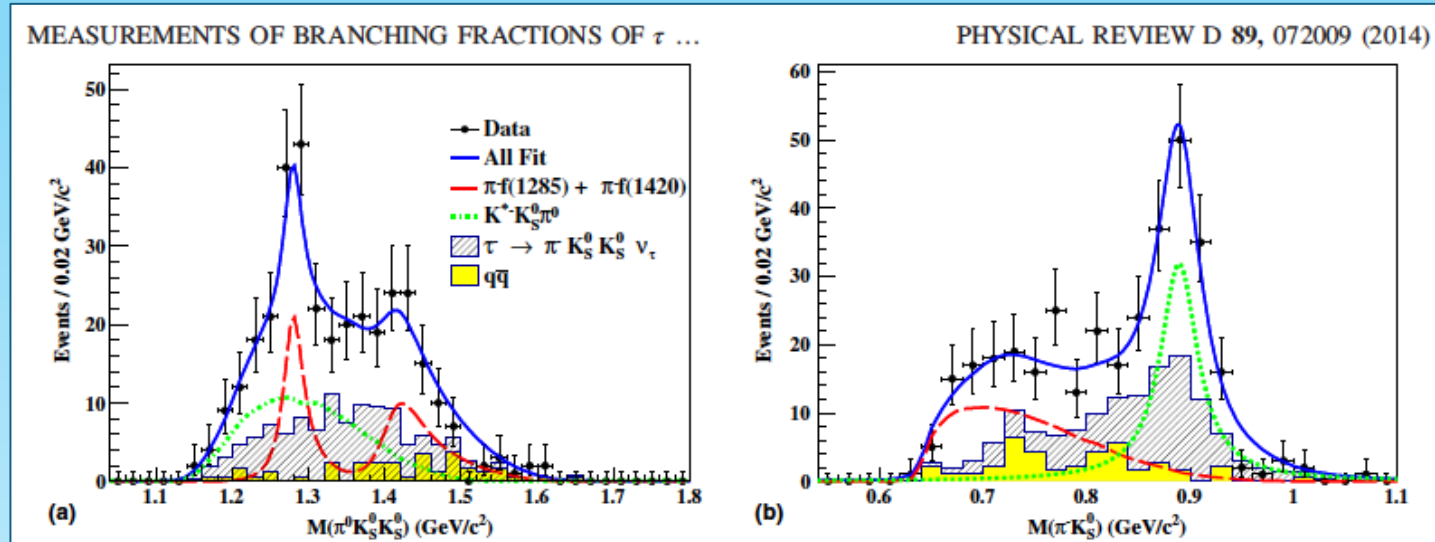
### Start assuming all decays are from a resonance

Ignore phase space contributions – e.g. tau to ( $\pi^- K^- K^+ \pi^0$  nut)

### Resonances: $\phi$ , $a_0(980)$ , $f_{1258}$ , $f_{1420}$ , $K^*(\pi^- K^+)$ , $K^*(K^+ \pi^-)$

Not clear whether data will yield results for all resonances

## Follow the Belle method to fit the mass distributions



### Belle did a simultaneous fit for the f1285, f1420 and $K^*K$

(including contributions of resonances to other mass distributions)

Note for Belle,  $K^* = (\pi^- K^+)$

**Plan is to include:**  $\phi$ ,  $a_0(980)$  and the  $K^*$  ( $\pi^- K^+$ ), ( $K^- \pi^0$ ), ( $K^+ \pi^0$ )  
Will generate phase space distributions using Pythia (like Belle)

# 3 Data and MC Samples

## Section 3: Data and MC samples I

### Data (Int Lum = 568 fb-1 according to gbasf2)

Peak	/belle/collection/Data/S16_proc16_4S_skim_79000100_v2
Scan	/belle/collection/Data/S16_proc16_5S_scan_skim_79000100_v2
Offpeak	/belle/collection/Data/S16_proc16_4S_offres_skim_79000100_v2
Peak prompt	/belle/collection/Data/S16_prompt_4S_skim_79000100_v1
Offpeak prompt	/belle/collection/Data/S16_prompt_4S_offres_skim_79000100_v1

### Monte Carlo

udsc	/belle/collection/MC/S16_MC16rd-proc16_4S_udsc_skim_79000100_v2
bb	/belle/collection/MC/S16_MC16rd-proc16_4S_BB_skim_79000100_v2

### Monte Carlo (no/few events pass preselection)

llXX	/belle/collection/MC/S16_MC16rd-proc16_4S_llXX_skim_79000100_v2
ee	/belle/collection/MC/S16_MC16rd-proc16_4S_ee_skim_79000100_v2
mumu	/belle/collection/MC/S16_MC16rd-proc16_4S_mumu_skim_79000100_v2

### Special note:

Although no ee MC events pass preselection, there is evidence for ee events in the e-tag data  
Cuts applied to remove  $P_{tag}(CMS) > 4$  GeV and in endcap regions when tag hemisphere has an electron track

## Section 3:

## Data and MC samples II

### Tau pair Monte Carlo

Peak	/belle/collection/MC/S16_MC16rd-proc16_4S_taupair_skim_79000100_v1
Offpeak	/belle/collection/MC/S16_MC16rd-proc16_4S_offres_taupair_skim_79000100_v1
Scan	/belle/collection/MC/S16_MC16rd-proc16_5S_scan_taupair_skim_79000100_v1

### Belle II Tauola decay modes

(mode number from Table 1 BELLE2-NOTE-PH-2020-055)

pi- K- K+ pi0	(mode 23)	Tauola (0.00061)	Phase space decay (CLEO results 30% error)
pi- K- K+	(mode 103)	Tauola (0.001435)	Well understood (K- K*)
K- phi K+K-	(mode 230)	Tauola (0.000023)	Well understood
pi- f1(1285):	eta pi+ pi- (with eta to gg, pi-pi+pi0) and 2pi- 2pi+		Well understood

### Not included:

pi- phi	PDG BR (3.4 10 <sup>-5</sup> )
pi- phi pi0	(mode 118)
pi- f1(1285)	PDG BR (f1 to pi- K K) = 11%
pi- f1(1420)	PDG (f1 to pi- K Kb) seen and (f1 to K K*b) seen

### Focused on a data-driven analysis (may not need signal MC samples)

No previous studies of (pi- K- K+ pi0) decay mechanism (only BF with 30% error)

Not yet sure what measurements will result (considering relative BF ratios)

**Sample of (pi- K- K+ pi0) is okay for topology variables but is not useful for studying the decay mechanism.**

## **4 Grid skim selection and lists**

## Section 4: Processing of the TauGeneric skim samples

### **Event requirements:**

Standard track, photon\*, pi0 lists  
2-6 tracks

### **Tau topology requirements:**

Event split into hemispheres based on thrust (total Q in event = 0)  
One hemisphere has only 1 track (tag hem)  
Signal hemisphere has at least two tracks (with opp Q) with  $KPID > 0.01$  and  $N(\pi^0) \geq 1$

### **Photon requirements:**

Following CPV analysis but refined cut to remove fake photons from hadronic interactions  
(they used a hard energy cut; smoothed it out but the impact in this work is negligible as  $E_{\pi^0}$  is high)

$E(\text{photons}) > 0.08 \text{ GeV}$   
(did not need to be so low for this work; developed for the  $(\pi^- K_S K_S \pi^0)$  selection)

Also added (recommendation of CHearty)  
`gammaCuts += ' and abs(clusterTiming/clusterErrorTiming) < 2'`

### **Last processing in September 2025**

Based on advice, little has changed; the grid (basf2) jobs are essentially skim jobs (into a private format)

## Section 4:

## Grid (my-skim) requirements

```
# track lists
# Sep 10 2025 - CDC hits > 20 (from 0)
trackCuts = 'p<10'
trackCuts += ' and pt>0.1'
trackCuts += ' and nCDCHits>20'
trackCuts += ' and abs(d0)<2'
trackCuts += ' and abs(z0)<4'
ma.fillParticleList(decayString='pi-:allhem', cut=trackCuts, path=my_path)
ma.fillParticleList(decayString='pi-:hem0', cut=trackCuts, path=my_path)
ma.fillParticleList(decayString='pi-:hem1', cut=trackCuts, path=my_path)

# photon lists (updated Jan 22 2024)
# Feb 11 2025: change E from 0.02 back to 0.10 (CPV selection criteria)
# Jun 4 2025: change to 0.05 from 0.10 (and remove cleanup)
# Jun 19 2025: energy cut to 0.08 (from 0.05) and restore a modified cleanup cut
# Jun 27 2025: added cut on significance of the cluster timing
gammaCuts = 'E>0.08'
gammaCuts += ' and clusterNHits > 1.5'
gammaCuts += ' and -0.8660 < cosTheta < 0.9563'
gammaCuts += ' and abs(clusterTiming) < 200'
gammaCuts += ' and abs(clusterTiming/clusterErrorTiming) < 2'
ma.fillParticleList(decayString='gamma:allhem', cut=gammaCuts, path=my_path)
ma.fillParticleList(decayString='gamma:hem0', cut=gammaCuts, path=my_path)
ma.fillParticleList(decayString='gamma:hem1', cut=gammaCuts, path=my_path)

# apply cuts to reduce processing time (50% reduction in time) - not needed for tau Kshort skim
event_cuts = "[nParticlesInList(pi-:allhem) > 1]"
ma.applyEventCuts( event_cuts, path=my_path )

# Sep 2025 - added for pi/K NN PID
b2.conditions.prepend_globaltag('pid_nn_release06_Kpi')

# pizeros
reconstructDecay('pi0:allhem -> gamma:allhem gamma:allhem', '0.119954<M<0.150000', 1,True, my_path)
reconstructDecay('pi0:hem0 -> gamma:allhem gamma:allhem', '0.119954<M<0.150000', 1,True, my_path)
reconstructDecay('pi0:hem1 -> gamma:allhem gamma:allhem', '0.119954<M<0.150000', 1,True, my_path)
```

Require two oppositely charged tracks to have KPID >0.01

E(photon) > 0.08 GeV could be higher as E(pi0) in (kkpz) decays is >0.5 GeV

## 5 Preselection

## Section 5: Analysis strategy

Using “kkpz” to mean  $(\pi^- K^- K^+ \pi^0)$

### Preselection

Loose preselection to select (lepton and hadron tagged) events with tau kkpz decay

### Two likelihoods (following section):

First LH separates the tau decays from qqbar

(separate LH for lepton and hadron tagged events as one has a lepton but the other does not)

Second LH separates kkpz decays from other tau background

(created using all tau decays but is mainly  $(\pi^- K^- K^+ \pi^0)$  and  $(\pi^- K^- K^+)$  decays)

LHs created using all selection criteria including the other LH

## Section 4: Preselection criteria I (select topology)

### Signal topology requirements

Three tracks with  $\text{abs}(Q) = 1$

(2 with the same charge as the tau (**Qtau-tracks**) and 1 with the opposite charge as the tau (**Qopp-track**))

#### Qopp-track

Must be a kaon ( $\text{KPIDNN} > 0.8$ ) \*

One of the **Qtau-tracks**

Must be a kaon ( $\text{KPIDNN} > 0.8$ ) – select track with highest  $\text{KPIDNN}$  as K-candidate

Remaining **Qtau-track**

Considered as the hadron-track (no Kaon/pion PID applied)

### Signal topology requirements

Determine type of tag track

Electron =  $\text{ePID} > 0.08$  Muon =  $\text{muPID} > 0.8$

Hadron != (electron or muon)

#### Bremsstrahlung photons in electron tag

Identify photons that have  $\text{abs}(\text{deltaTheta}) < 0.02$  radians and  $\text{deltaPhi}$  in ( $=0.025$  to  $0.200$ ) radians as **brem-photons**

Then apply preselection criteria

#### \* PID calculation

(PID for electron/muon and PIDNN for kaon/pion)

Will change to e/mu PIDNN soon

Kaon/pion PID

eg.  $K = K / (K + \text{Pi})$

Electron/muon PID

eg  $E = E / (E + M + K + \text{Pi})$

## Section 4: Preselection criteria II

Apply selection criteria that minimize cuts on signal

### Event properties:

Thrust	(0.91 – 0.995)	
Evisible	(4-11) GeV	
M(kkpz)	(1.4 – 1.76) GeV	
M(tag)	< 1 GeV	(if there are photons in the tag hem)

### Tracks (one hadron track and two charge K tracks)

Hadron track is a pion	$\pi$ PIDNN( hadron-track ) < 0.8
Tighter PID on kaon tracks	KPIDNN( kaon-tracks ) > 0.95
cosTheta of kaon tracks	cosTheta > -0.5
None of the tracks are an electron	

### Photons

Nphotons (tag)	< 1 (lepton-tag) and < 2 (hadron-tag) *
Nphotons (event)	< 5 (lepton-tag) and < 6 (hadron-tag)
Energy (extra photons)	< 1 GeV (residual photon energy in sig-hem after excluding photons from $\pi^0$ )

### Bhabbas in electron-tag events

Reject event if tag track is an electron with  $P(\text{CMS}) < 3.8$  GeV and  $\cos\Theta(\text{lab}) < -0.3$  or  $> 0.5$

\* Nphotons (tag) – allow 2 photons if mass of the two photons is between (0.11 – 0.16) GeV

## 6 Likelihoods for minimizing $q\bar{q}$ and non-signal tau decays

## Section 6:

## qqbar Likelihood selection – lepton tag

### Use the samples after the Prelection and the pi0 LH

Combine all tau decays into one sample

#### Variables:

thrust, Evisible

Nphotons(sig), Nphotons(tag), Nphotons (sum), Ephoton(extra)

PT (sig-hadron track), PT (tag-track)

#### For reference:

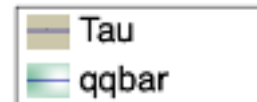
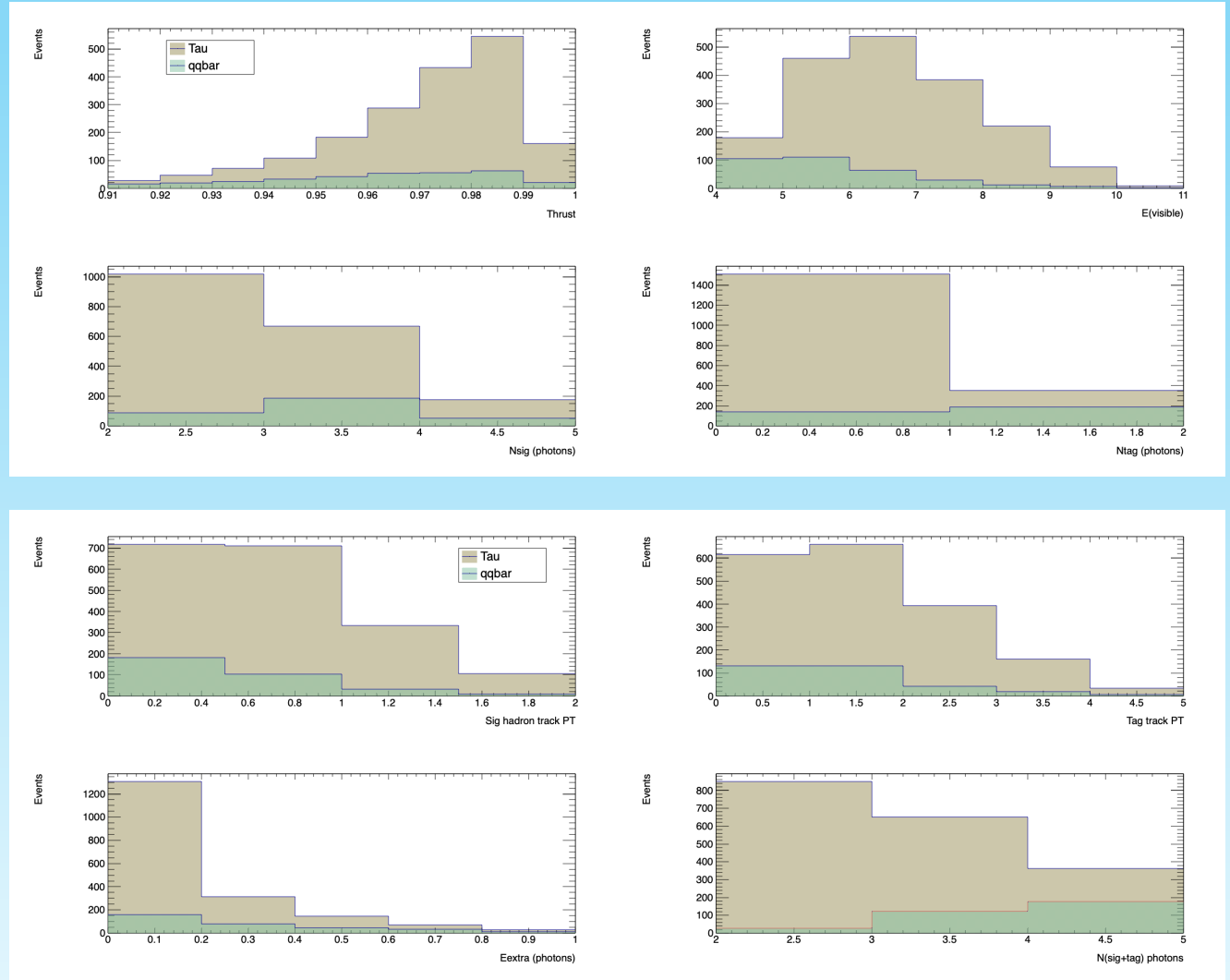
Probability of variable  $V$  having a value  $v_i$  for hypothesis  $H$  is:  $P_V(v_i)$

Probability of a set of variables having values  $\vec{V}$  is:  $L_H(\vec{V}) = \prod_i P_{V_i}$

Likelihood to separate tau decays from qq is:  $LH(qq) = \frac{L_{\tau}}{L_{\tau} + W L_{H_{qq}}}$

Likelihood to separate tau-kkpz decays from other tau decays is:  $LH(kkpz) = \frac{L_{\tau-kkpz}}{L_{\tau-kkpz} + W L_{H_{\tau bkgd}}}$

where  $W$  is constant factor that can adjust the relative contribution



## Section 6: qqbar Likelihood selection – hadron tag

Made a new LH for the hadron-tag sample (different type of qqbar events – no lepton)

### Use the samples after the Prelection and the pi0 LH

Combine all tau decays into one sample

#### Variables:

thrust, Evisible

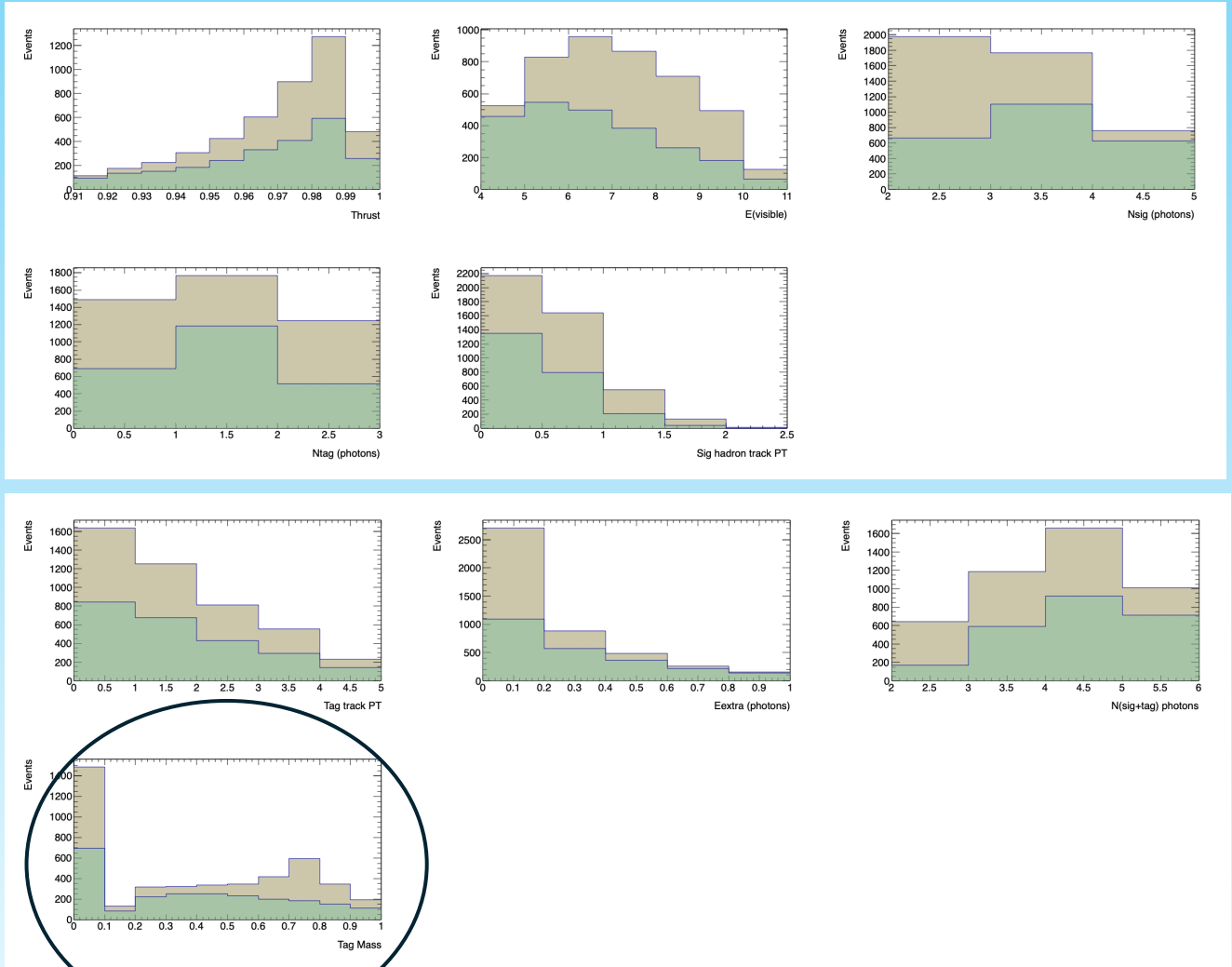
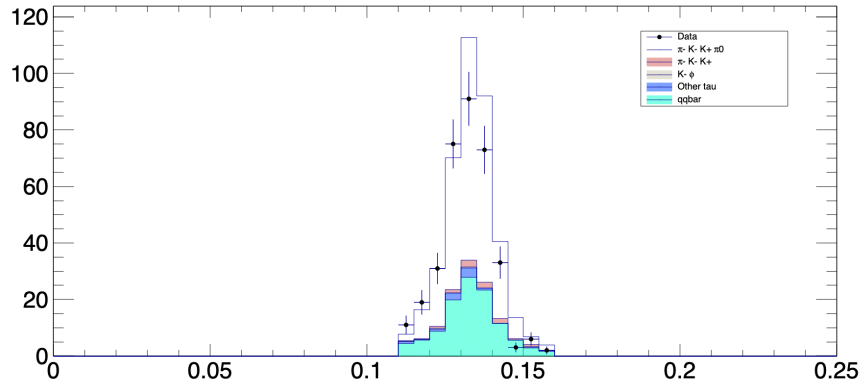
Nphotons(sig), Nphotons(tag), Nphotons (sum)

PT (sig-hadron track), PT (tag-track)

Ephoton(extra)

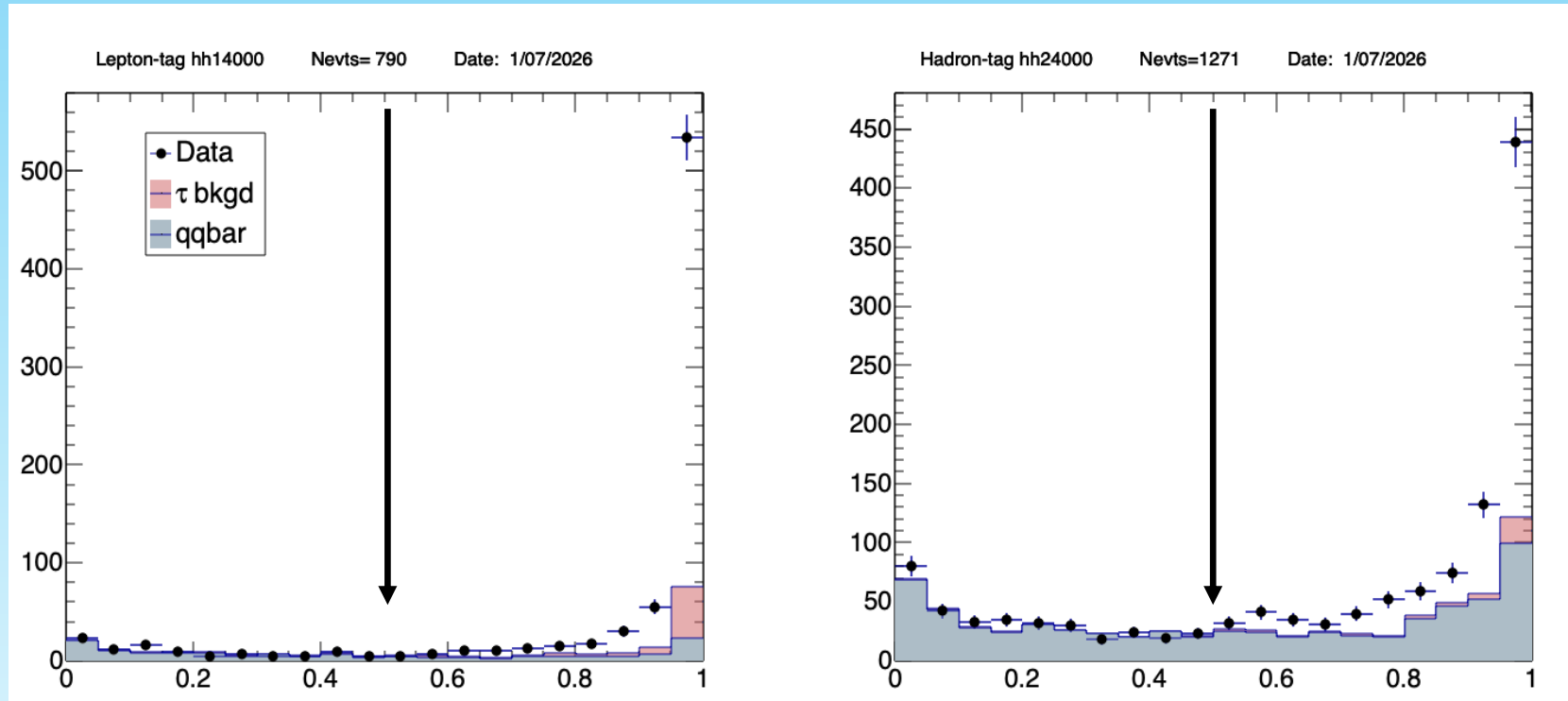
Mass of tag-track and photons in tag hemisphere

Allow hadron-tagged data to have 1 or 2 photons in the tag hemisphere but the mass of the 2 photons must be (0.11-0.16) GeV



## Section 6: qqbar Likelihood selection – lepton and hadron tag

Includes rescaling factor on qqbar and kkp MC samples (in next pages)



## Section 6:

# Tau Likelihood (pzLH) – separate signal (pi- K- K+) from other tau decays (mainly pi-K-K+)

### Use the samples after the Prelection and the qqLH

Combine all tau decays into one sample

The pzLH first adds cuts to define boundaries of the pzLH where there is no signal.

We use the excess of DATA over MC background to define (kkpz) signal

### Variables:

Pi0: mass, energy

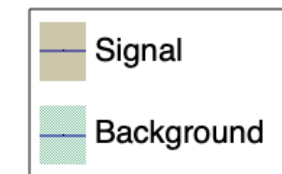
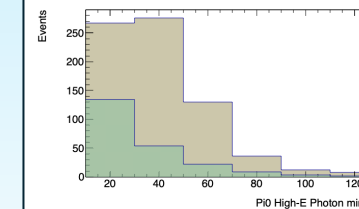
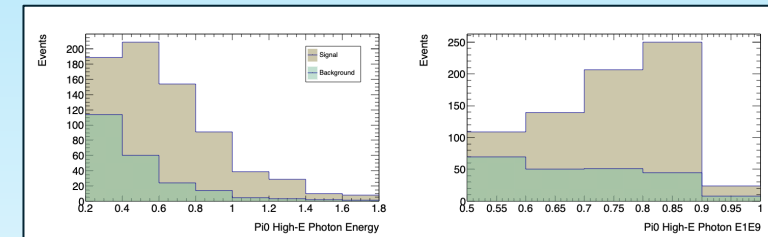
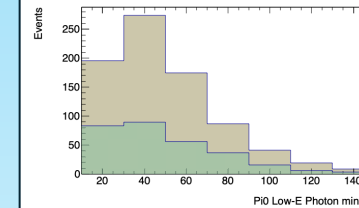
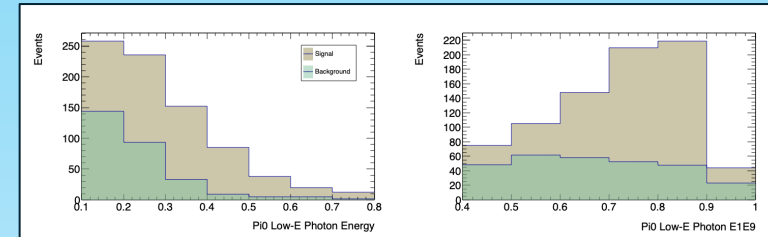
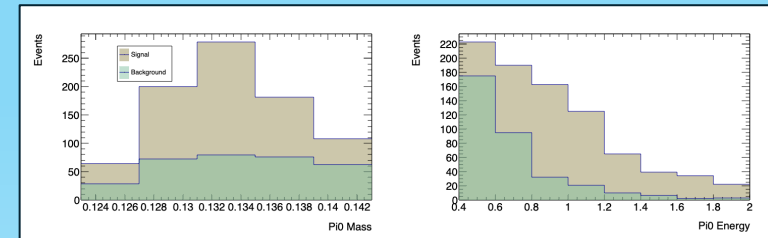
Photon1: energy, E1E1, minCD distance

Photon2: energy, E1E1, minCD distance

Signal = (pi- K- K+ pi0)

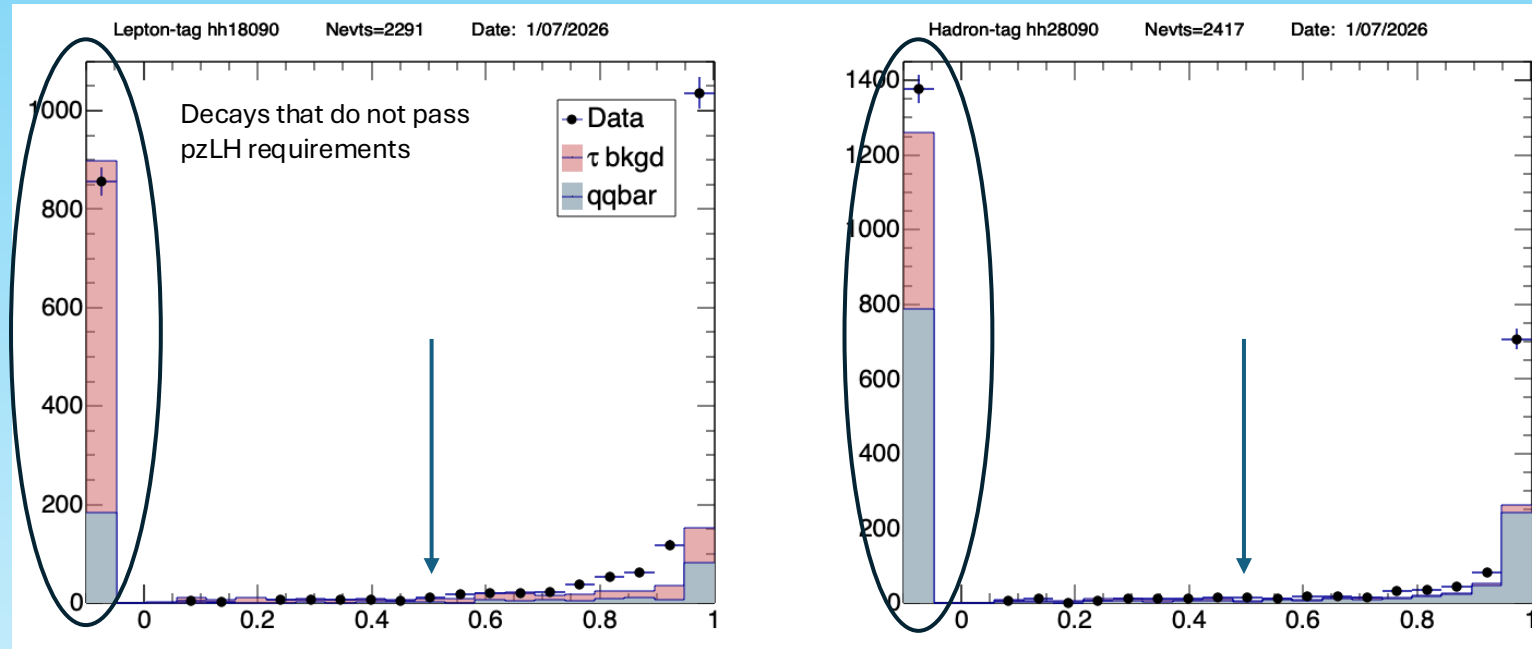
Background = other tau decays

(qqbar not included)



## Section 6: Tau Likelihood – separate signal ( $\pi^- K^- K^+ \pi^0$ ) from other tau decays

Non-signal tau decays are mainly  $\pi^- K^- K^+ \pi^0$  with a fake  $\pi^0$



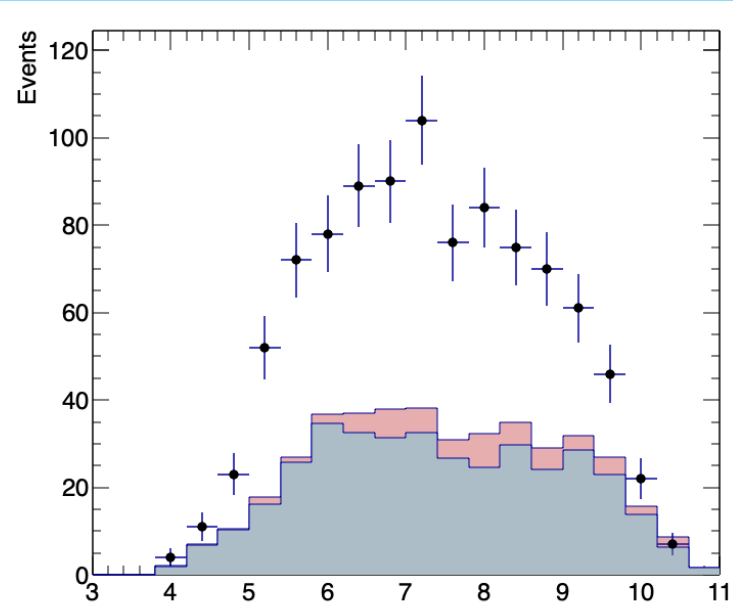
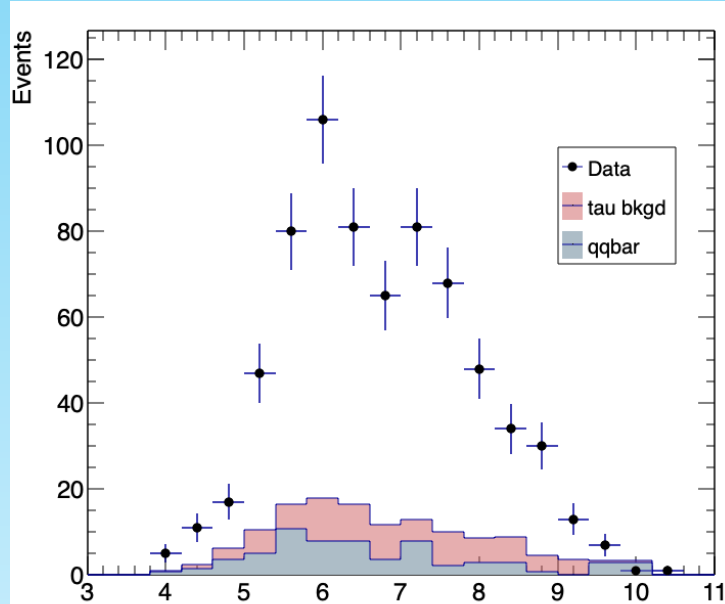
**Require  $p_{zLHv2} > 0.5$**

Background from other tau decays is small and the impact of this requirement is minimal.

## Section 6: Results after preselection and LH selections

Visible energy in the lepton and hadron tag selection

All cuts applied including  $3 < E_{vis} < 11$  GeV



MC backgrounds have been validated using D/MC control sample studies

Lepton-tag

qqbar 9%  
Tau 11%

K- K+ pi-	44.4
K- phi	3.0
Other tau decays	23.1

Hadron-tag

qqbar 38%  
Tau 6%

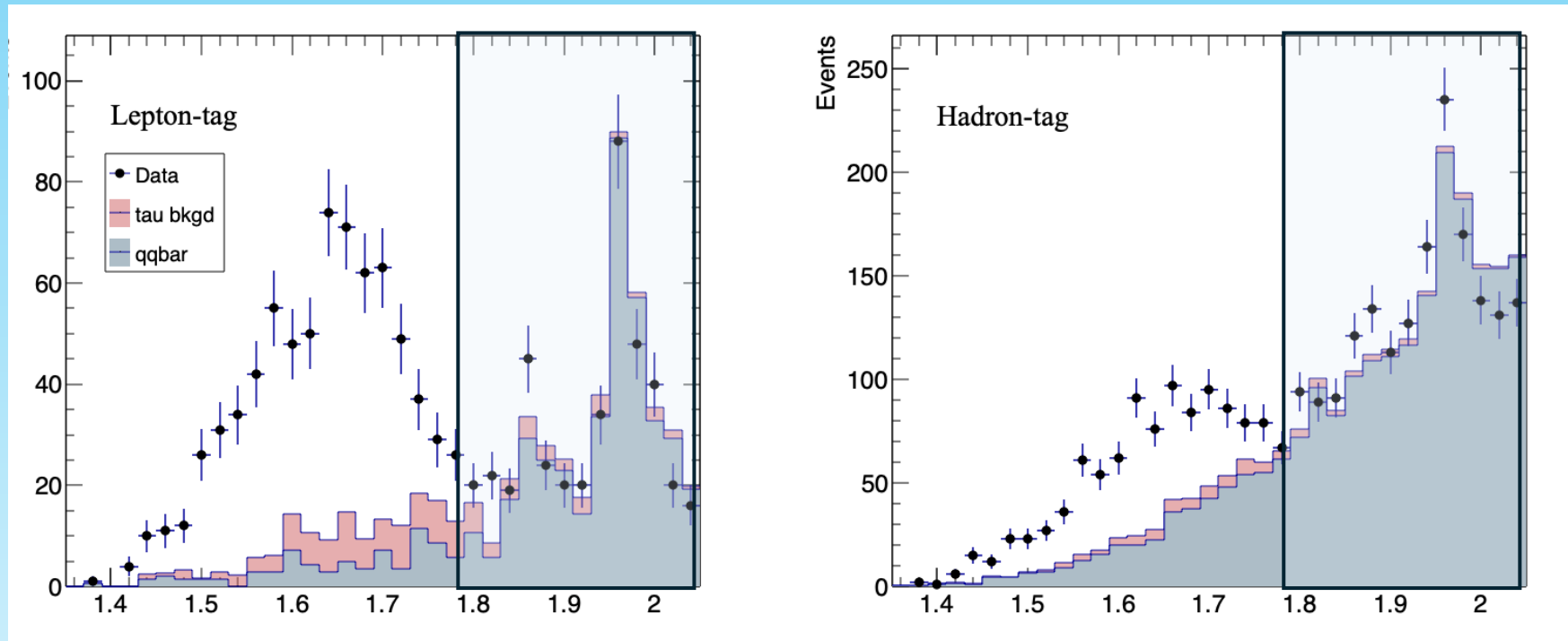
K- K+ pi-	21.6
K- phi	2.4
Other tau decays	16.7

pi- pi0	1.0
pi- pi+ pi- pi0	1.8
K- KS pi0	4.6
K- omega	5.1
pi0 2pi- K-	4.1

## **7 Validation of qqbar and tau backgrounds with control samples**

## Section 7: validate qqbar background with control samples

Mass distributions for ( $\pi^- K^- K^+ \pi^0$ ) and ( $\pi^- K^- K^+$ )



### Mass ( $\pi^- K^- K^+ \pi^0$ ) with all selection criteria

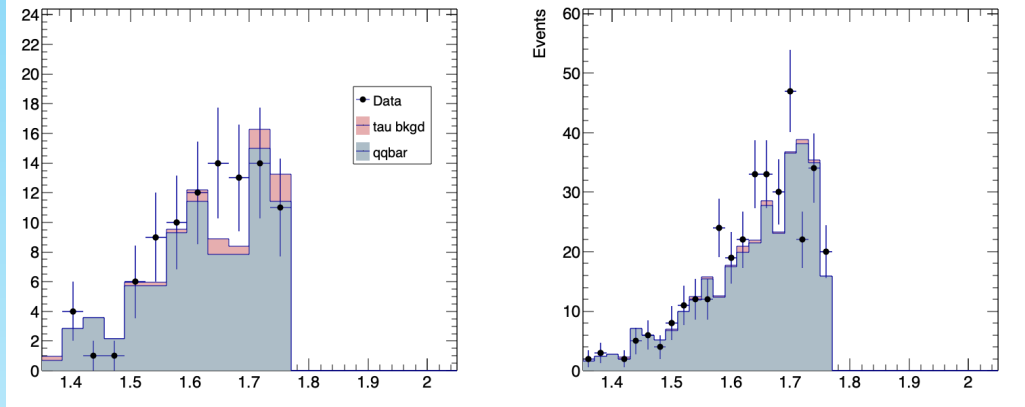
qqbar background in lepton-tag was scaled upward by 1.7

qqbar background in hadron-tag did not require a scale factor

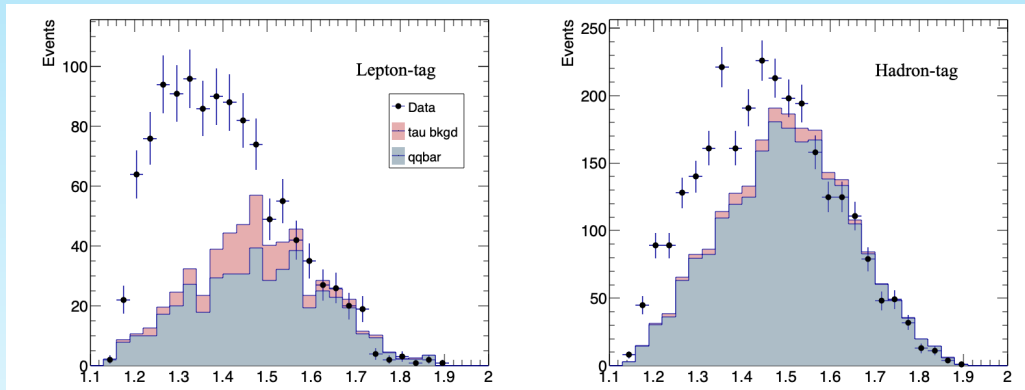
## Section 7: cross check qqbar background with other distributions

Cross check of qqbar background

Further confirmation that qqbar is understood



Mass ( $\pi^- K^- K^+ \pi^0$ ) with all selection criteria and qqLH cut reversed (lepton and hadron tag)



Mass ( $\pi^- K^- K^+$ ) with all selection criteria and qqLH cut reversed and no cut on  $M(\text{pkkz})$  (lepton and hadron tag)

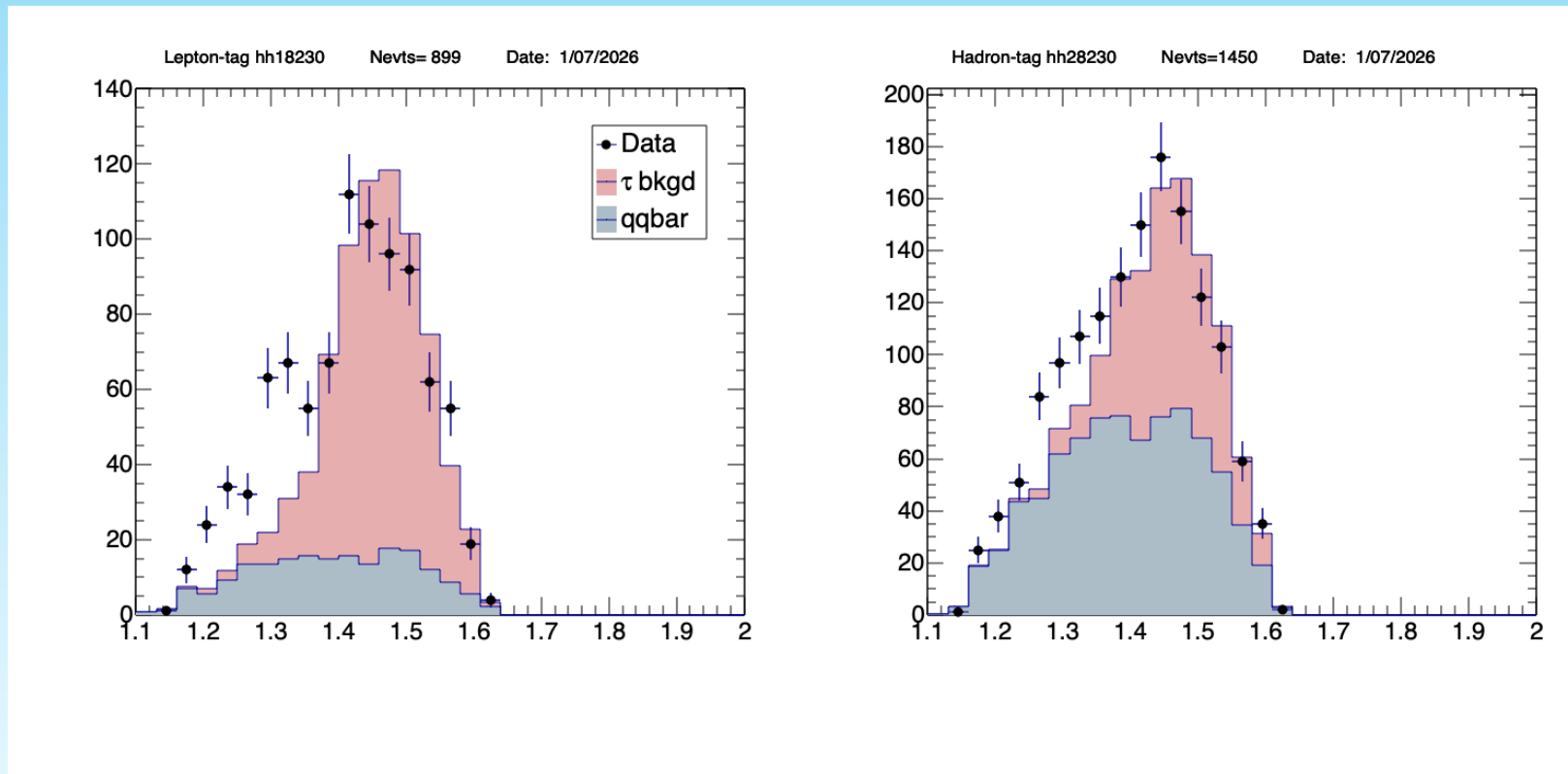
## Section 7:

## Tau background – check results with control samples

All selection criteria except flip the pzLH requirement (pzLH<0.5 instead of pzLH>0.5)

Plotting the mass ( $\pi^- K^- K^+$ ) for lepton and hadron tag sample (some signal in the sample)

Results indicate that tau and qqbar background are well-modelled and good to <10%  
(need to determine specific error and add another check)



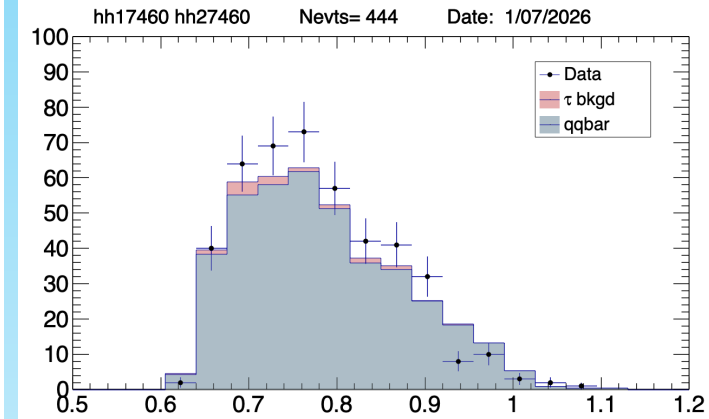
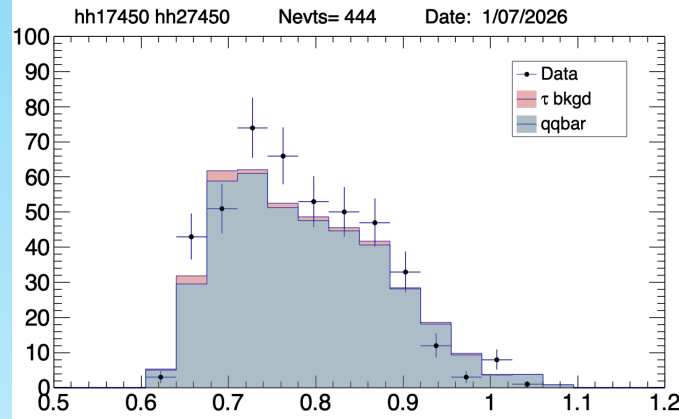
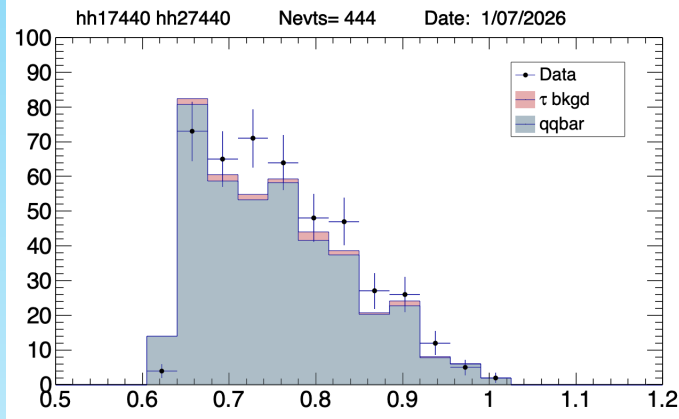


## Section 8: Resonant backgrounds from qqbar events

All selection criteria (both lepton and hadron tags) with qqLH cut reversed

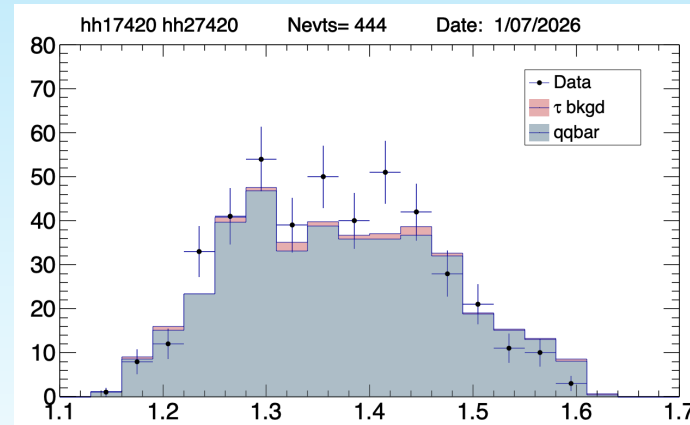
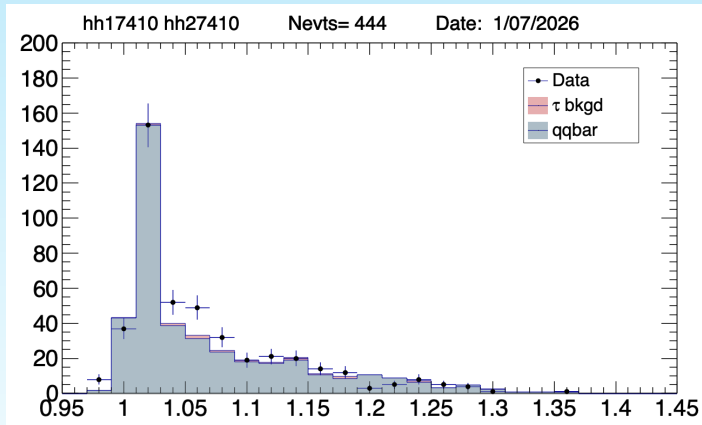
### Masses of ( $\pi^- K^+$ ), ( $K^- \pi^0$ ) and ( $K^+ \pi^0$ ) where the charge of the tau is MINUS

No evidence of  $K^*$  from qqbar events



### Masses of the ( $K^- K^+$ ) and ( $K^- K^+ \pi^0$ )

Evidence of the phi meson and no evidence of the f1 mesons

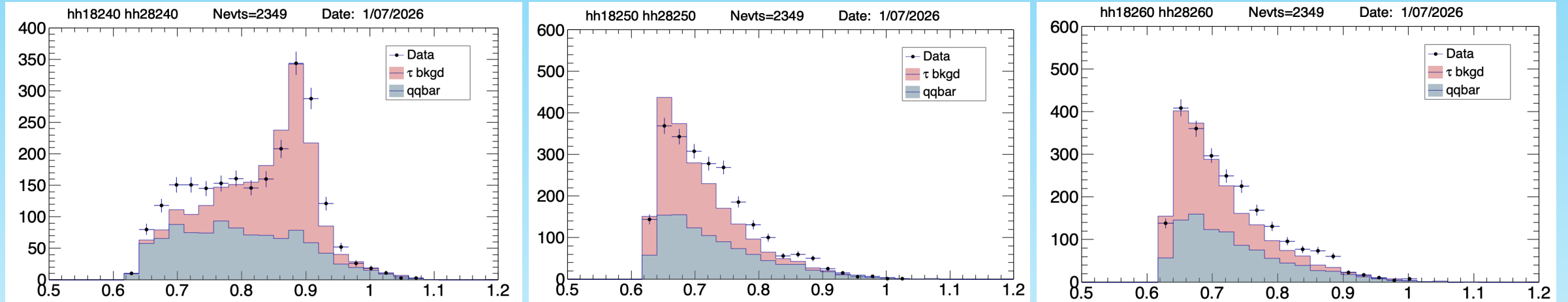


## Section 8: Resonant backgrounds other tau decays

All selection criteria (both lepton and hadron tags) with qqLH cut reversed  
Primary tau background is (pi- K- K+) which cannot contain an f1 meson

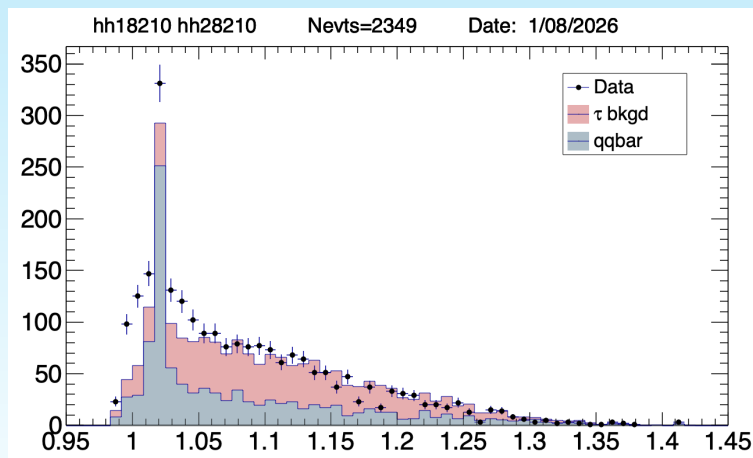
### Masses of (pi- K+), (K- pi0) and (K+ pi0) where the charge of the tau is MINUS

K\* (pi- K+) from (pi- K- K+) decays

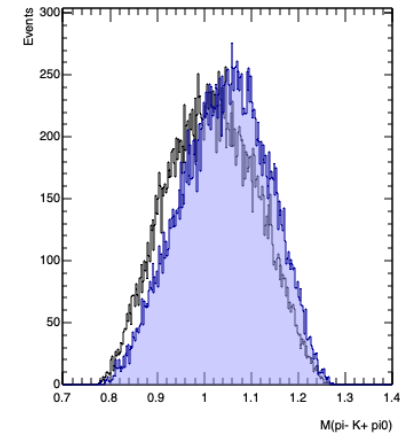
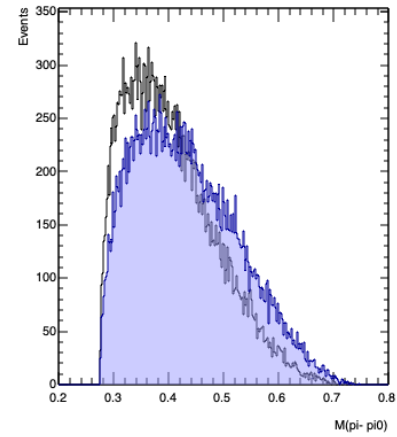
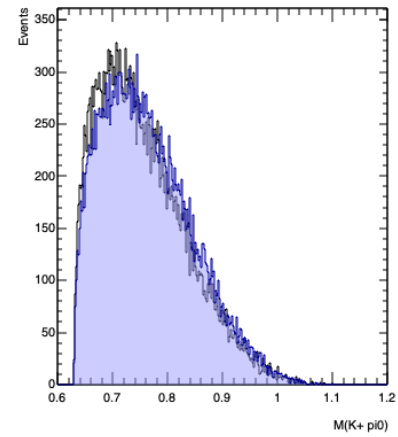
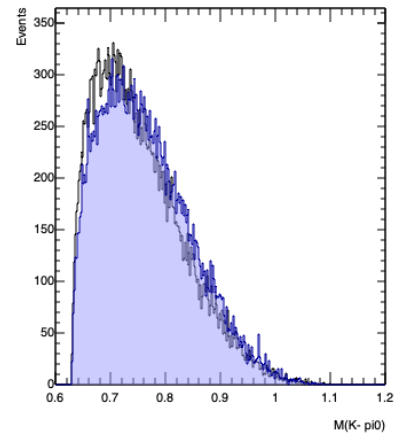
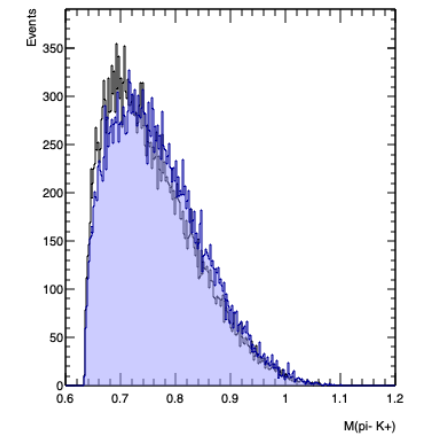
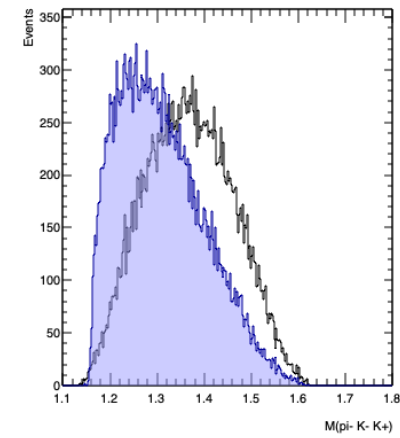
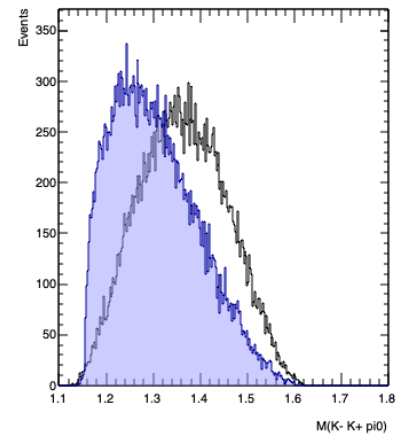
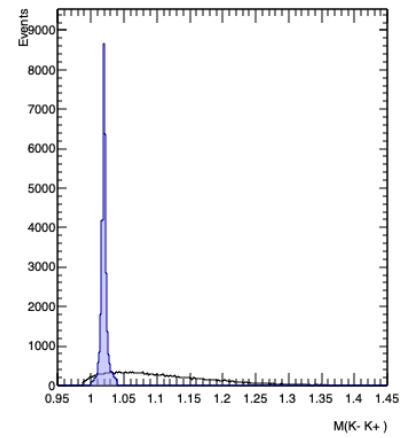
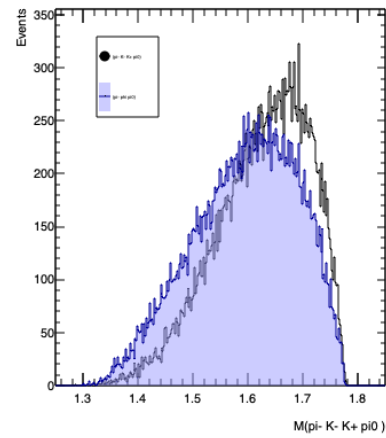


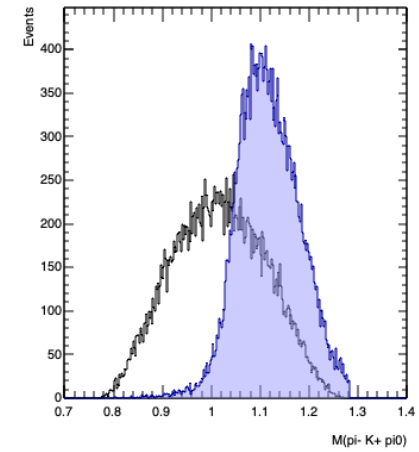
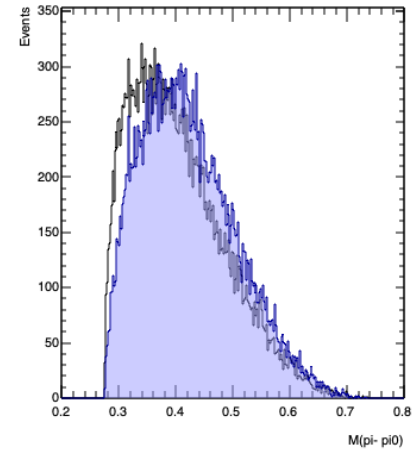
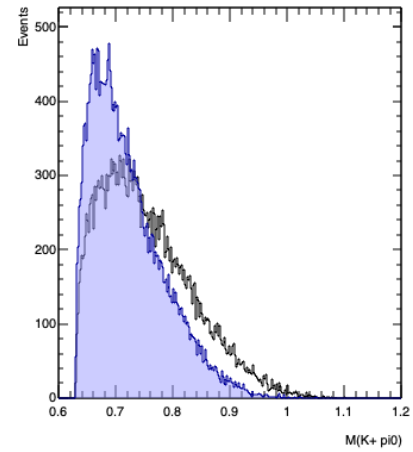
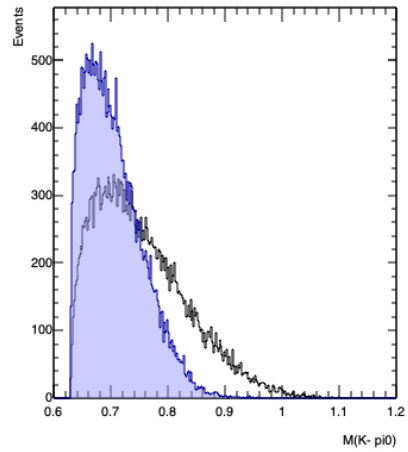
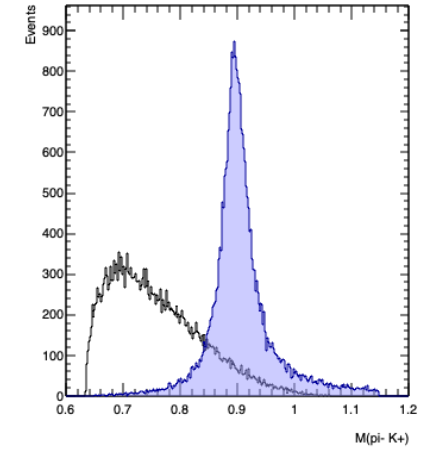
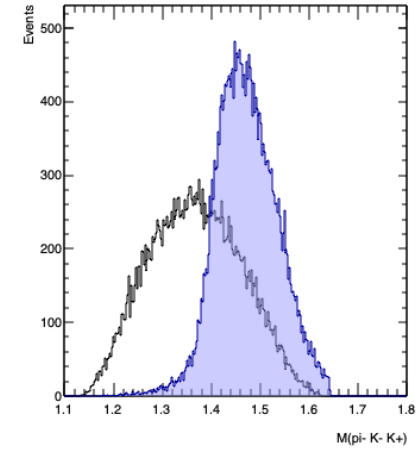
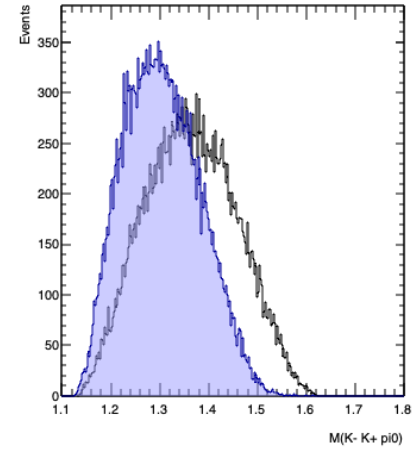
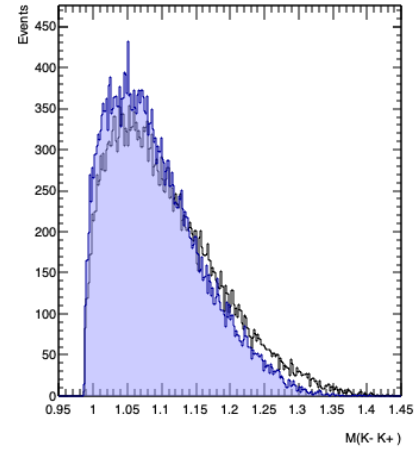
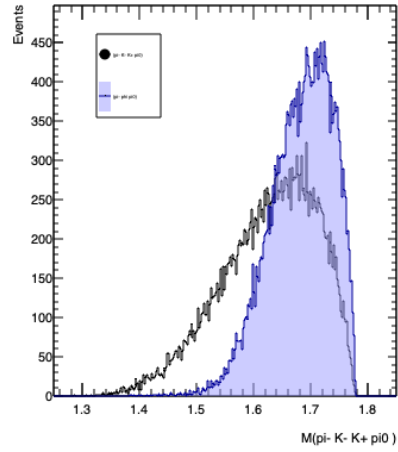
### Mass of the (K-K+)

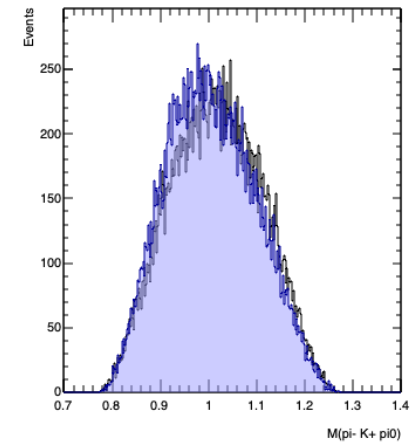
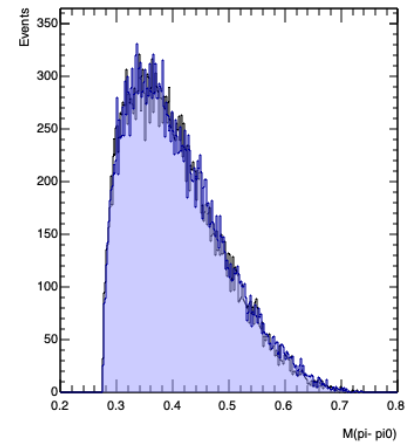
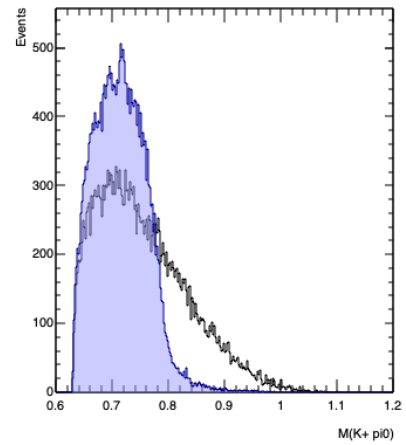
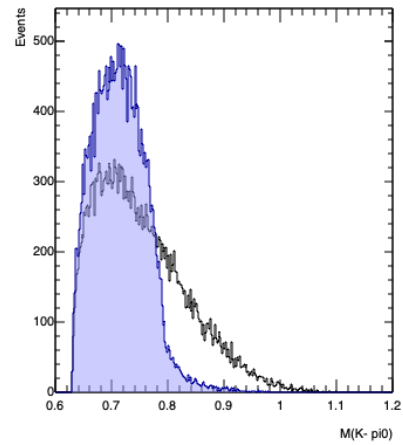
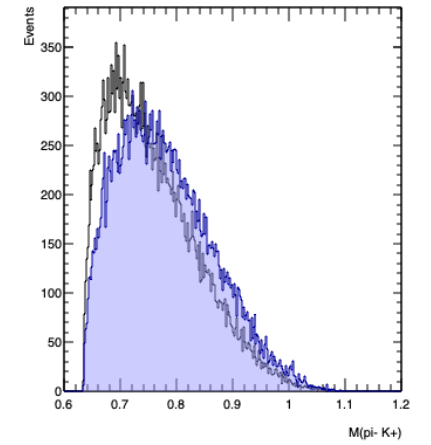
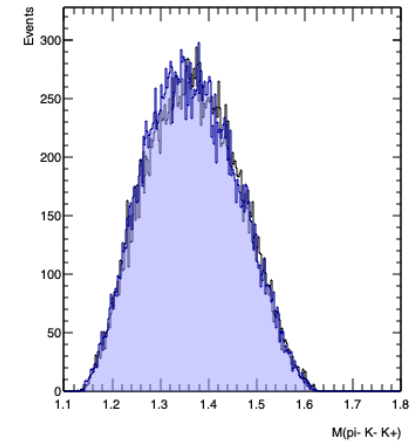
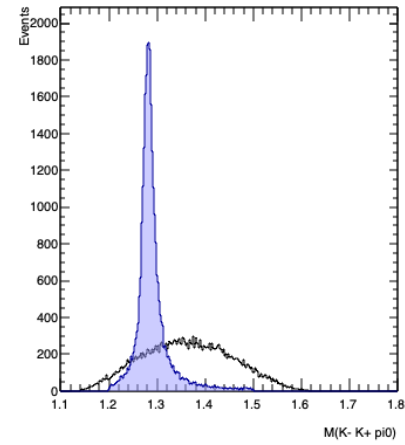
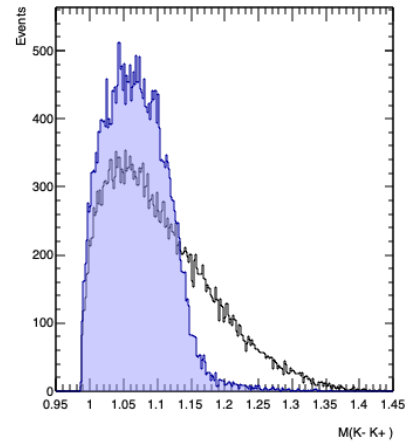
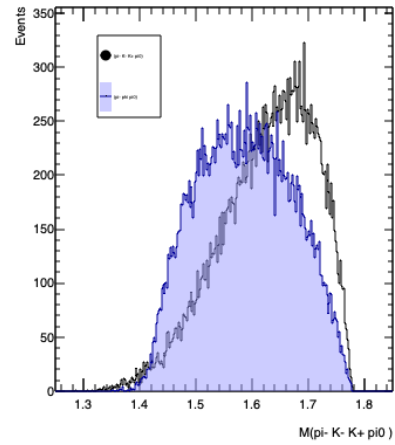
Tau decay to (pi- phi) has been observed (see PDG) – the MC predicts 8 decays. Needs further investigation.

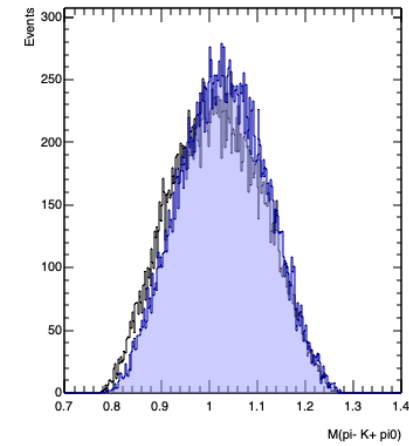
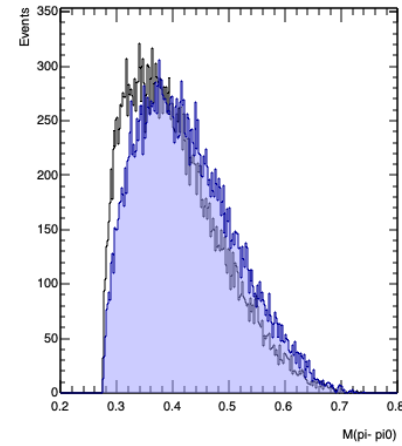
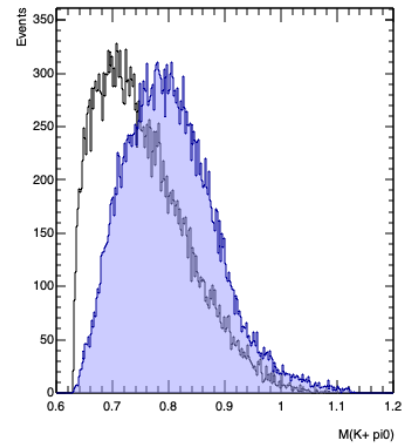
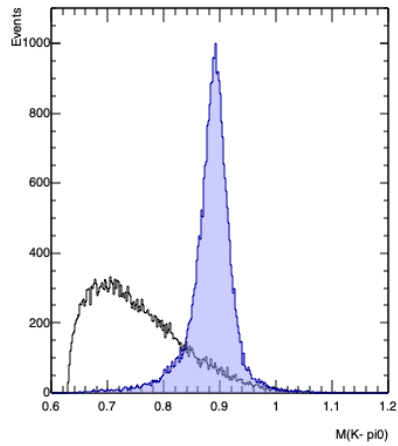
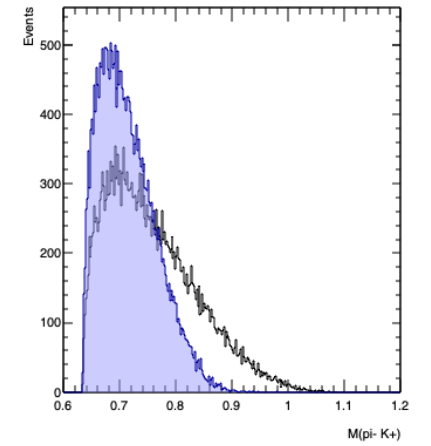
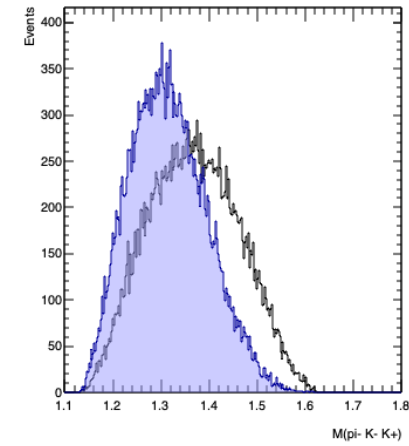
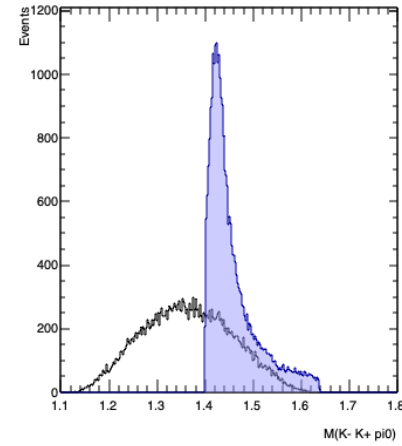
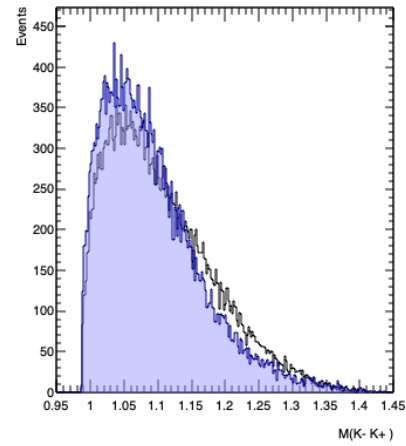
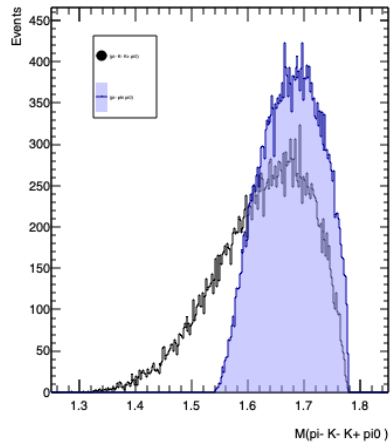


## **Section 9      Pythia 4-vector samples for fit studies**







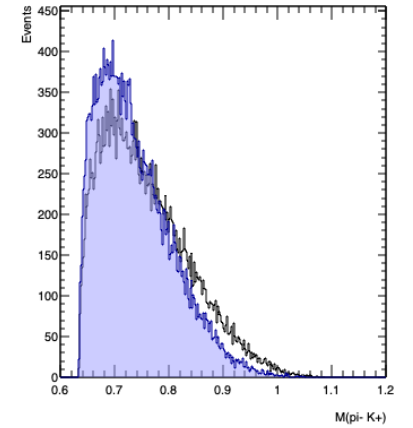
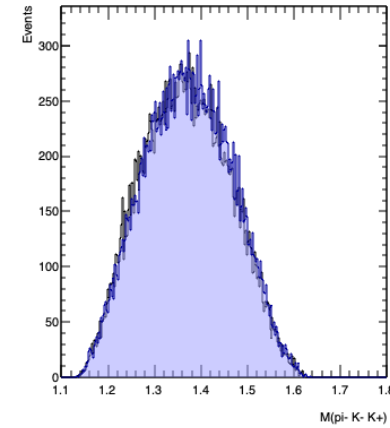
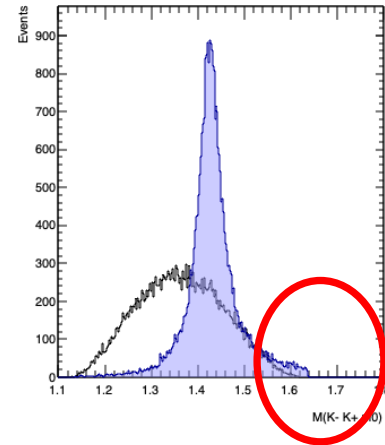
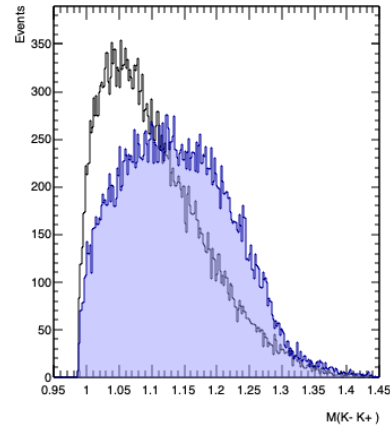
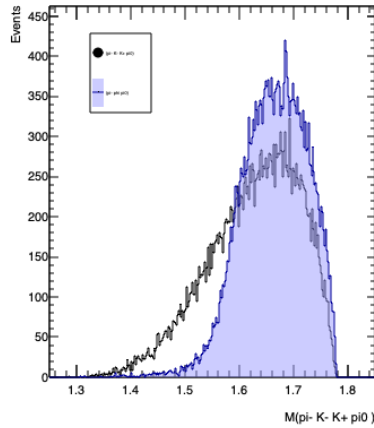


# Section 9

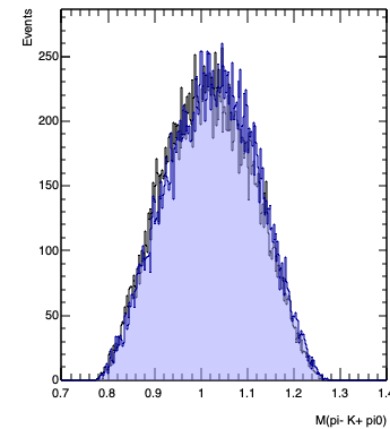
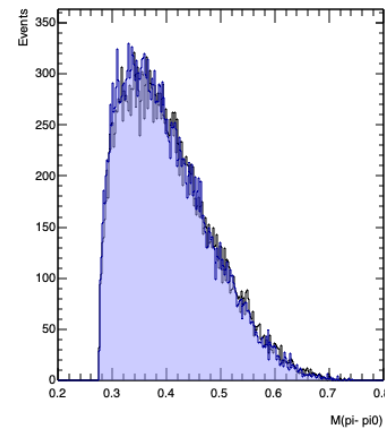
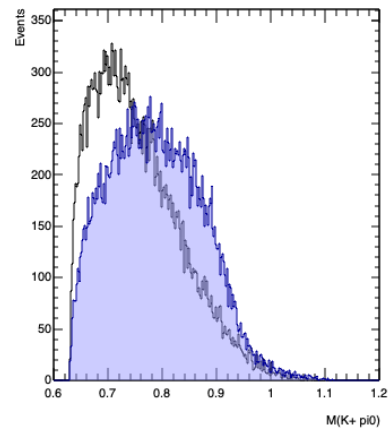
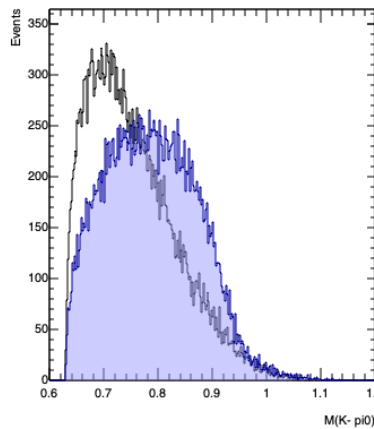
# Pythia 4vectors (no selections)

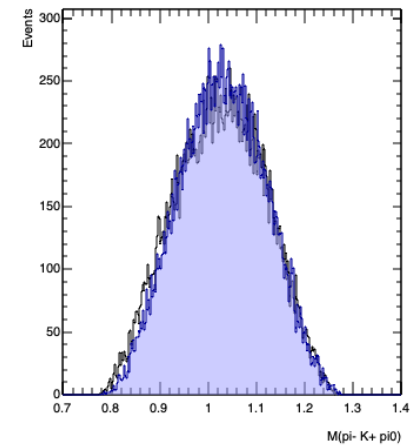
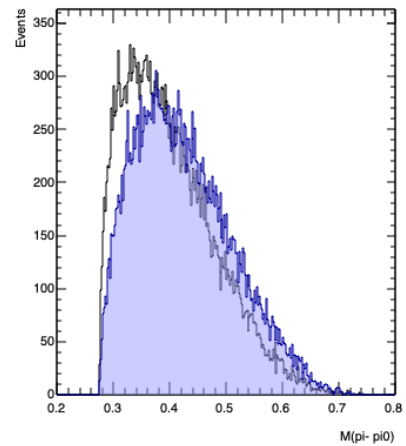
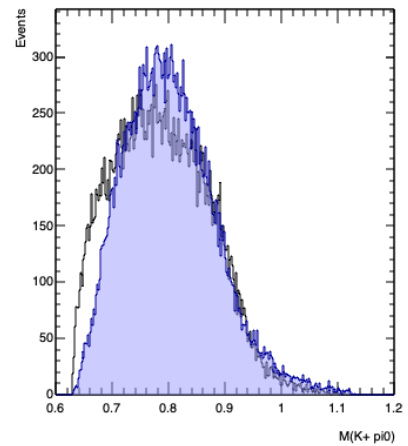
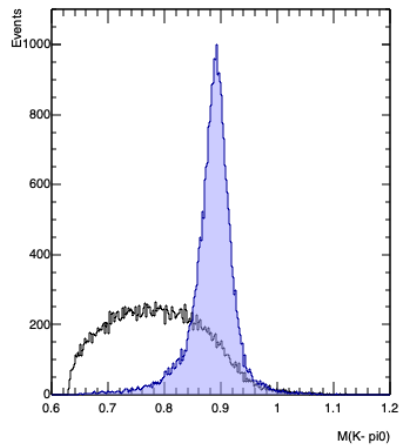
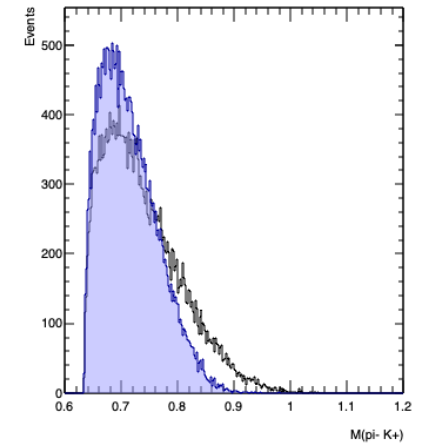
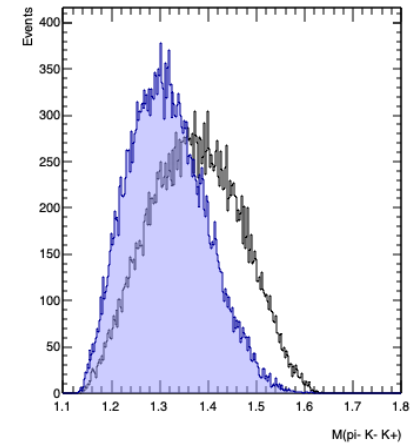
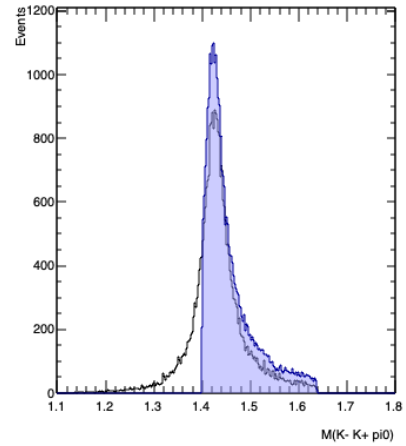
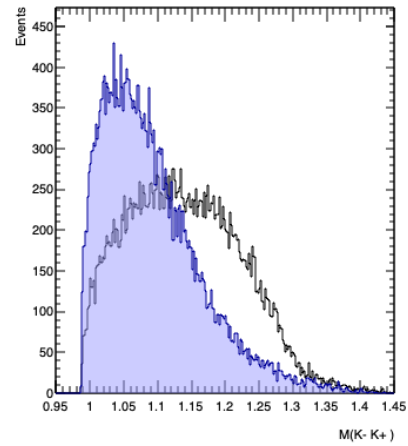
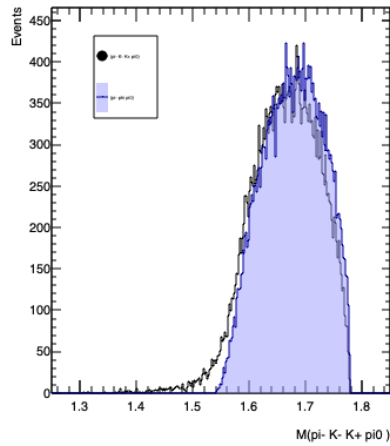
# PS and f1420 to (K+K- pi0)

Not the default mode for the f1420 (needed to be added)



Note abrupt cut off



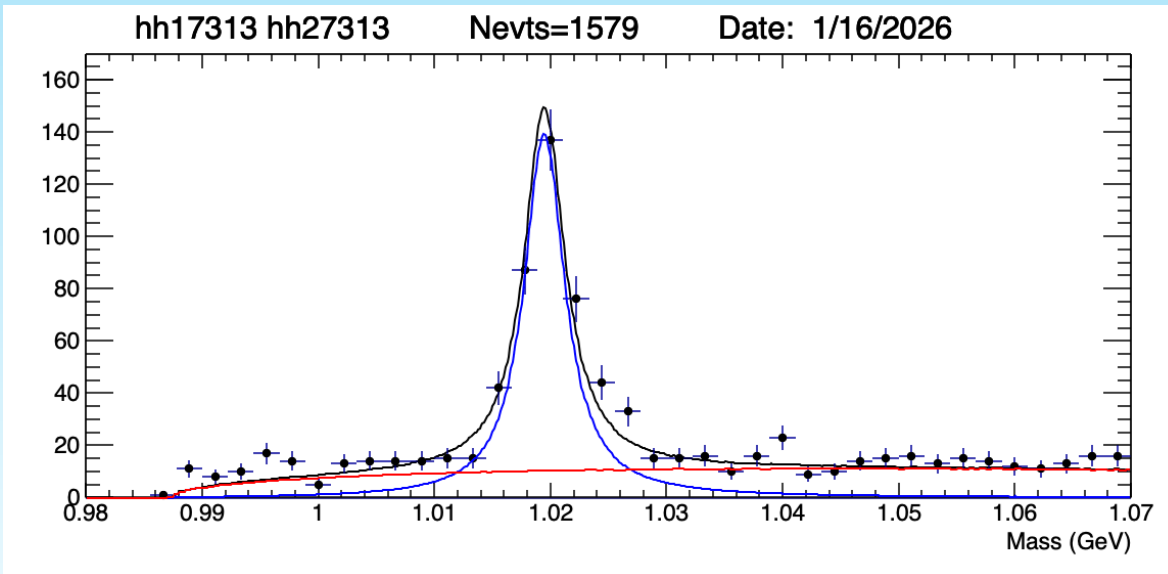




**Fit phi meson with qqbar sample**

(pzLHv2\_nominal&gt;0.5 &amp;&amp; qqLHv2\_ltag&lt;0.5 &amp;&amp; mpkkz0&lt;2.05)

PDG phi M/W = (1.01946 and 0.00425)

**Fit gives M/W consistent with the PDG values**

Fixed, M-free, MW-free

NO.	NAME	VALUE	ERROR
1	p0	1.01404e-01	5.92870e-03
2	p1	1.01946e+00	fixed
3	p2	4.25000e-03	fixed
4	p3	1.12017e+00	4.78208e-02

Chisq/NDF = 93.02 for NDF 72  
0.06  
Area from integration : 401 +/- 23

NO.	NAME	VALUE	ERROR
1	p0	1.01680e-01	5.90955e-03
2	p1	1.01986e+00	2.09899e-04
3	p2	4.25000e-03	fixed
4	p3	1.11991e+00	4.78103e-02

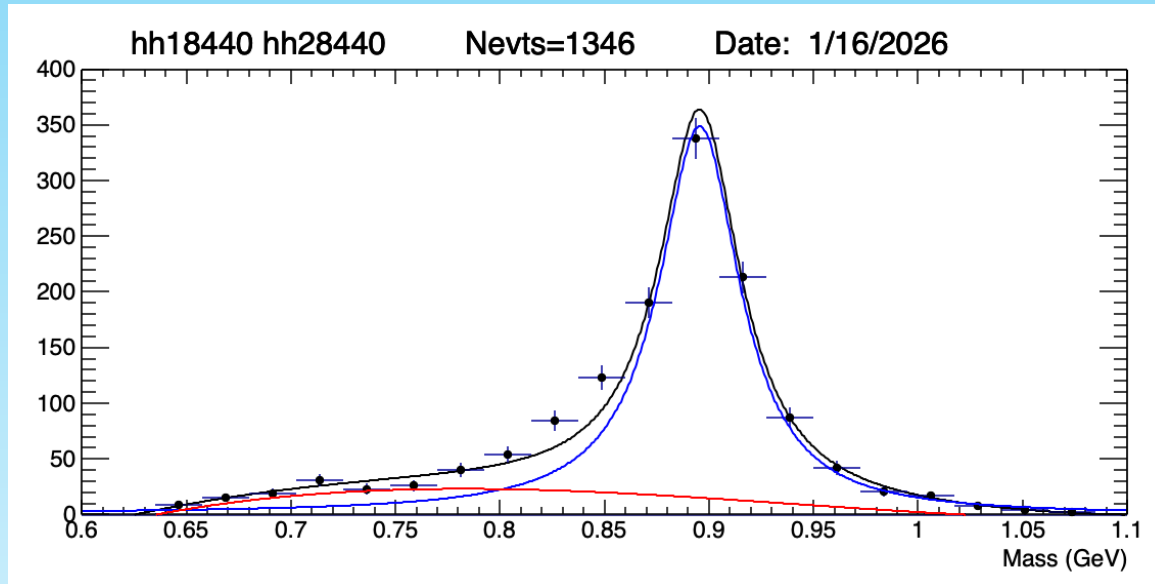
Chisq/NDF = 88.92 for NDF 71  
0.06  
Area from integration : 404 +/- 23

NO.	NAME	VALUE	ERROR
1	p0	1.05135e-01	6.51510e-03
2	p1	1.01990e+00	1.96775e-04
3	p2	4.71302e-03	3.96256e-04
4	p3	1.10346e+00	4.97742e-02

Chisq/NDF = 87.4 for NDF 70  
0.06  
Area from integration : 416 +/- 26

## Section 10

## Mass and width of $K^*$ ( $\pi$ - $K^+$ ) with tau control samples



NO.	NAME	VALUE	ERROR
1	p0	2.58822e+01	9.33563e-01
2	p1	8.95550e-01	fixed
3	p2	4.73000e-02	fixed
4	p3	-1.30794e+03	6.32881e+00
5	p4	4.46467e+03	1.13689e+01
6	p5	-4.88937e+03	1.09011e+01
7	p6	1.73492e+03	8.23878e+00
8	p7	0.00000e+00	fixed

Chisq/NDF = 23.8 for NDF 15  
0.04  
Area from integration : 1087 +/- 39

PDG parameters for  $K^*$  are fine

