Status of the tau to (K K pi pi) decay analysis

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Motivation

Data and MC Samples

Grid skim selection and lists

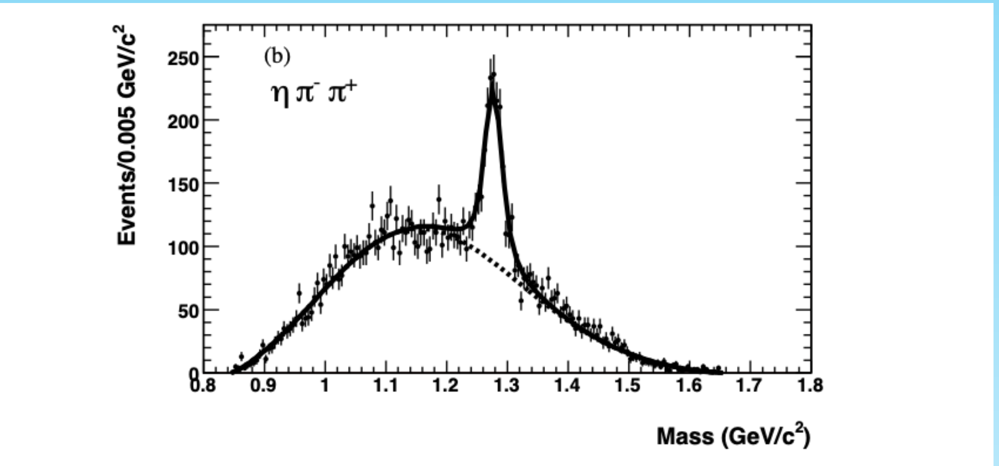
Preselection

Likelihoods for minimizing qqbar and non-signal tau decays

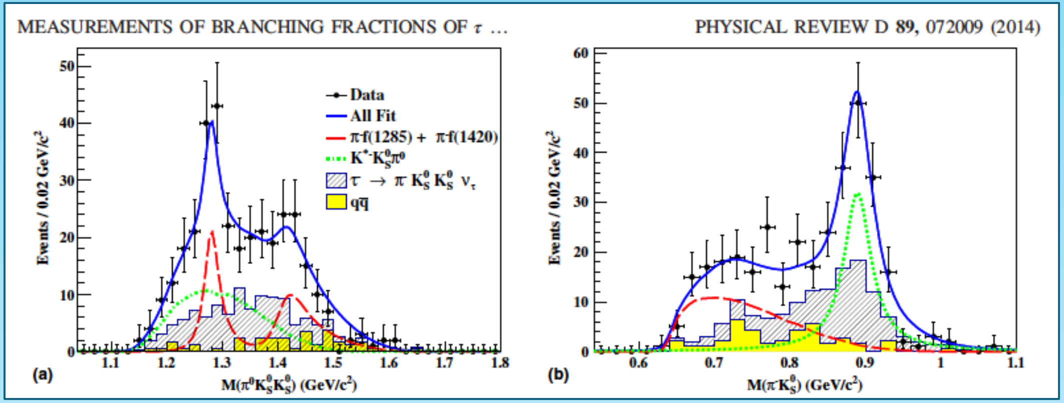
Understanding the decay mode (mass distributions and fits)

Motivation for studying these decays

Our BaBar analysis of tau decays to (4pi) and (2pi eta) showed clear evidence of the f1(1285) resonance



Belle found a signal for the f1(1285) and f1(1420) in tau decays to (pi- KS KS pi0)



$f_1(1285)$ $\quad I^G(J^{PC}) = 0^+(1^{++})$

Mass $m = 1281.8 \pm 0.5$ MeV (S = 1.7)
Full width $\Gamma = 23.0 \pm 1.1$ MeV (S = 1.6)

$f_1(1285)$ DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
4π	(32.7 ± 1.8) %	S=1.2	568
$\pi^0\pi^0\pi^+\pi^-$	(21.8 ± 1.2) %	S=1.2	566
$2\pi^+2\pi^-$	(10.9 ± 0.6) %	S=1.2	563
$\rho^0\pi^+\pi^-$	(10.9 ± 0.6) %	S=1.2	336
$\rho^0\rho^0$	seen		†
$4\pi^0$	$< 7 \times 10^{-4}$	CL=90%	568
$\eta\pi^+\pi^-$	(35 ± 15) %		479
$\eta\pi\pi$	(52.2 ± 1.9) %	S=1.2	482
$a_0(980)\pi$ [ignoring $a_0(980) \to K\bar{K}$]	(38 ± 4) %		238
$\eta\pi\pi$ [excluding $a_0(980)\pi$]	(14 ± 4) %		482
$K\bar{K}\pi$	(9.0 ± 0.4) %	S=1.1	308
$K\bar{K}^*(892)$	not seen		†

$f_1(1420)$ $\quad I^G(J^{PC}) = 0^+(1^{++})$

See the review on "Spectroscopy of Light Meson Resonances."
Mass $m = 1428.4^{+1.5}_{-1.3}$ MeV (S = 1.8)
Full width $\Gamma = 56.7 \pm 3.3$ MeV (S = 1.3)

$f_1(1420)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$K\bar{K}\pi$	seen	440
$K\bar{K}^*(892)$+ c.c.	seen	167
$\eta\pi\pi$	possibly seen	574
$\phi\gamma$	seen	350

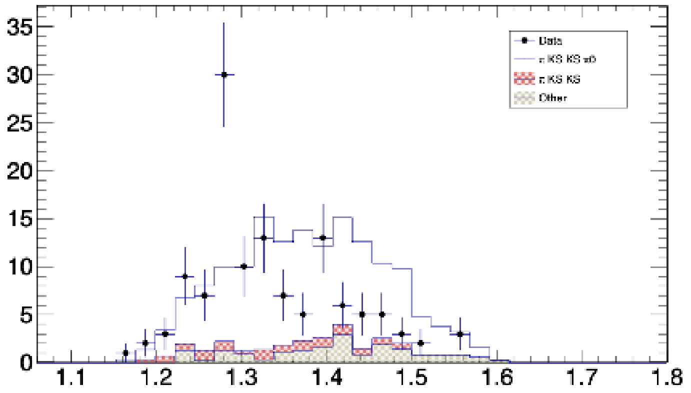
Did BaBar miss the f1(1420) ?

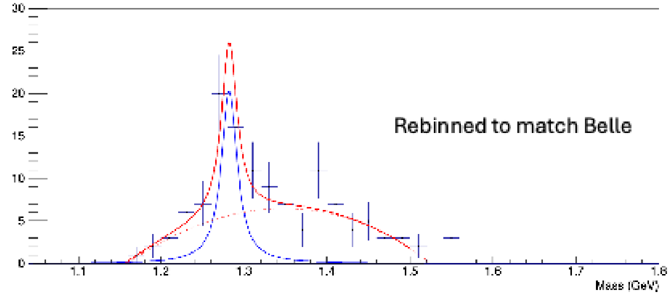
Or is it only visible in channels with kaons?

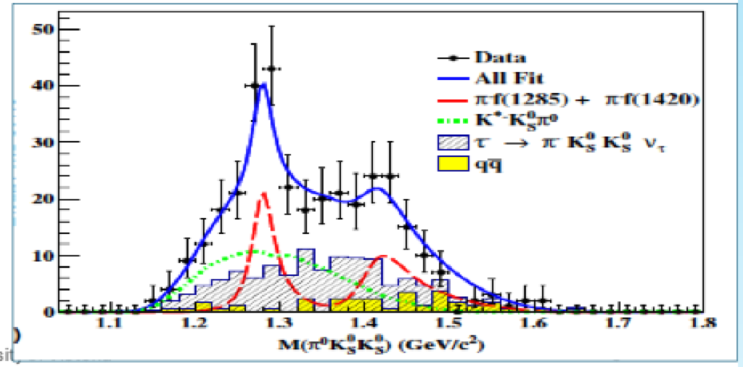
At the Sept B2GM, the BelleII analysis of the tau to (pi- KS KS pi0) channel was presented

Even with all the data samples (prod and prompt), the number of events is small

f1 meson in (pKKz) decays
Evidence for the f1(1285) but not the f1(1420)
Fit has fixed f1(1285) mass and width Breit-Wigner (no smearing function) Npeak = 37 +/- 6 decays







Based on peak height, B2 and Belle have same number of f1 (1285) candidates

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However, isospin symmetry says (pi- K0 K0b pi0) = (pi- K+ K- pi0)

CLEO and ALEPH measurements (30% error) and no detailed studies of the decay mechanism

$\Gamma(K^- K^+ \pi^- \pi^0 \nu_\tau)/\Gamma_{\text{total}}$ Γ_{107}/Γ

VALUE (units 10^{-4})	CL%	EVTS	DOCUMENT ID		TECN	COMMENT
0.61±0.18 OUR FIT		(Produced by HFLAV)				
0.60±0.18 OUR AVERAGE						
$0.55 \pm 0.14 \pm 0.12$		48	ARMS	05	CLE3	7.6 fb^{-1}, E^{ee}_{cm}=10.6 GeV
$7.5 \pm 2.9 \pm 1.5$			BARATE	98	ALEP	1991–1995 LEP runs

The (K K pi pi) channel has been studied by other experiments in fixed target and ISR measurements

Observation of the D (f1(1285)) and E (f1(1420)) mesons

PHYSICAL REVIEW D VOLUME 22, NUMBER 7 1 OCTOBER 1980

Observations of the D and E mesons and possible three-kaon enhancements in $\pi^- p \to K^0 K^{\pm} \pi^{\mp} X$, $K^0 K^+ K^- X$ at 50 and 100 GeV/c

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Indiana University, Bloomington, Indiana 47401

(Received 14 April 1980)

We present results from the study of multikaon final states in the reactions $\pi^- p \to K^0 K^{\pm} \pi^{\mp} \tilde{X}$, $K^0 K^+ K^- \tilde{X}$ at 50 and 100 GeV/c. Here $\tilde{X}$ is semi-inclusive, that is, a specific forward topology and an interaction registering in the counters surrounding the target are required. The $D(1285)$ meson is seen in the $\delta\pi$ mode, while the $E(1420)$ meson is observed in both the $\delta\pi$ and the K^*K modes. In addition, there are two possible enhancements in the three-kaon final state. The first ($M \sim 1840$ MeV) is associated with the $K^0\phi$ mode and is consistent with being the charmed D^0. The second, a K^* state ($M = 2003 \pm 14$ MeV, $\Gamma = 87 \pm 43$ MeV), decays into KA_2.

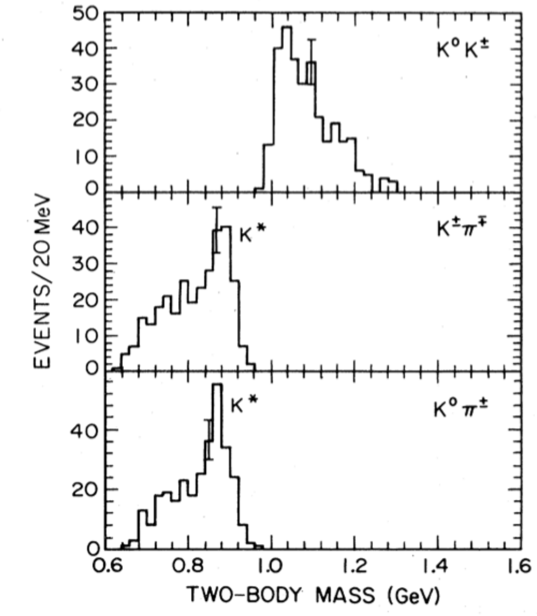


FIG. 10. Two-body mass distribution with E cut $[1.356 \leq M(KK\pi) \leq 1.476$ GeV].

Results are low statistics and the PDG does not give branching fractions

$f_1(1420)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$K\overline{K}\pi$	seen
Γ_2	$K\overline{K}^*(892)$ + c.c.	seen
Γ_3	$\eta\pi\pi$	possibly seen
Γ_4	$a_0(980)\pi$	
Γ_5	$\pi\pi\rho$	
Γ_6	4π	
Γ_7	$\rho^0\gamma$	
Γ_8	$\phi\gamma$	seen

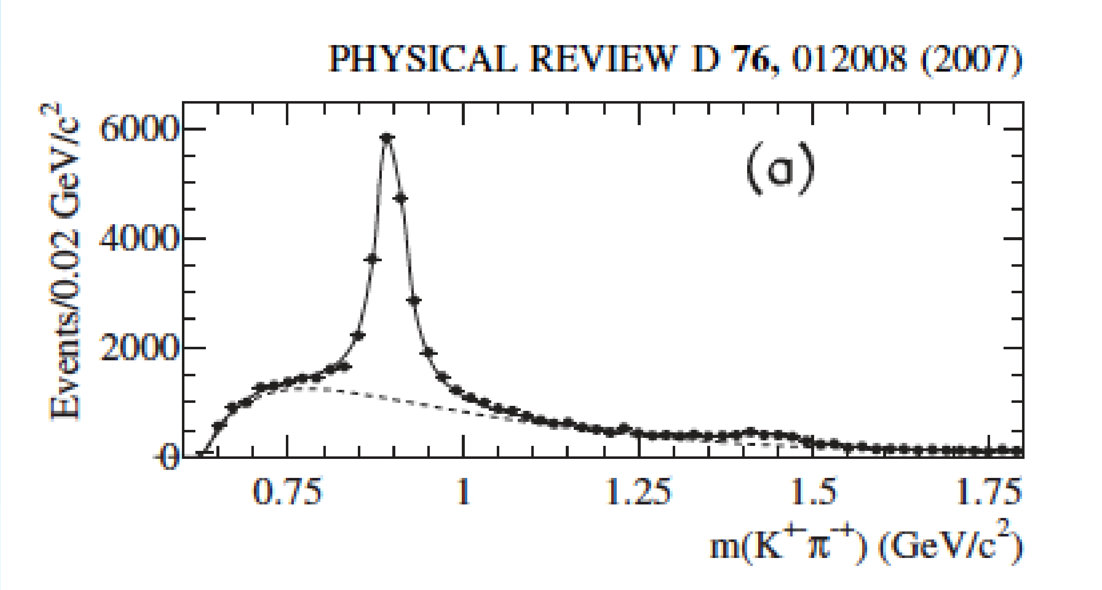
The (K K pi pi) channel has been studied by other experiments in fixed target and ISR measurements

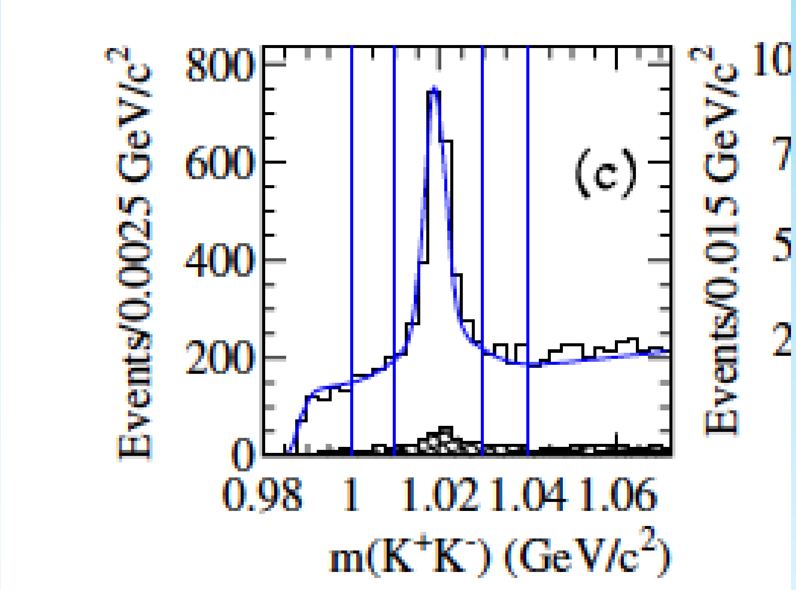
BaBar ISR measurement

PHYSICAL REVIEW D 76, 012008 (2007)

$e^+e^- \rightarrow K^+K^-\pi^+\pi^-$, $K^+K^-\pi^0\pi^0$ and $K^+K^-K^+K^-$ cross sections measured with initial-state radiation

BaBar ISR result was not constrained to systems with final states < tau mass





Results from previous measurements suggests that the tau to (K K pi pi) should see resonances even with the kinematic constraint of the tau mass

Data and MC samples I

Data (Int Lum = 568 fb-1 according to gbasf2)

Peak	/belle/collection/Data/S16_proc16_4S_skim_79000100_v2
Scan	/belle/collection/Data/S16_proc16_5S_scan_skim_79000100_v2
Offpeak	/belle/collection/Data/S16_proc16_4S_offres_skim_79000100_v2
Peak prompt	/belle/collection/Data/S16_prompt_4S_skim_79000100_v1
Offpeak prompt	/belle/collection/Data/S16_prompt_4S_offres_skim_79000100_v1

Monte Carlo

udsc	/belle/collection/MC/S16_MC16rd-proc16_4S_udsc_skim_79000100_v2
bb	/belle/collection/MC/S16_MC16rd-proc16_4S_BB_skim_79000100_v2

Monte Carlo	**(no/few events pass preselection)**
llXX	/belle/collection/MC/S16_MC16rd-proc16_4S_llXX_skim_79000100_v2
ee	/belle/collection/MC/S16_MC16rd-proc16_4S_ee_skim_79000100_v2
mumu	/belle/collection/MC/S16_MC16rd-proc16_4S_mumu_skim_79000100_v2

Special note:
Although no ee MC events pass preselection, there is evidence for ee events in the e-tag data
Cuts applied to remove Ptag(CMS) > 4 GeV and in endcap regions when tag hemisphere has an electron track

Data and MC samples II

Tau pair Monte Carlo

Peak	/belle/collection/MC/S16_MC16rd-proc16_4S_taupair_skim_79000100_v1
Offpeak	/belle/collection/MC/S16_MC16rd-proc16_4S_offres_taupair_skim_79000100_v1
Scan	/belle/collection/MC/S16_MC16rd-proc16_5S_scan_taupair_skim_79000100_v1

Belle II Tauola decay modes

(mode number from Table 1 BELLE2-NOTE-PH-2020-055)

pi- K- K+ pi0	(mode 23)	Tauola (0.00061)	Phase space decay (CLEO results 30% error)
pi- K- K+	(mode 103)	Tauola (0.001435)	Well understood (K- K*)
K- phi K+K-	(mode 230)	Tauola (0.000023)	Well understood
pi- f1(1285):	eta pi+ pi- (with eta to gg, pi-pi+pi0) and 2pi- 2pi+		Well understood

<u>**Not included:**</u>

pi- phi	PDG BR (3.4 10-5)
pi- phi pi0	(mode 118)
pi- f1(1285)	PDG BR (f1 to pi- K K) = 11%
pi- f1(1420)	PDG (f1 to pi- K Kb) seen and (f1 to K K*b) seen

Focused on a data-driven analysis (may not need signal MC samples)

No previous studies of (pi- K- K+ pi0) decay mechanism (only BF with 30% error)
Not yet sure what measurements will result (considering relative BF ratios)

Sample of (pi- K- K+ pi0) is okay for topology variables but is not useful for studying the decay mechanism.

Processing of the TauGeneric skim samples

Event requirements:
Standard track, photon*, pi0 lists
2-6 tracks

Tau topology requirements:
Event split into hemispheres based on thrust (total Q in event = 0)
One hemisphere has only 1 track (tag hem)
Signal hemisphere has at least two tracks (with opp Q) with KPID > 0.01 abnd N(pi0) >= 1

Photon requirements:
Following CPV analysis but refined cut to remove fake photons from hadronic interactions
(they used a hard energy cut; smoothed it out but the impact in this work is negligible as Epi0 is high)

E(photons) > 0.08 GeV
(did not need to be so low for this work; developed for the (pi- KS KS pi0) selection)

Also added (recommendation of CHearty)
gammaCuts += ' and abs(clusterTiming/clusterErrorTiming) < 2'

Things to do
Need to reprocess the samples with the recent PID recommendations (might need help)
Currently using “ceres” release – update to latest release
Use electron and muon PIDNN (already using kaon and pion PIDNN)

Analysis strategy   Using “kkpz” to mean (pi- K- K+ pi0)

Preselection

Loose preselection to select (lepton and hadron tagged) events with tau kkpz decay

Two likelihoods:

First LH separates the <u>tau decays from qqbar</u>

(separate LH for lepton and hadron tagged events as one has a lepton but the other does not)

Second LH separates <u>kkpz decays from other tau background</u>

(created using all tau decays but is mainly (pi- K- K+ pi0) and (pi- K- K+) decays)

LHs created using all selection criteria including the other LH

Preselection criteria I (select topology)

Signal topology requirements
Three tracks with abs(Q) = 1
(2 with the same charge as the tau (**Qtau-tracks**) and 1 with the opposite charge as the tau (**Qopp-track**)

Qopp-track	Must be a kaon (KPIDNN >0.8) *
One of the **Qtau-tracks**	Must be a kaon (KPIDNN >0.8) – select track with highest KPIDNN as K-candidate
Remaining **Qtau-track**	Considered as the hadron-track (no Kaon/pion PID applied)

Signal topology requirements
Determine type of tag track
Electron = ePID > 0.08   Muon = muPID > 0.8   Hadron != (electron or muon)

<u>Bremsstrahlung photons in electron tag</u>
Identify photons that have abs(deltaTheta) < 0.02 radians and deltaPhi in (=0.025 to 0.200) radians as **brem-photons**

Then apply preselection criteria

*** PID calculation**	**(PID for electron/muon and PIDNN for kaon/pion)**	**Will change to e/mu PIDNN soon**
Kaon/pion PID	eg. K = K / (K+Pi)	
Electron/muon PID	eg E = E / (E+M+K+Pi)	

Preselection criteria II

Apply selection criteria that minimize cuts on signal

Event properties:

Thrust	(0.91 – 0.995)	
Evisible	(4-11) GeV	
M(kkpz)	(1.4 – 1.76) GeV	
M(tag)	< 1 GeV	(if there are photons in the tag hem)

Tracks (one hadron track and two charge K tracks)

Hadron track is a pion	piPIDNN(hadron-track) < 0.8
Tighter PID on kaon tracks	KPIDNN(kaon-tracks) > 0.95
cosTheta of kaon tracks	cosTheta > -0.5
None of the tracks are an electron	

Photons

Nphotons (tag)	< 1 (lepton-tag) and < 2 (hadron-tag) *
Nphotons (event)	< 5 (lepton-tag) and < 6 (hadron-tag)
Energy (extra photons)	< 1 GeV (residual photon energy in sig-hem after excluding photons from pi0)

Bhahbas in electron-tag events

Reject event if tag track is an electron with P(CMS) < 3.8 GeV and cosTheta(lab)< -0.3 or >0.5

* Nphotons (tag) – allow 2 photons if mass of the two photons is between (0.11 – 0.16) GeV

Preselection criteria III – Thrust plot

All topology and preselection criteria applied (except on the variable plotted)

MC tau bkgd scaled by 0.82 (both tags) — Based on D/MC studies

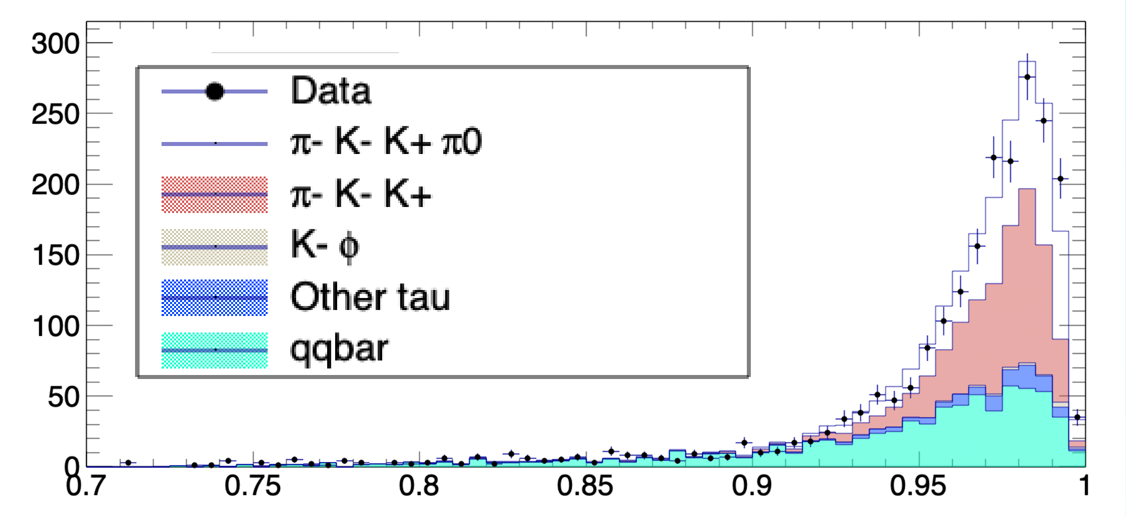
(primarily pi- K- K+ decays with a fake pi0 but tau background is found to be small in both lepton and hadron tag samples)

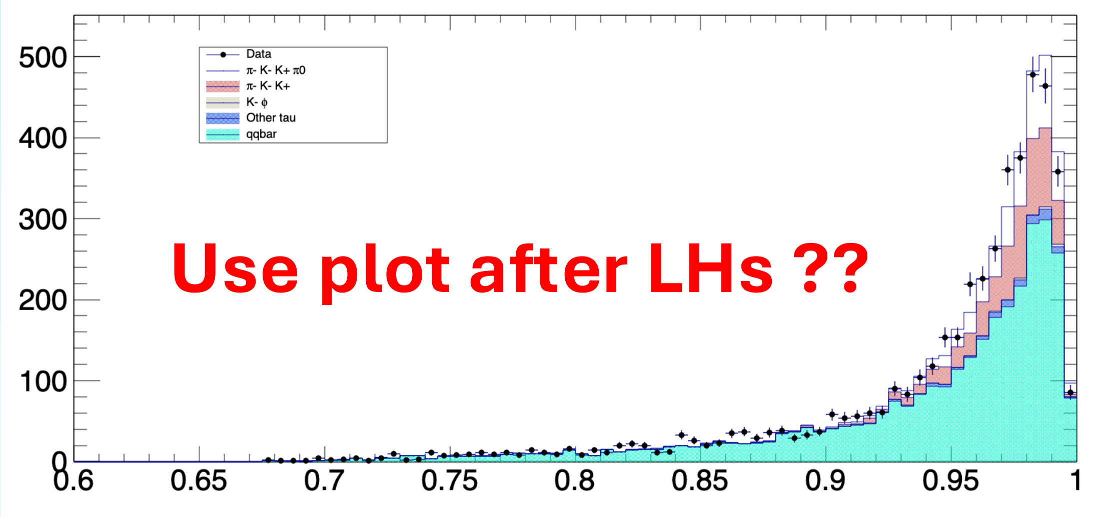
MC udsc/bb scaled by 1.74 (lepton-tag) and 1.00 (hadron-tag) — Based on D/MC studies

MC kkpz (pi- K- K+ pi0) scaled by 0.4 — Arbitrary normalization to match number of events in data

Thrust (nominal selection = 0.910 - 0.995)

Lepton-tag sample (left) — Hadron-tag sample (right)





qqbar Likelihood selection – lepton tag

Use the samples after the Prelection and the pi0 LH

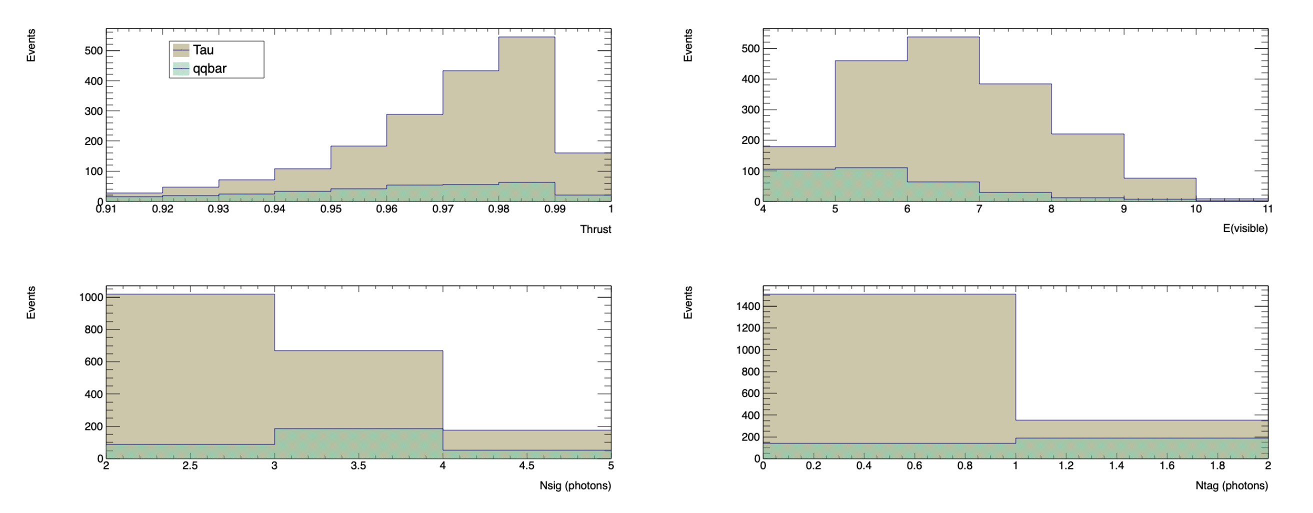
Combine all tau decays into one sample

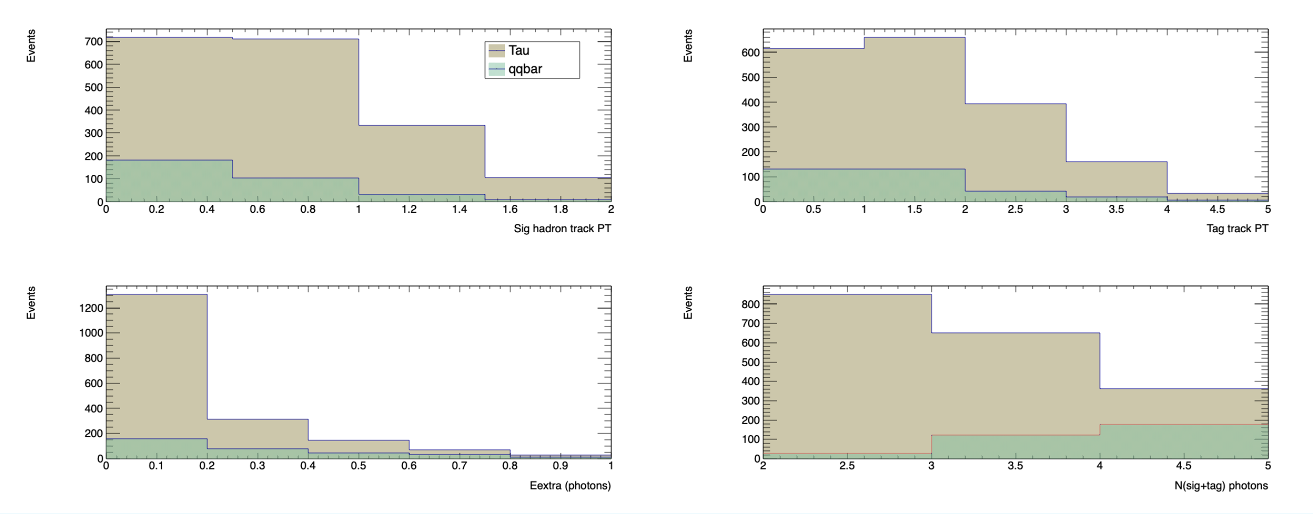
Variables:

thrust, Evisible

Nphotons(sig), Nphotons(tag), Nphotons (sum), Ephoton(extra)

PT (sig-hadron track), PT (tag-track)





For reference:

Probability of variable V having a value v_i for hypothesis H is: $P_V(v_i)$

Probablility of a set of variables having values $\vec{V}$ is: $L_H(\vec{V}) = \prod_i P_{V_i}$

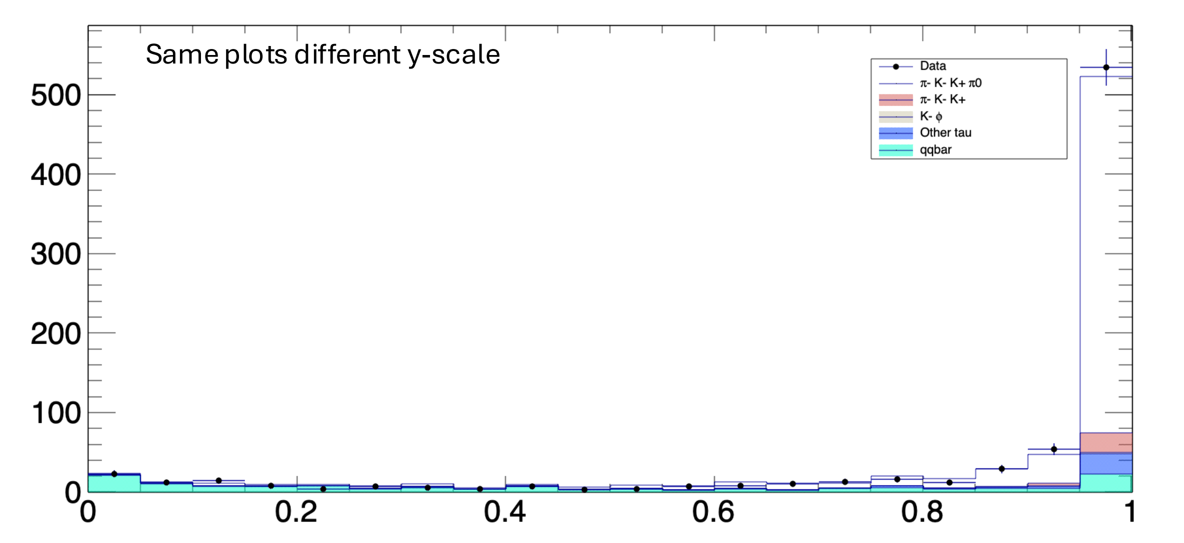
Likelihoold to separate tau decays from qq is: $LH(qq) = \dfrac{L_{tau}}{L_{tau} + W L_{H_{qq}}}$

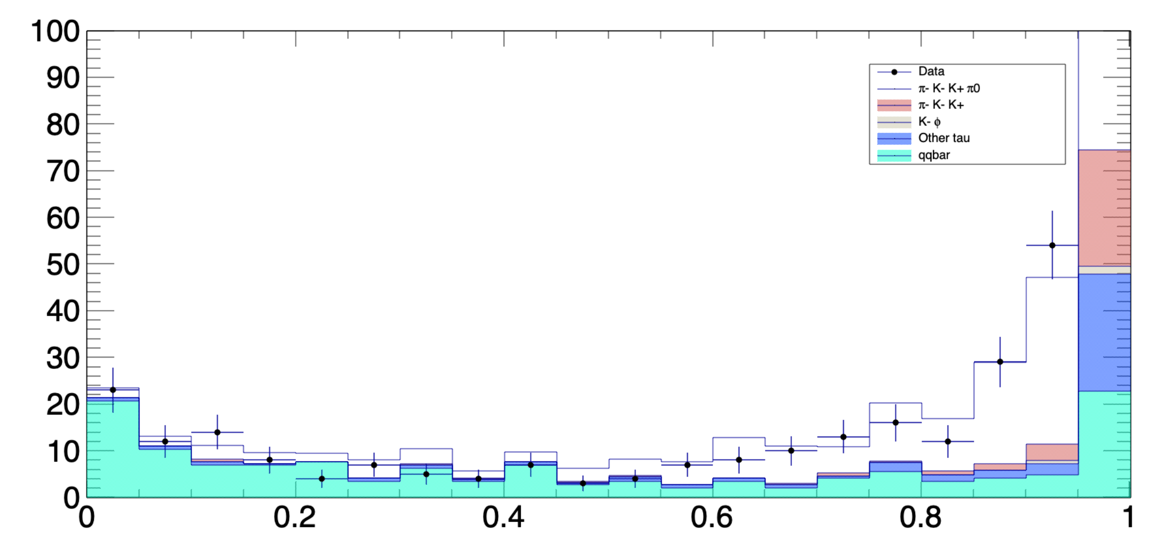
Likelihold to separate tau-kkpz decays from other tau decays is: $LH(kkpz) = \dfrac{L_{tau\text{-}kkpz}}{L_{tau\text{-}kkpz} + W L_{H_{tau\ bkgd}}}$

where *W* is constant factor that can adjust the relative contribution

qqbar Likelihood selection – lepton tag

Includes rescaling factor on qqbar and kkp MC samples (in next pages)
qqLH-ltag uses LH weighting factor of 0.1 (variations result in the stretching or compressing of x-axis)

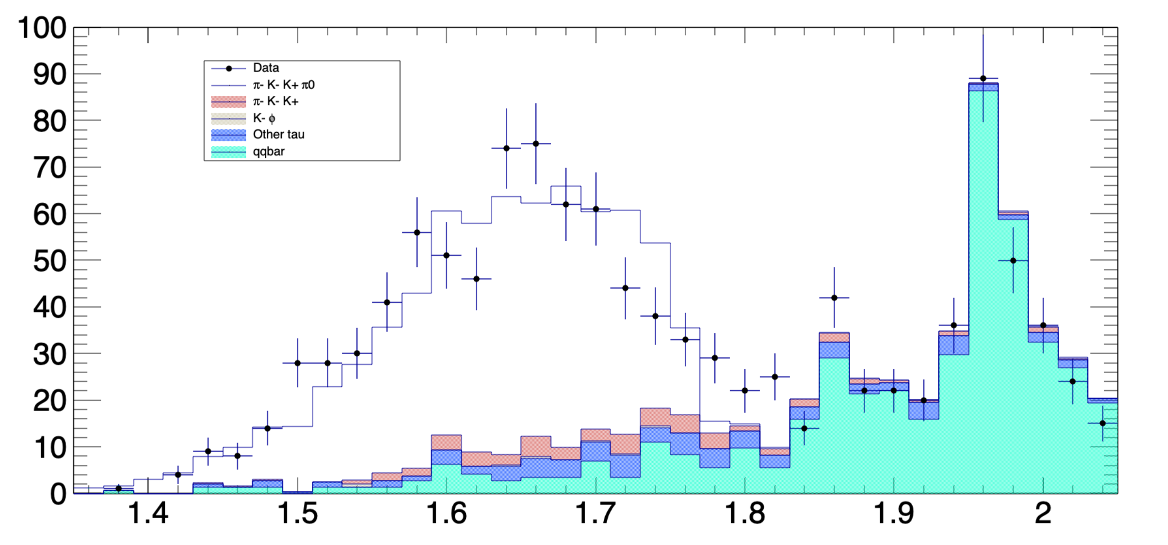


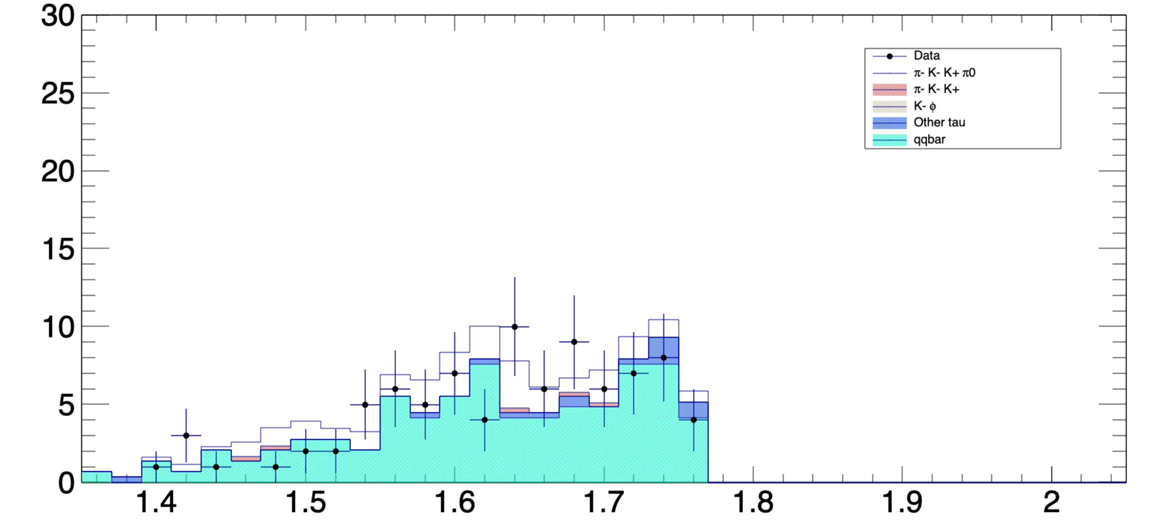


Require qqLH-ltag > 0.5
Reduces qqbar background from 17% to 8%

qqbar Likelihood validation – lepton tag

Use mass of the (pi- K- K+ pi0) system to cross check qqbar modelling





All selection criteria applied except M(kkpz)

Require qqbar scaling of 1.74 in M(kkpz) region of (1.76-2.05) GeV

Cross check by looking at M(kkpz) with qqLH-ltag<0.5

Ratio of qq (data/MC) = 0.75 (0.08)

qqbar background modeled within 10-20% in lepton-tag sample

(Recall qqbar background is 8% - critical if it is “resonant” background)

qqbar Likelihood selection – hadron tag

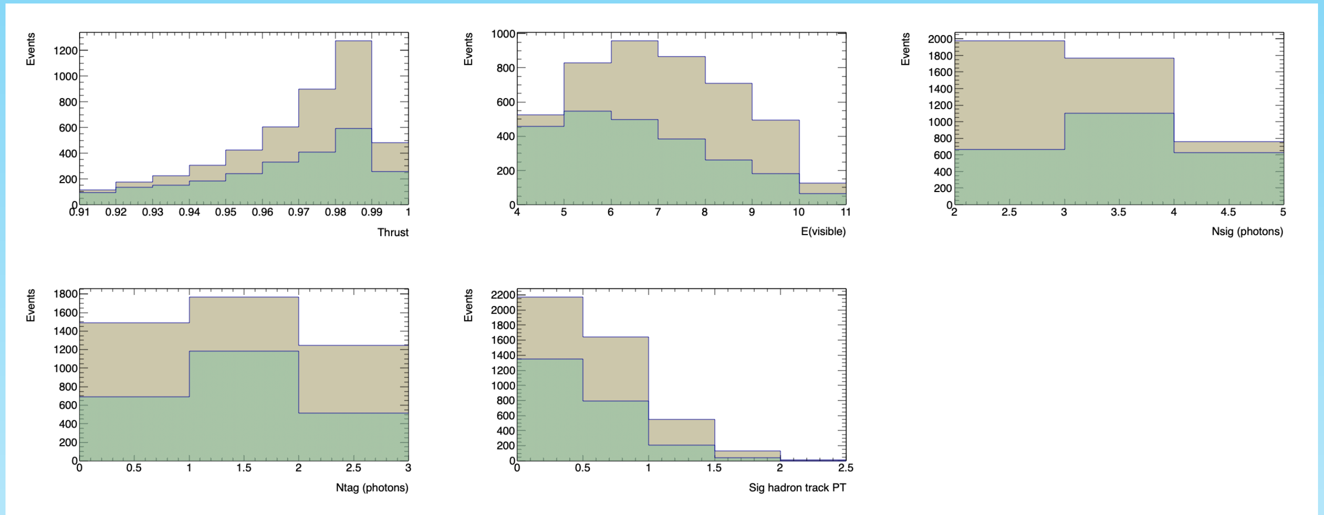
Made a new LH for the hadron-tag sample (different type of qqbar events – no lepton)

Use the samples after the Prelection and the pi0 LH

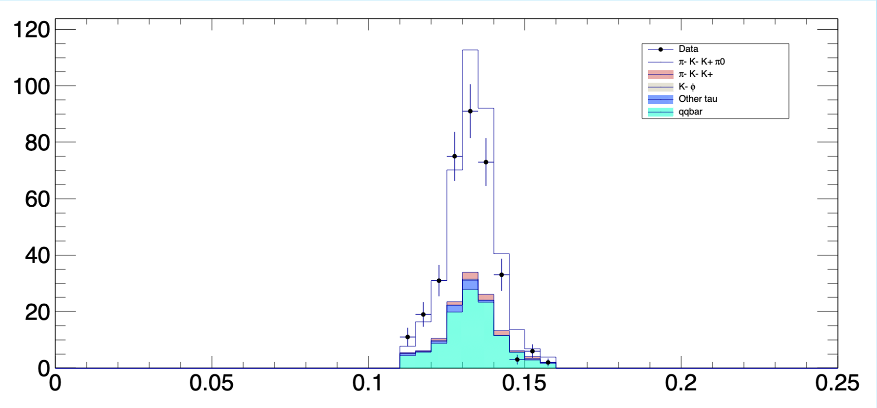
Combine all tau decays into one sample

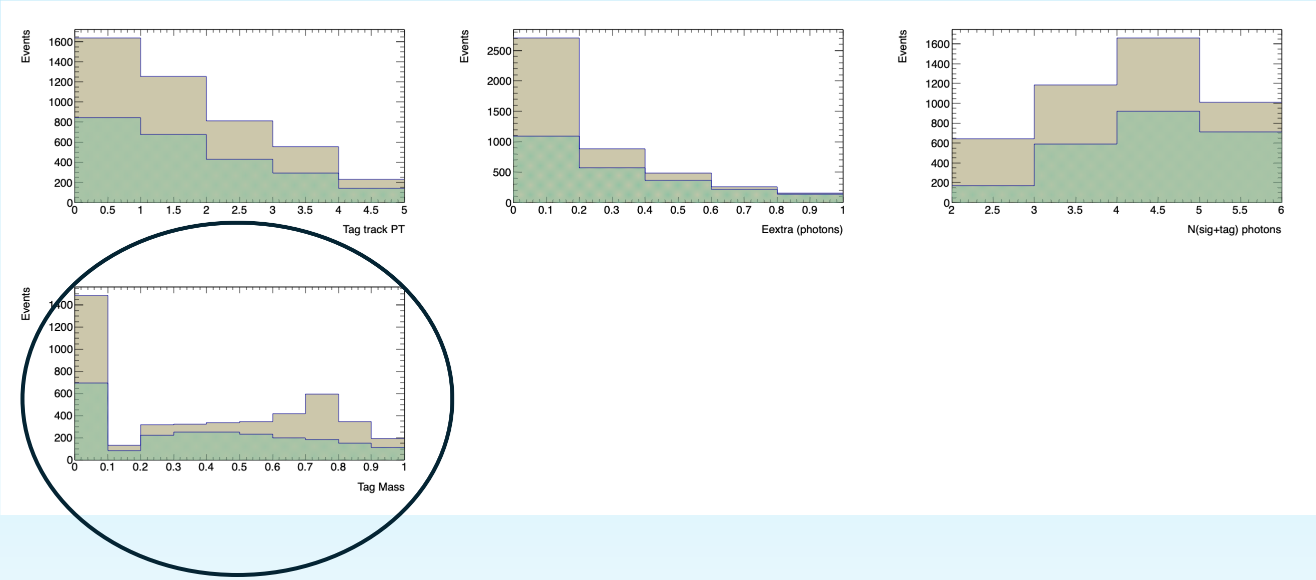
Variables:

thrust, Evisible
Nphotons(sig), Nphotons(tag), Nphotons (sum)
PT (sig-hadron track), PT (tag-track)
Ephoton(extra)
Mass of tag-track and photons in tag hemisphere



Allow hadron-tagged data to have 1 or 2 photons in the tag hemisphere but the mass of the 2 photons must be (0.11-0.16) GeV

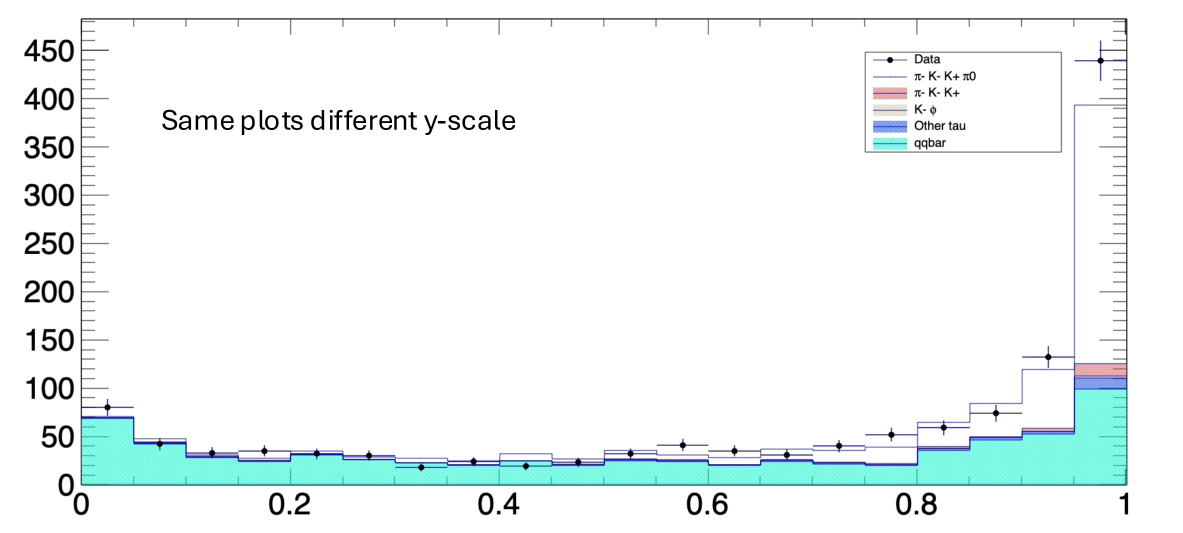


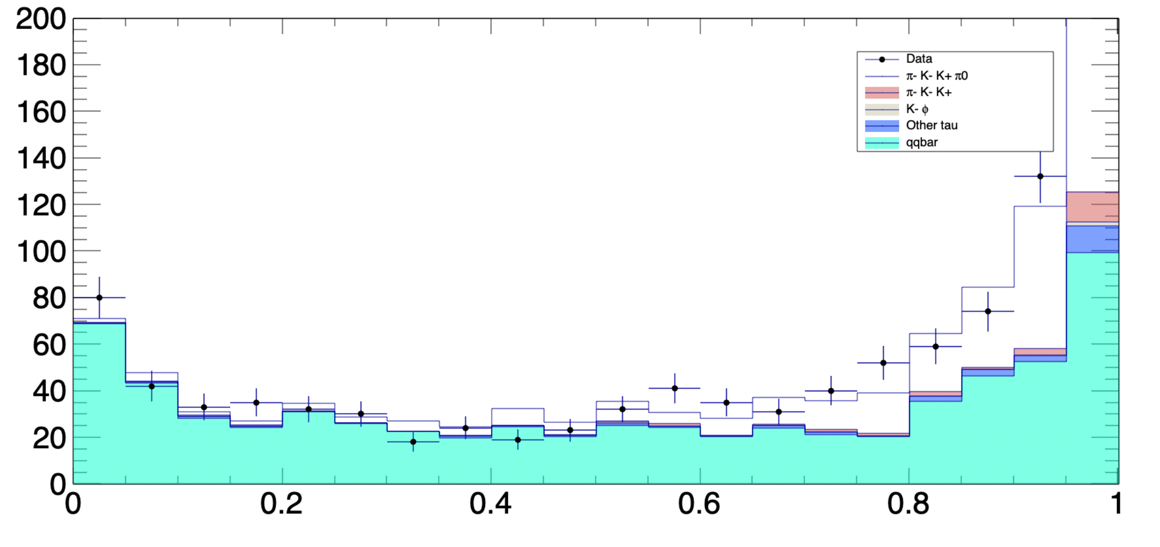


qqLH–htag (hadron tag)

Includes rescaling factor on qqbar and kkp MC samples

qqLH-htag uses LH weighting factor of 0.25 (variations result in the stretching or compressing of x-axis)

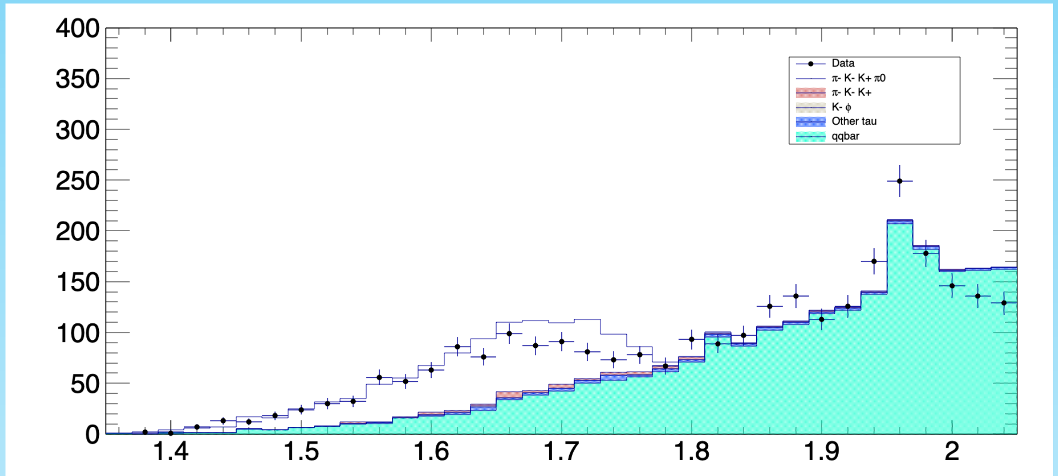




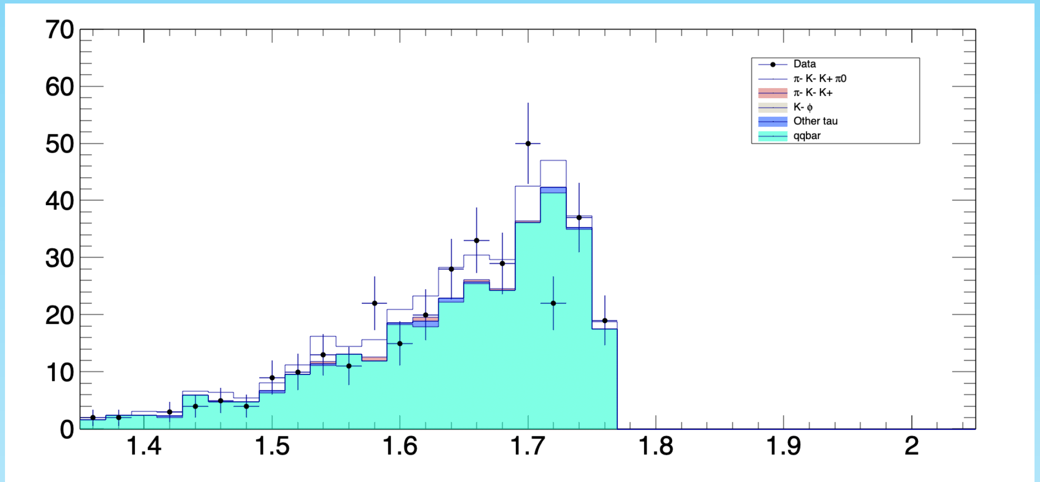
Require qqLH-htag > 0.5

Reduces qqbar background from 53% to 39%

qqbar Likelihood validation – hadron tag



All selection criteria applied except M(kkpz)
Ratio of D/MC = 1.00 (0.02) in M(kkpz) region of (1.76-2.05) GeV



Cross check by looking at M(kkpz) with qqLH-ltag<0.5
Ratio of qq (data/MC) = 0.89 (0.05)

qqbar background modeled within 5% in hadron-tag sample

Tau Likelihood (pzLH) – separate signal (pi- K- K+) from other tau decays (mainly pi-K-K+)

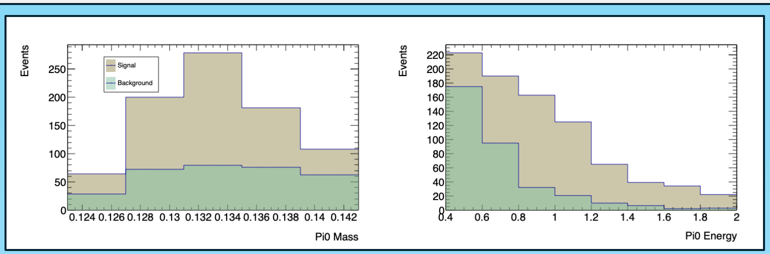
Use the samples after the Prelection and the qqLH
Combine all tau decays into one sample

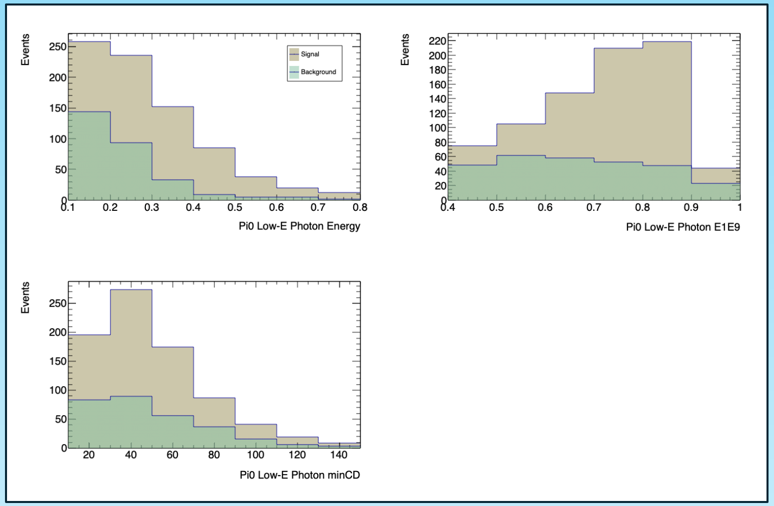
The pzLH first adds cuts to define boundaries of the pzLH where there is no signal.

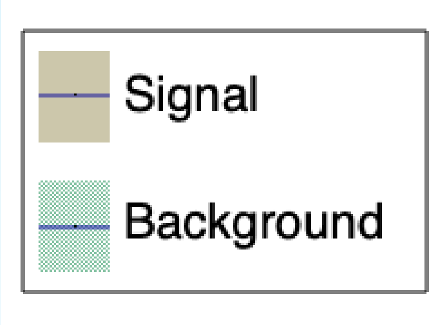
We use the excess of DATA over MC background to define (kkpz) signal

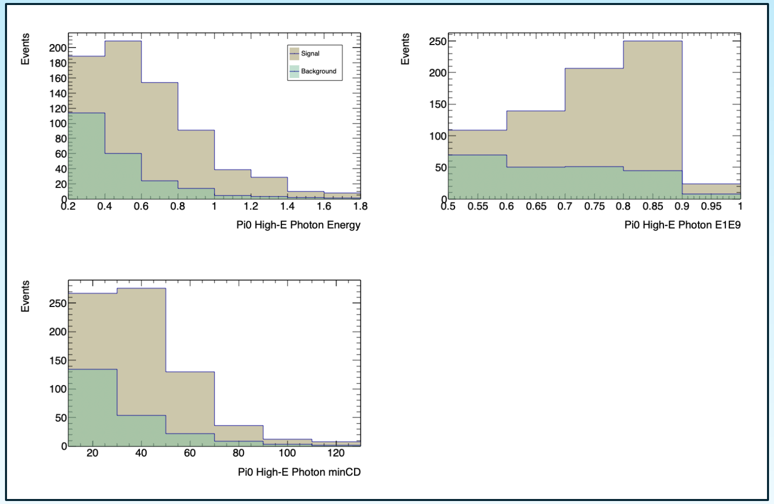
Variables:
Pi0: mass, energy
Photon1: energy, E1E1, minCD distance
Photon2: energy, E1E1, minCD distance

Signal = (pi- K- K+ pi0)
Background = other tau decays
(qqbar not included)





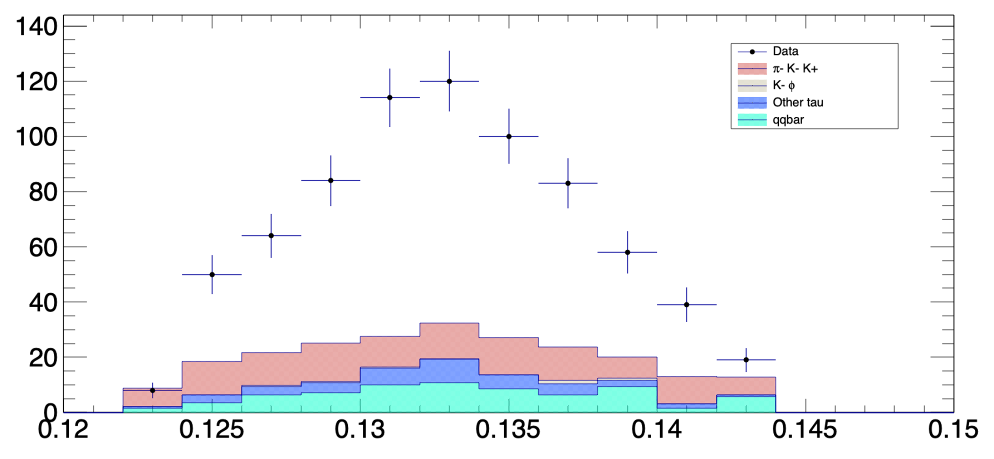


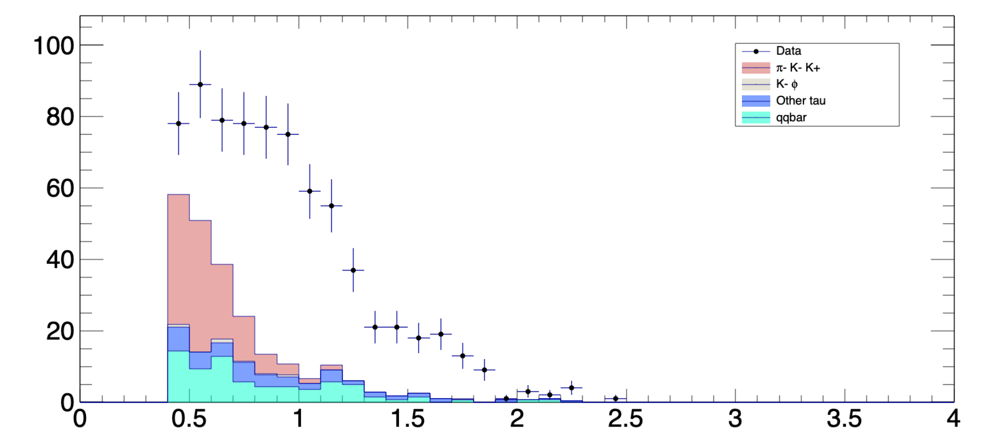


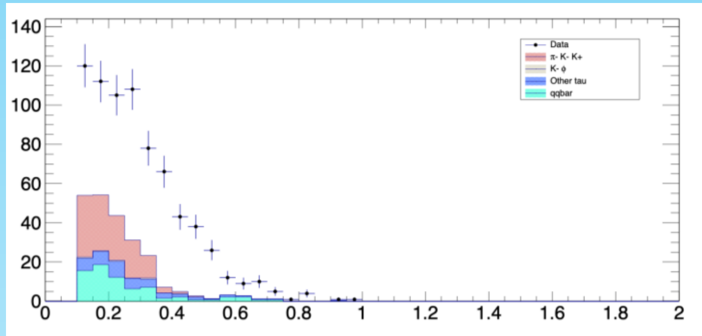
Tau Likelihood (pzLH) – selected histograms

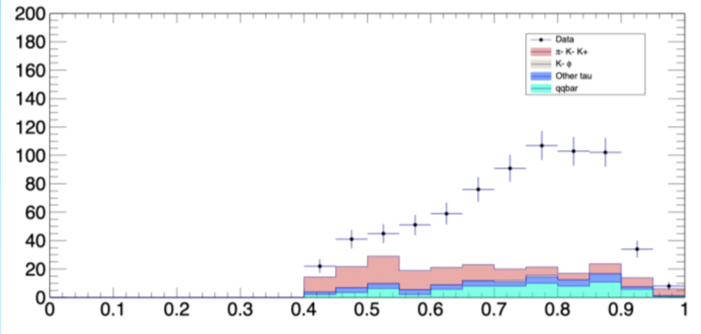
Left pi0 (mass and energy)

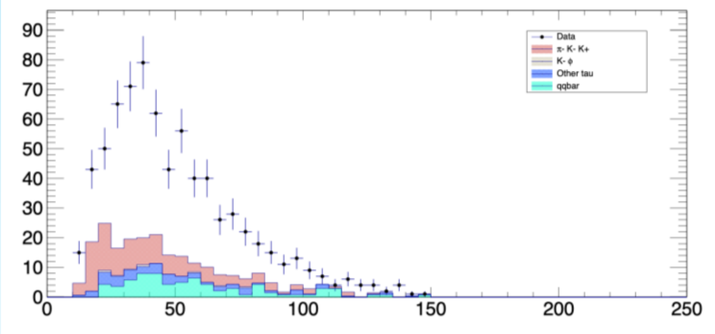
Right: lowE photon in pi0 (E, E1E9, minCD)



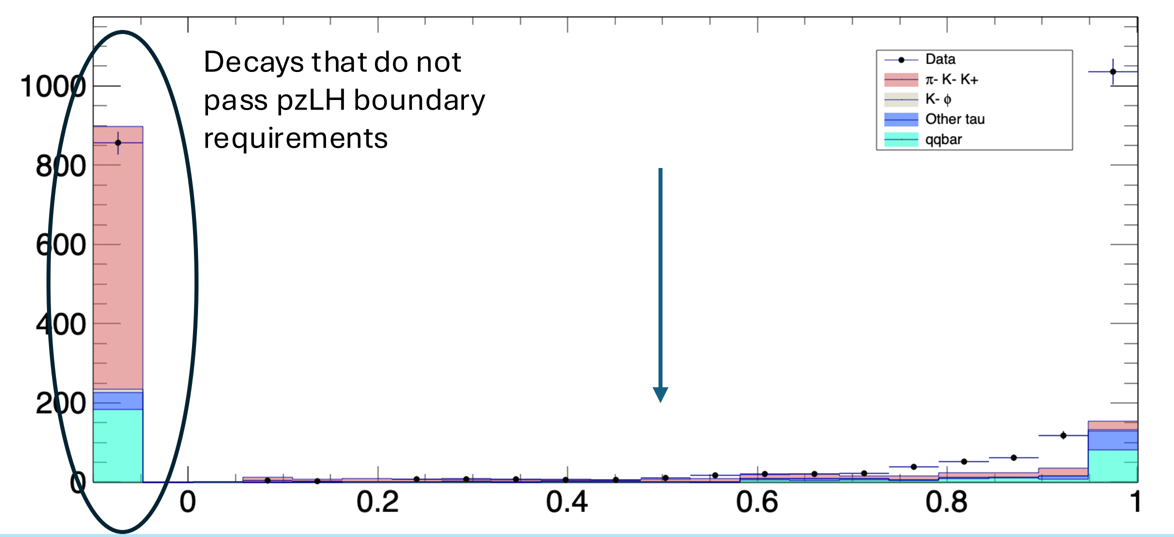


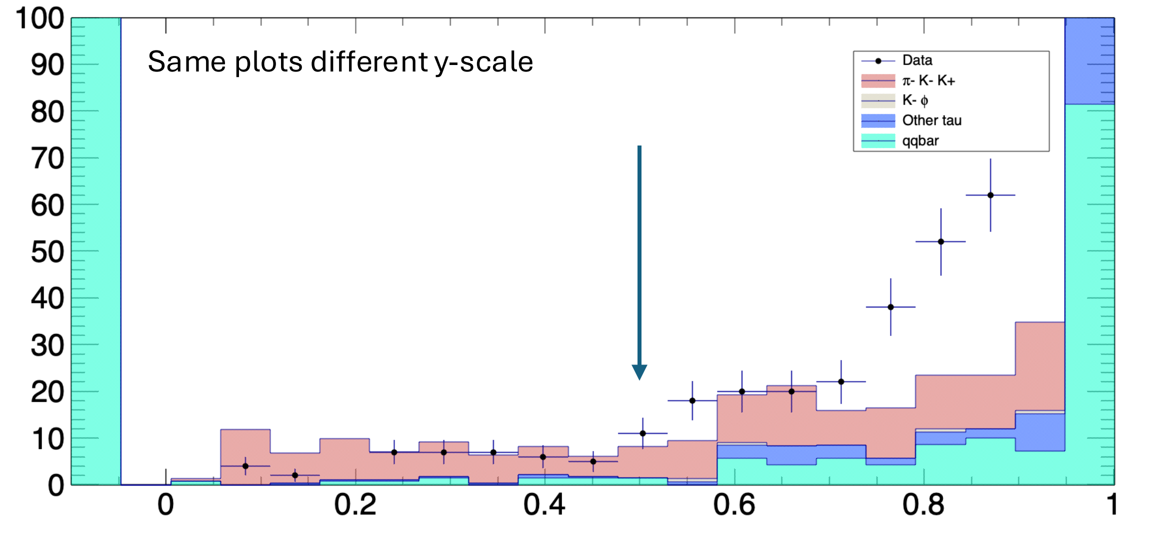






Tau Likelihood – separate signal (pi- K- K+) from other tau decays (mainly pi-K-K+)





Require pzLHv2 > 0.5

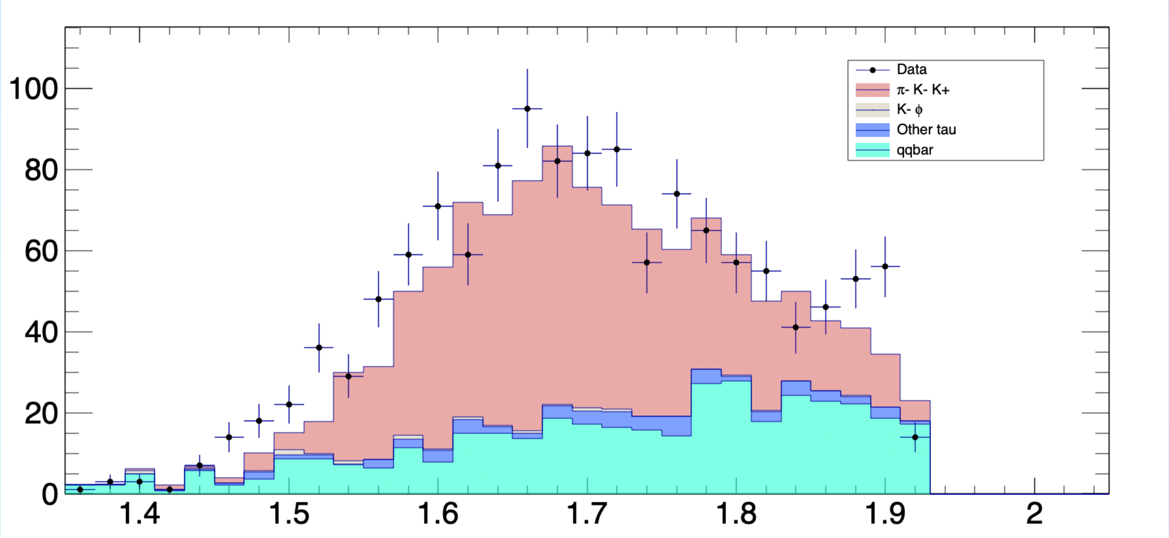
Phase space signal MC not plotted

As will be shown, the background from other tau decays is small and the impact of this requirement is minimal.

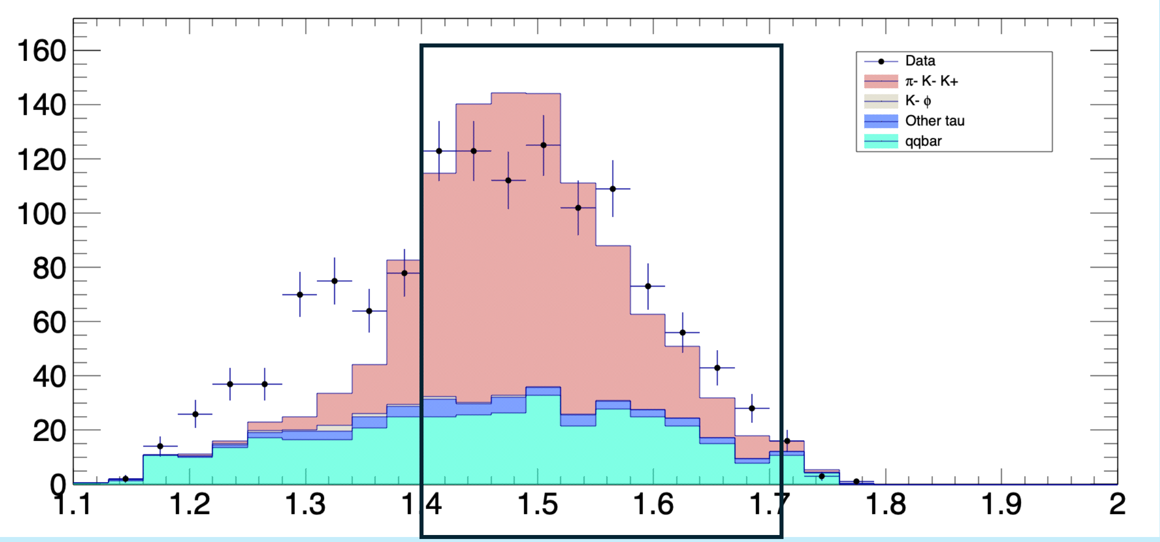
Tau Likelihood – validation for lepton-tag

The Mkkp (K- K+ pi-) distribution is better as it separates the kkpz/kkp decays
The tau background needs to be scaled down by 0.76 (scaling is applied in plots)

Mkkpz with no pzLHv2_FAIL and Mkkpz<1.92



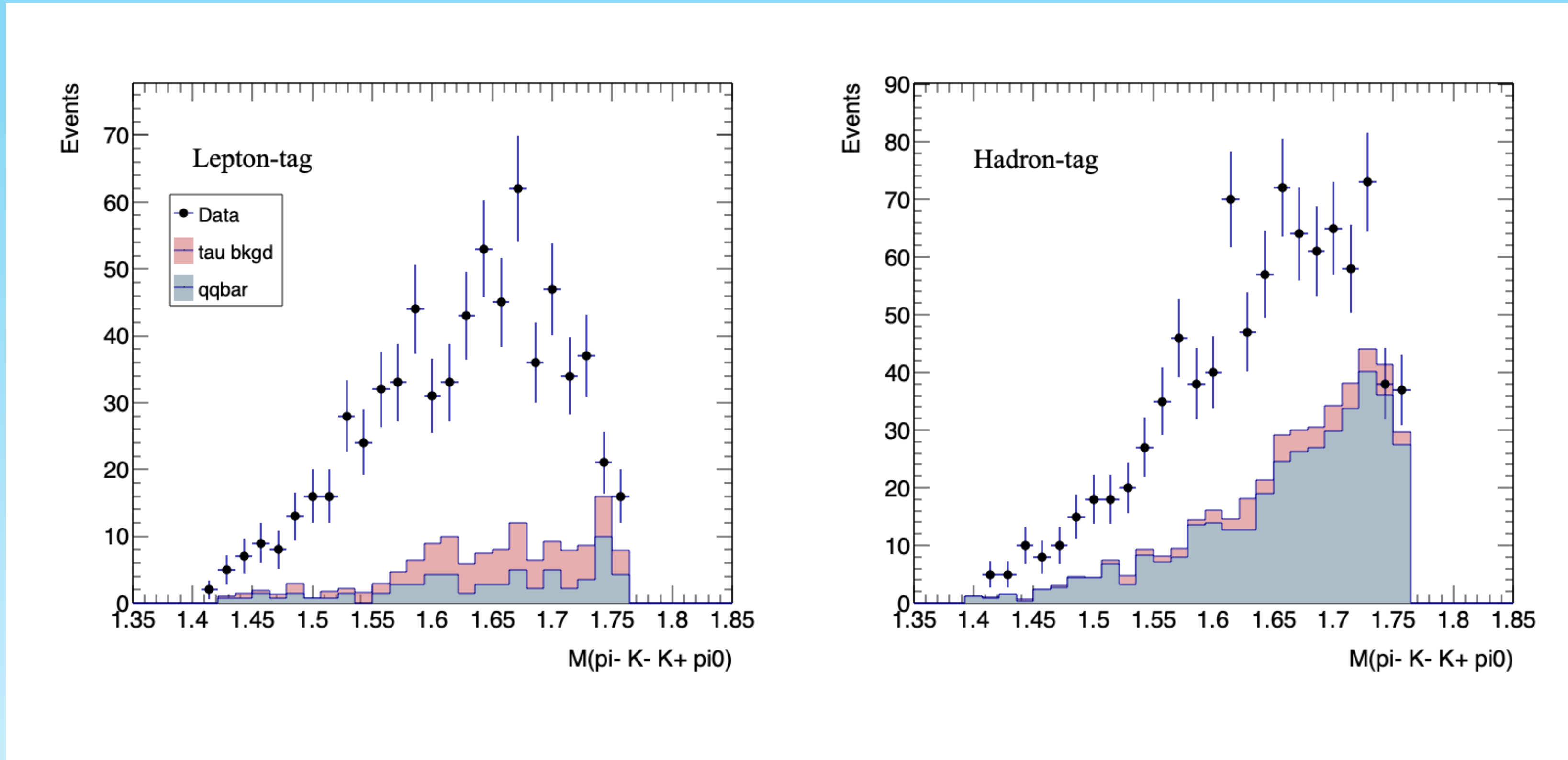
Mkkp for same requirements



<u>After rescaling the tau backgrounds</u>
M(kkp) = (1.4 – 1.7), D/MC = 665/677 = 0.98 (0.04)
(after subtracting qqbar and error is statistical only)

Tau Likelihood – validation for hadron-tag

Mass of (pi- K- K+ pi0) after all selections

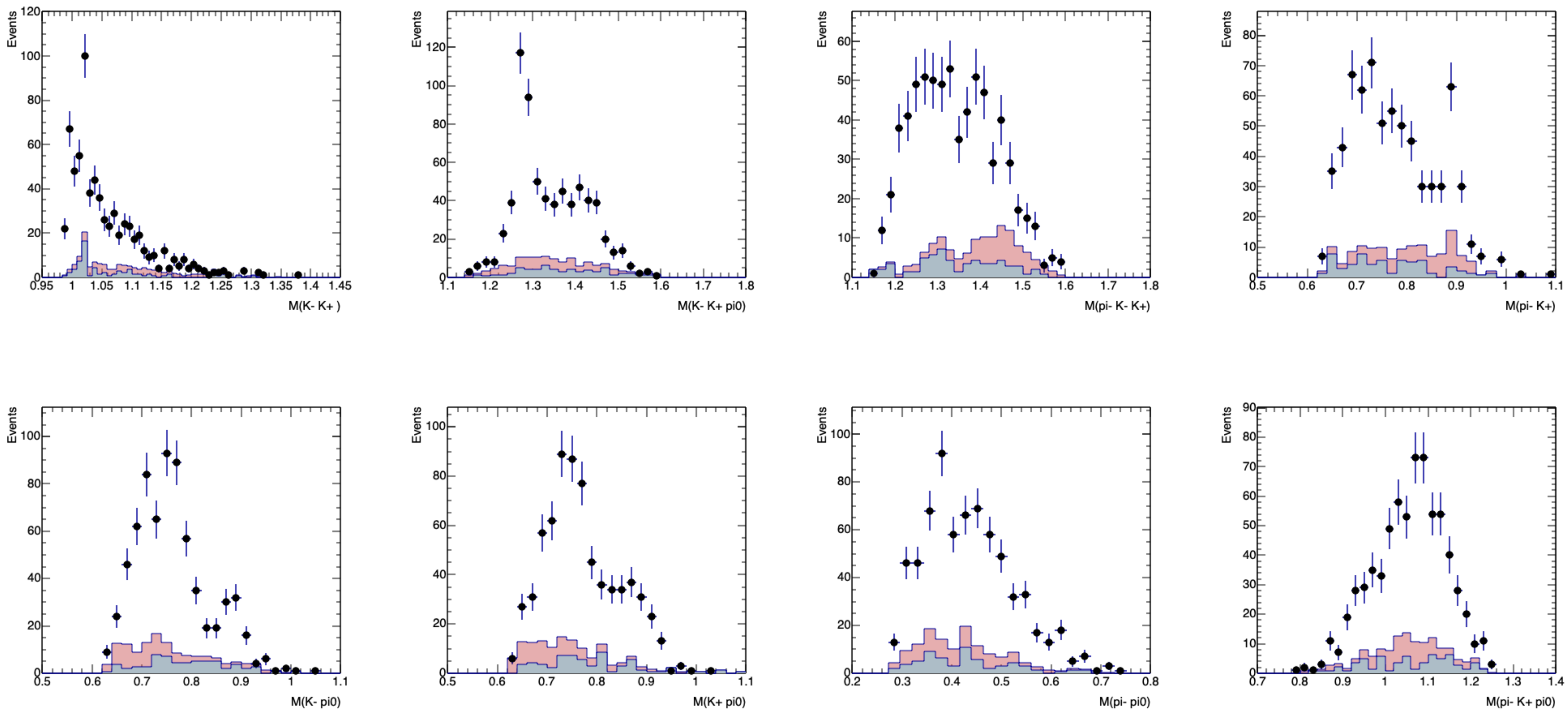


Lepton tag:	
Data	687 events
Tau bkgd fraction	0.11
QQ bkgd fraction	0.09

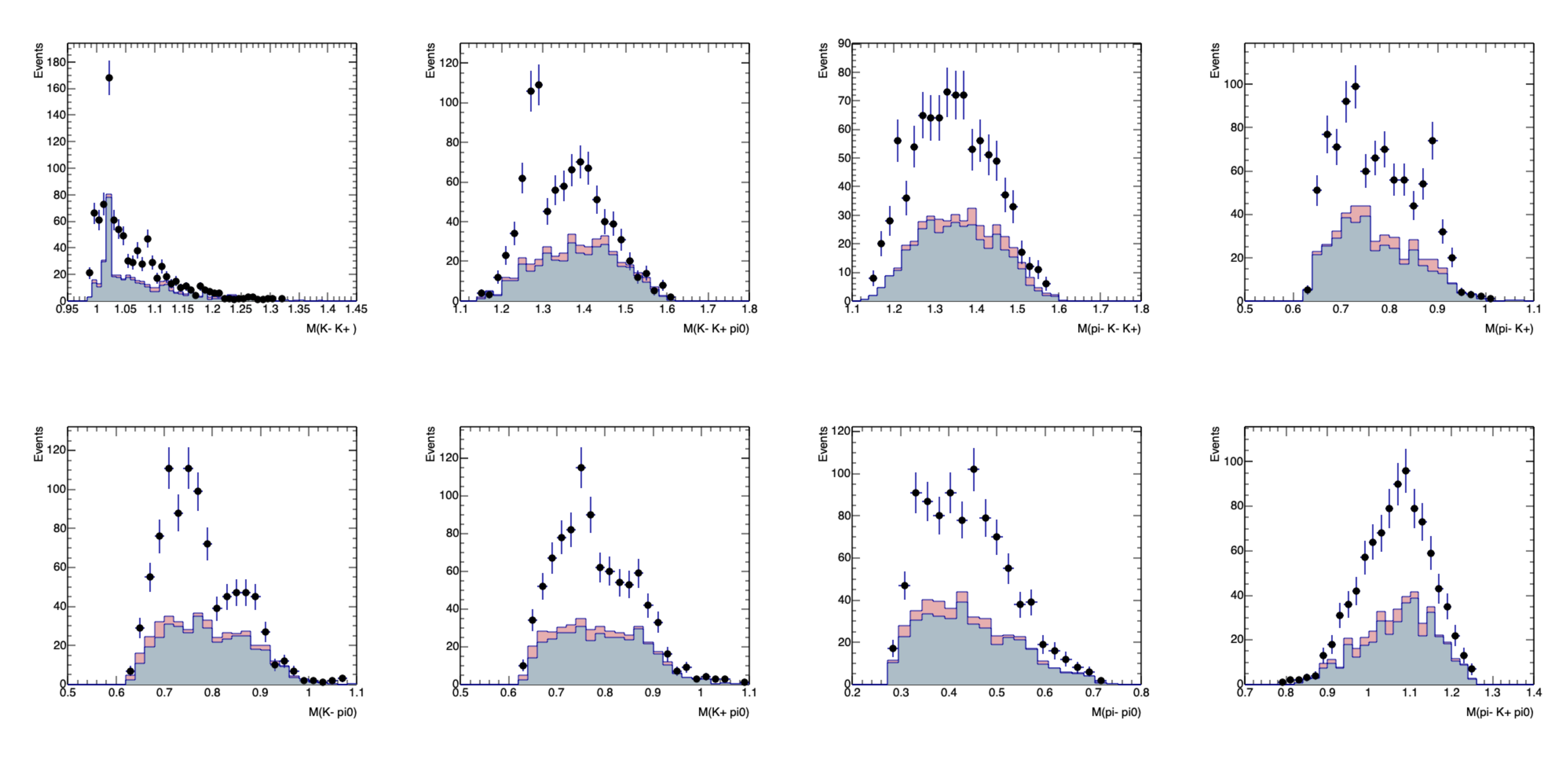
Hadron tag:	
Data	937 decays
Tau bkgd fraction	0.06
QQ bgkd fraction	0.39

Recall modelling of tau and qqbar background validated with data

Mass of particle combinations - lepton tag

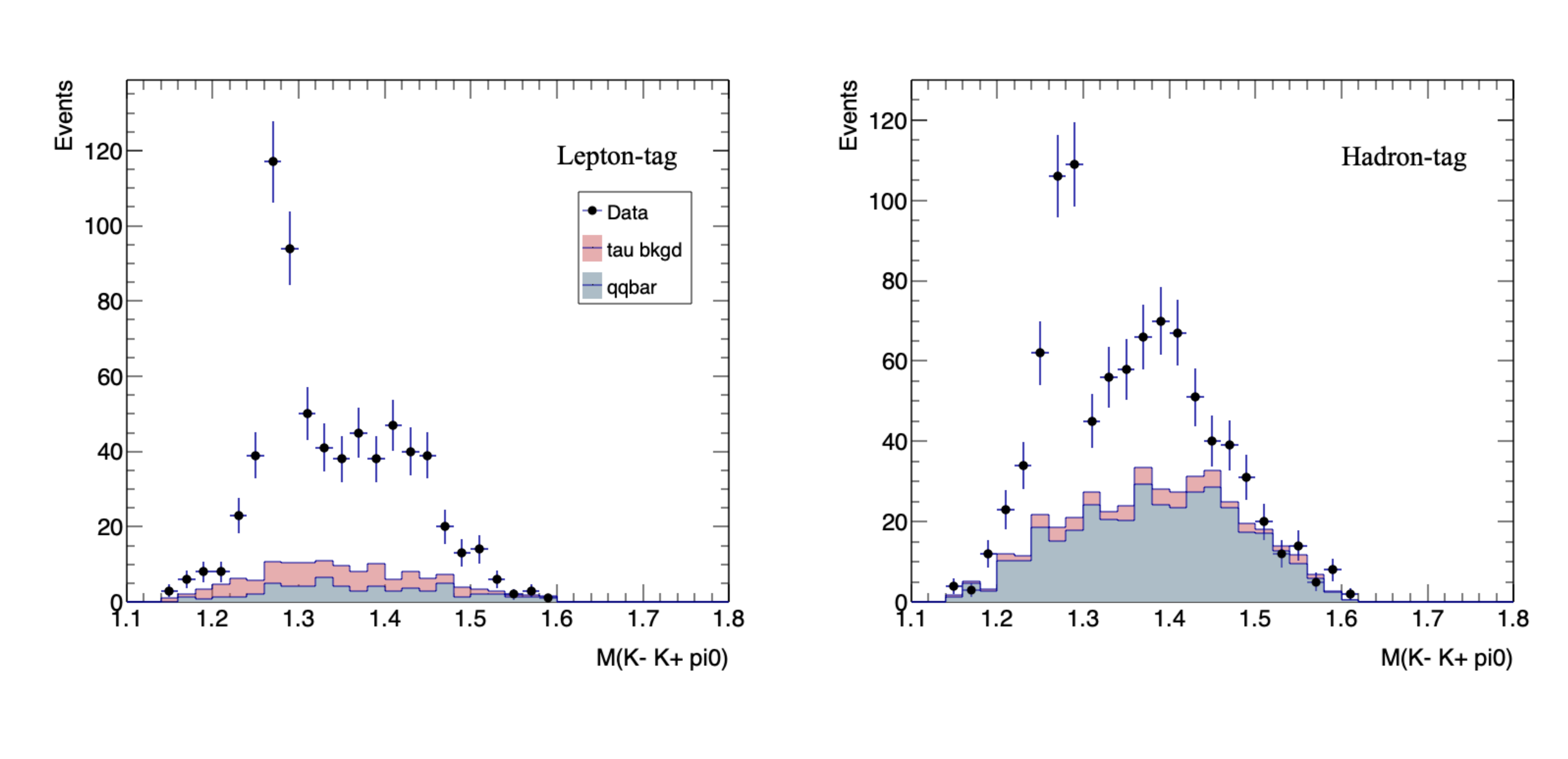


Mass of particle combinations - hadron tag



Mass (K- K+ pi0)

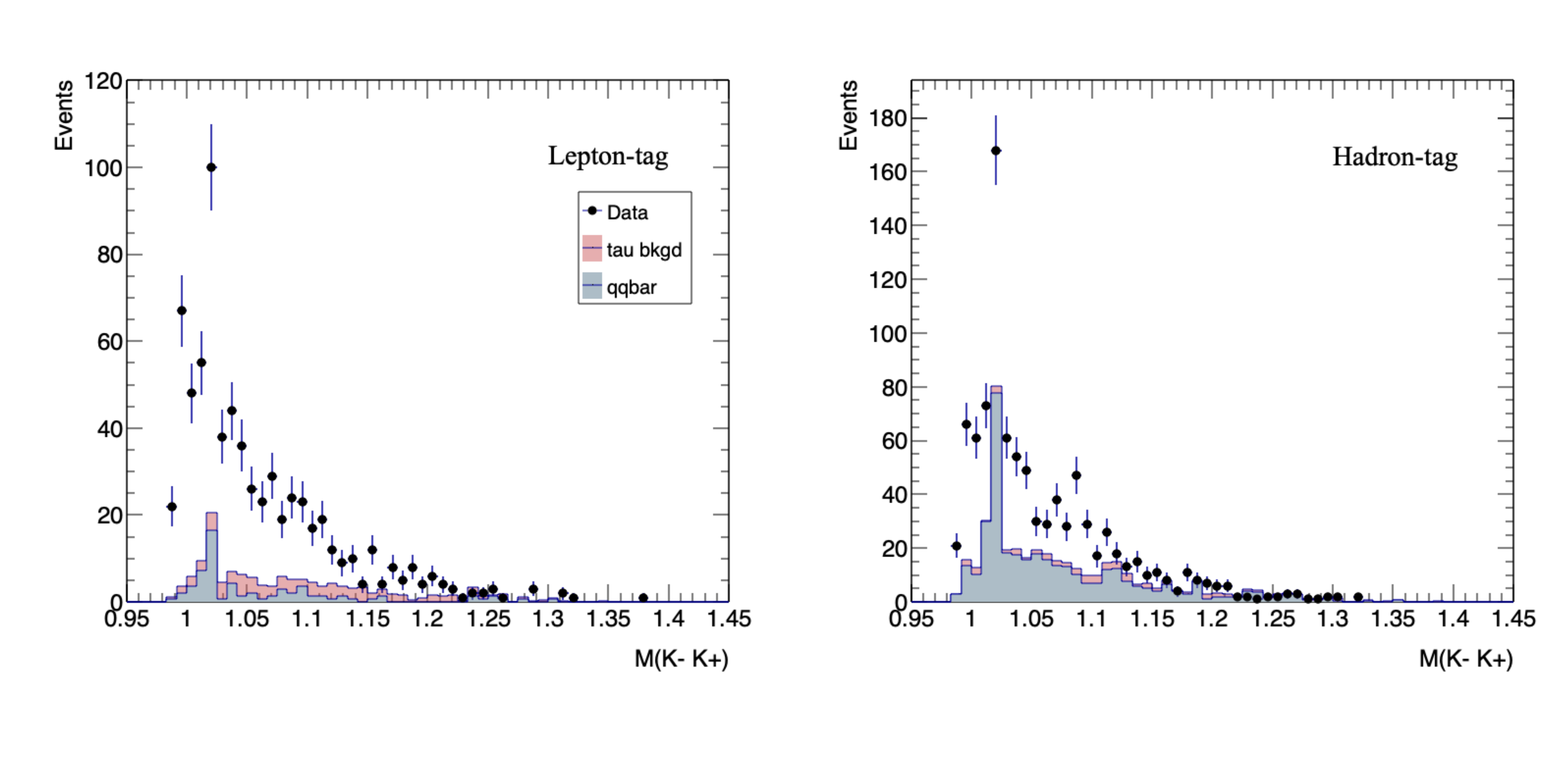
Lepton-tag (top), hadron-tag (bottom)



Evidence of f1(1285) in (pi- K- K+ pi0) decays

Mass (K- K+)

Lepton-tag (top), hadron-tag (bottom)

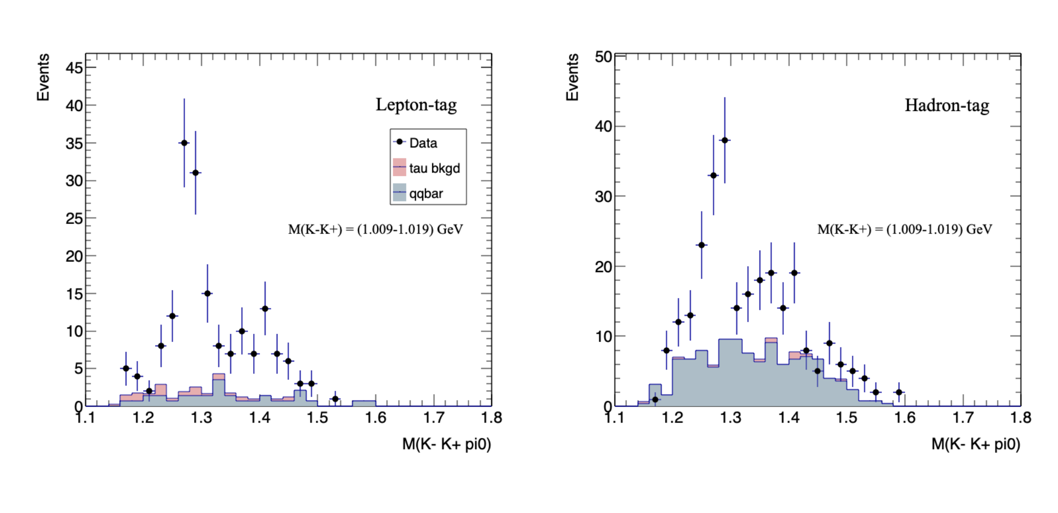


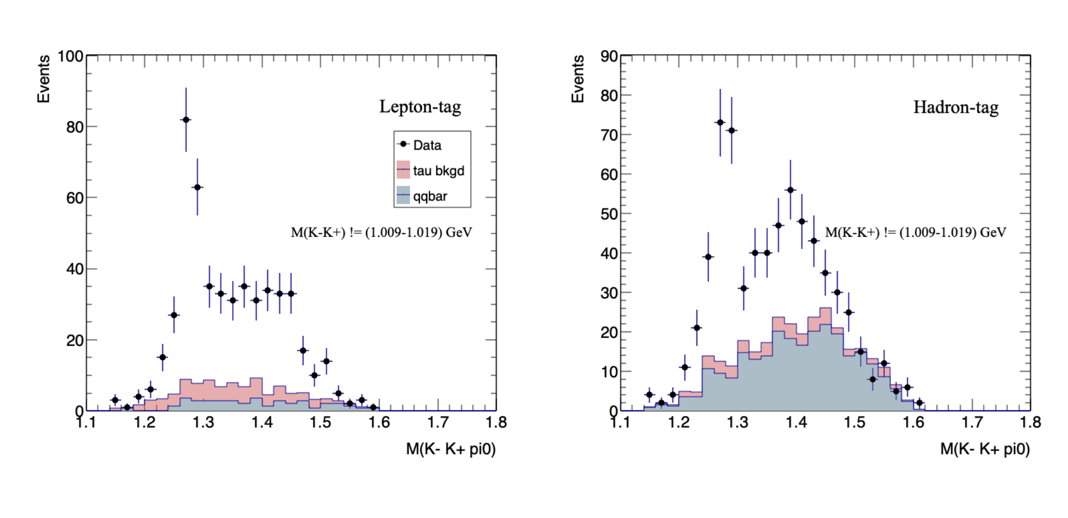
phi resonance in (pi- K- K+ pi0) decays

No measurements of (pi- phi pi0) in PDG

Mass (K- K+ pi0) for decays on and off the phi peak

Lepton-tag (top), hadron-tag (bottom)



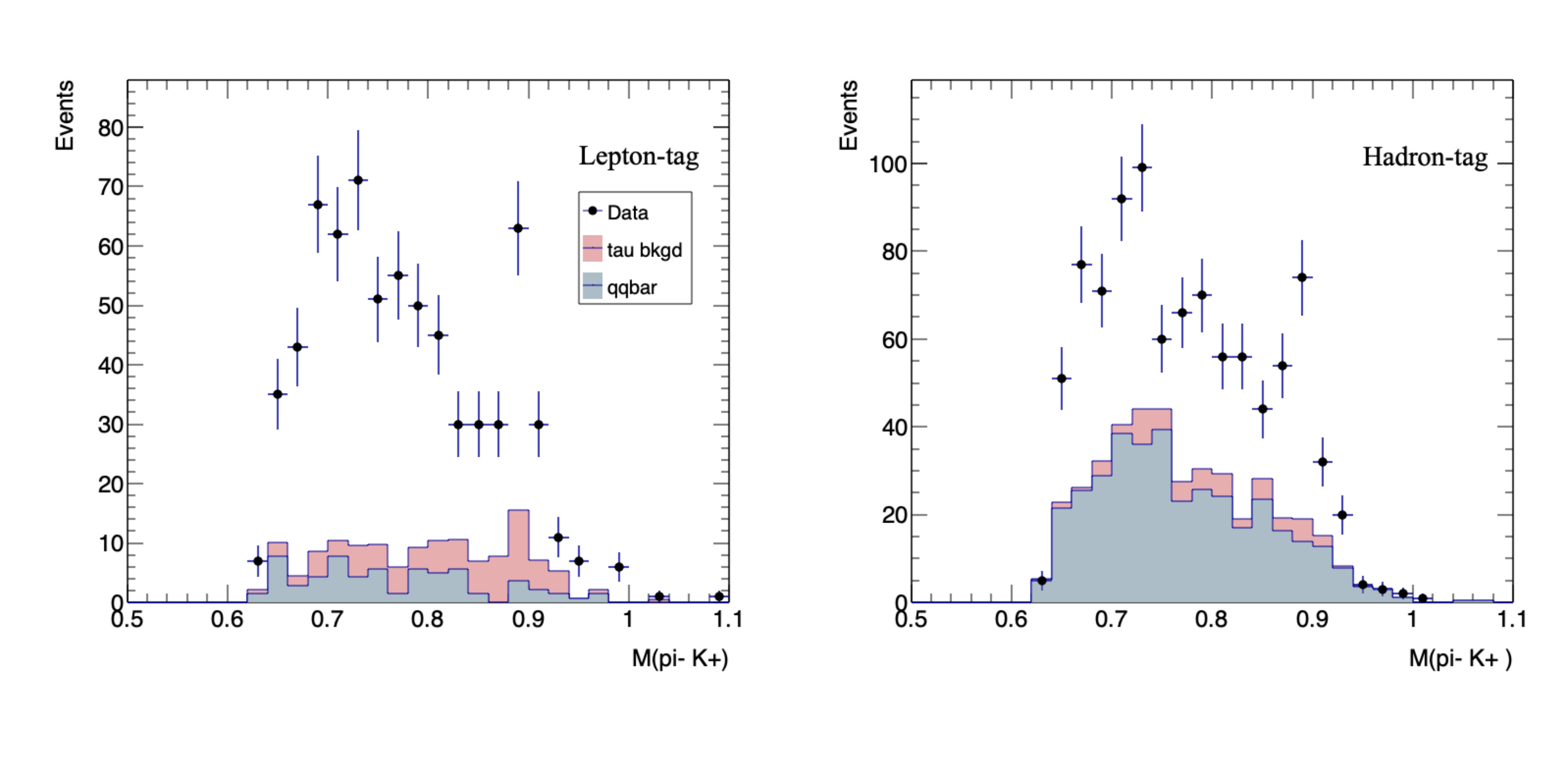


Possible evidence for the f1(1285) and f1(1420) decays to (phi pi0)

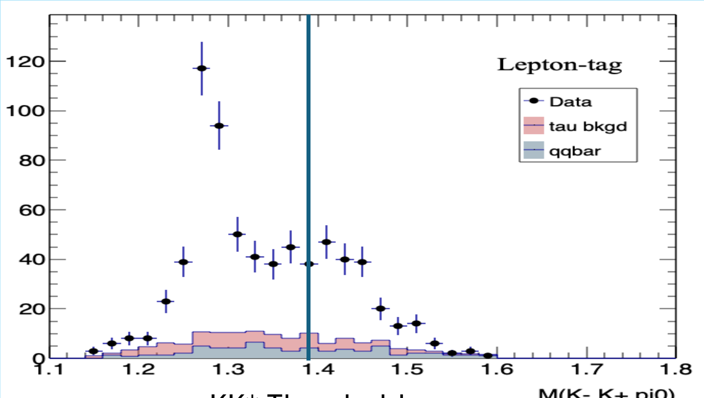
The f1 listing in the PDG does not list any decay modes with the phi meson

Mass (pi- K+)

Lepton-tag (top), hadron-tag (bottom)



Note the f1(1285) cannot decay to the K* but the f1(1420) is allowed



KK* Threshold

$f_1(1420)$ $\quad I^G(J^{PC}) = 0^+(1^{++})$

See the review on "Spectroscopy of Light Meson Resonances."

Mass $m = 1428.4^{+1.5}_{-1.3}$ MeV (S = 1.8)

Full width $\Gamma = 56.7 \pm 3.3$ MeV (S = 1.3)

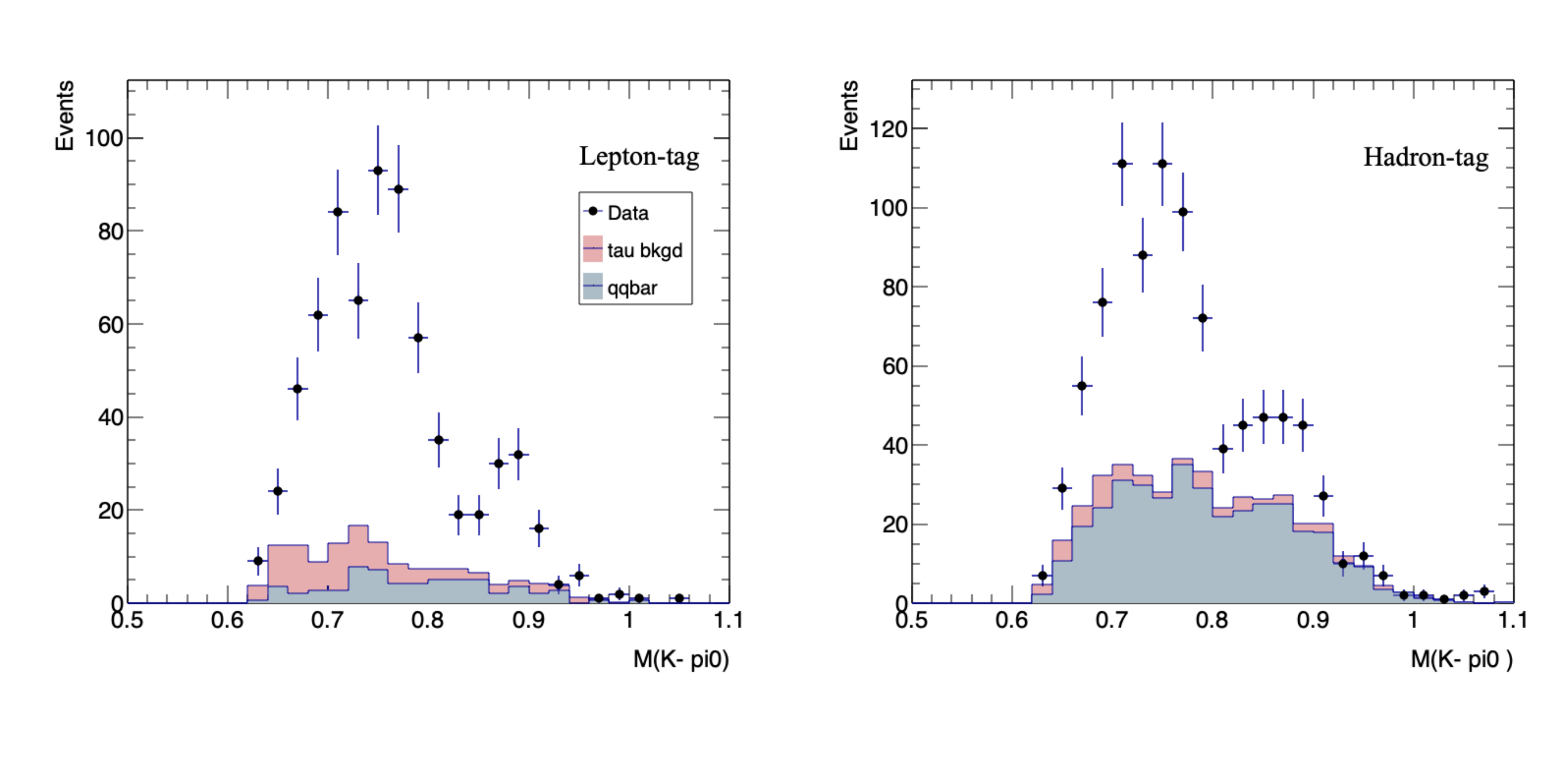
$f_1(1420)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$K\overline{K}\pi$	seen	440
$K\overline{K}^*(892)$ + c.c.	seen	167
$\eta\pi\pi$	possibly seen	574
$\phi\gamma$	seen	350

Hint of tau decay to (K- K* pi0) where K* to (pi- K+)

PDG: BF(K- K* >= 0 neutrals) = (3.2 +/- 1.4) E-3

Mass (K- pi0) where tau charge is -1

Lepton-tag (top), hadron-tag (bottom)

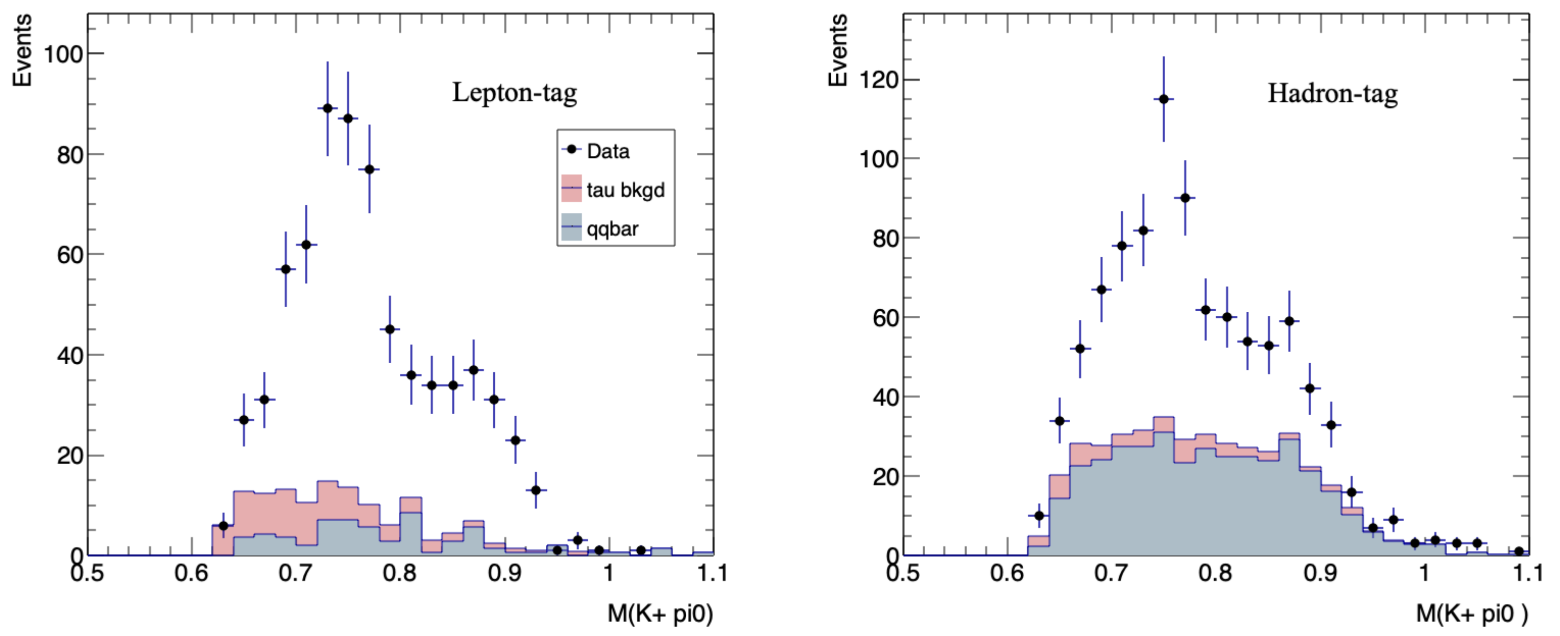


Tau decay to (pi– K*- K+) K* to (K- pi0)

Decay not in the PDG

Mass (K+ pi0) where tau charge is -1

Lepton-tag (top), hadron-tag (bottom)



Tau to (pi- K- K*+) K*+ to (K+ pi0)

Decay not in the PDG

Current state

Modes to study:

pi- K- K+ pi0	phase space
pi- f1(1285)	
pi- f1(1420)	
pi- phi pi0	
K- K* pi0	K* to (pi- K+)
pi– K*- K+	K* to (K- pi0)
pi- K- K*+	K* to (K+ pi0)

f1(1285) to (phi pi0)
f1(1420) to (phi pi0), (K K*)

Measure f1 properties

f1(1285) mass (width already measured precisely)
f1(1420) mass and BFs
Mixing amplitude of f1s

Measurement strategy

Relative BF measurements B(X)/B(kkpz)

PDG error on BF(f1 to KKpi0) is 5%
Further, there are no BF(f1 to phi pi0) measurements

Next step:

<u>Mass (K-K+ pi0):</u>
Determine the correct lineshape function
Is there an f1(1420)?
Do the f1's account for all the (pi- K- K+ pi0) decays?
How does one deal with the (KK*) threshold?
Can one make a competitive measurement of the f1(1285) mass?

Lineshape for the f1 resonances

Study impact of the three mass thresholds and amplitude with interfering resonances

On the ambiguity of determination of interfering resonances parameters

https://arxiv.org/pdf/1505.01509

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Abstract

The general form of solutions for parameters of interfering Breit-Wigner resonances is found. The number of solutions is determined by the properties of roots of corresponding characteristic equation and does not exceed 2^{N-1}, where N is the number of resonances. For resonances of more complicated form, provided that their amplitudes satisfy certain conditions, for any $N \geq 2$ multiple solutions also exist.

B. Relativistic form of resonance amplitude

Let us consider more accurate dependence of the resonance amplitude on energy:

$$\sigma(E) = \frac{m_1^4 \sqrt{E^2 - 4\mu^2}^3}{E^4 \sqrt{m_1^2 - 4\mu^2}^3} \cdot \left| \frac{2m_1 a}{E^2 - m_1^2 + i\Gamma_1 m_1} + \frac{2m_2 b e^{i\psi}}{E^2 - m_2^2 + i\Gamma_2 m_2} \right|^2$$